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A NEW EFFICIENCY INDEX FOR THE EVALUATION OF SEPARATOR PERFORMANCE

Separator performance is evaluated by the so-called separation efficiency. The concept of efficiency in mineral dressing operations must be interpreted with considerable care. Although qualitative use of term efficiency is valid and useful, difficulties arise in expressing it quantitatively because two conflicting criteria, grade and recovery are normally involved in its evaluation. It is undesirable to express efficiency as a single value, even though this is frequently done in practice, since there are strictly speaking an infinite number of combinations of grade and recovery that can give that one value. Inefficiencies in processes can be attributed to three broad sources: mineral limitations as represented by the separability curves, equipment limitations (which prevent the maximum separation indicated by the separability curves), and human failure to minimize these inefficiencies. In the present paper, the efficiency of two-constituents separation process has been examined. The separation process has been evaluated by comparing the sharpness factor of the actual, the absolutely efficient and the absolutely inefficient operations.

INTRODUCTION

Mineral processing is concerned primarily with the separation of mineral particles based on variations in size or composition. Separation is brought about by suspending the particles in a medium and passing the suspension through an appropriate piece of equipment termed a separator. There a suitable force is applied to the particles; and because the materials have different properties depending on their composition or size, they are affected to varying degrees by the applied force (Figure 1). Those that are moved by the force have a positive response, those unaffected a negative response. Obviously liberation is a prerequisite for perfect separation, although in most cases the ideal of complete liberation may not be practical (Kelly 1982).

For convenience, a separation can be considered to depend on three factors: the properties of the minerals, separator characteristics, and production requirements of grade and recovery. Some mineral properties that may be exploited are size, shape, density, magnetic susceptibility, electrical conductivity, or surface properties.

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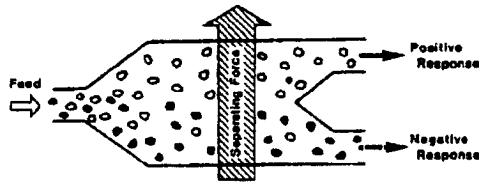


Fig. 1. The basic principle of a separator

Rys. 1. Podstawowe zasady działania separatora

A separation process ideally is one that exploits a single mineral property, although in practice this situation is not achieved, thereby placing a limit on the separation. Sometimes the desired constituent is an element that occurs in more than one mineralogical form in the ore, and it may be necessary to use a different separator to recover each mineral. Since particles are normally produced by a size reduction process, any set of particles will have a range of particle sizes. To give a quantitative measure of the property that is being exploited for the purpose of separation, the value of the property of an individual particle is termed β . The critical value of the property that determines the response (or split) in the equipment is called the separator setting β (Kelly 1982).

As illustrated in Figure 2, the range of property values of a given set may be presented as frequency or cumulative curves. These curves have been applied mostly to specific gravity separations, and because they were originally used to study coal washing, they are often referred to as washability curves (Coe 1938). However, since they represent the mineral property characteristics and can be applied to most separations, they are better termed separability curves.

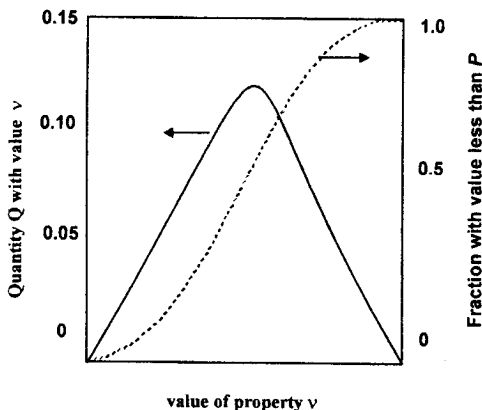


Fig. 2. The cumulative and frequency forms of the separability curve of a set of particles

Rys. 2. Wykres krzywych kumulacyjnej i częstości procesy separacji zbioru ziaren

It must always be remembered that the separation predicted from separability curves represents the maximum separation that can be achieved on the material as it exists by exploiting that articular property. If the separation is unsatisfactory, use of an alternative device exploiting the same property will generally not improve the separation because the maximum separation is limited by the properties of the particles alone. In practice, engineering limitations means that a separator never achieves the maximum separation indicated by the separability curves, but since there may be differences in the degree to which different separators approach the maximum separation, one may give a better performance than another.

In the present paper, the efficiency of two-constituents separation process have been examined. The separation process has been evaluated by comparing the sharpness factor of the actual, the absolutely efficient and the absolutely inefficient separation operations.

METHOD DEVELOPMENT

Imperfection in the performance of any real separation equipment can be characterized by the separation efficiency. A single-stage separational apparatus can be schematically drawn as in Figure 4, where: M is the mass flow of the feed (in kg s^{-1}), M_c is the mass flow rate of the coarse material in the underflow (in kg s^{-1}) and M is the mass flow rate of the fine material in the overflow (in kg s^{-1}). The total mass of the feed must be equal to the sum of the total masses of the products if there is no accumulation of material in the equipment, (Svarovsky 1977), i.e.

$$M = M_c + M_f \quad (1)$$

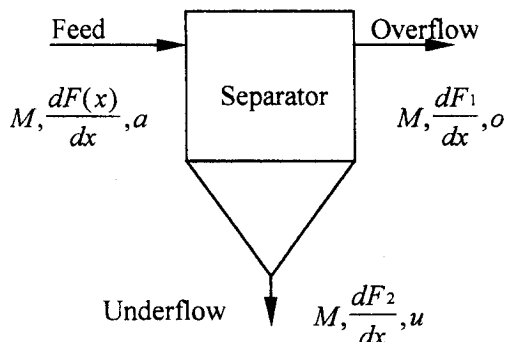


Fig. 3. Schematic diagram of a separator

Rys. 3. Schemat separatora

Mass balance must also apply to any size fractions present in the feed if there is no change in particle size of the solids inside the separator (no agglomeration or comminution). Hence for particles of size between x_1 and x_2 .

$$(M)_{x_1/x_2} = (M_c)_{x_1/x_2} + (M_f)_{x_1/x_2} \quad (2)$$

and also for each particle size x present in the feed:

$$(M)_x = (M_c)_x + (M_f)_x \quad (3)$$

If the property (the physical characteristic of the grains to be separated) at which the separation process will be conducted is denoted by X , the distribution and probability density (frequency) functions of the raw material (i.e. the feed to the separator) are $F(X)$ and $f(X)$, then the mass yield of concentrate (m) and tailing products (m_t) in the interval X and $X + dX$ is related to $f(X)$ as follows (Pethó, 1982)

$$F(X)dX = m_c f_c(X)dX + m_t f_t(X)dX; X_{\min} < X < X_{\max} \quad (4)$$

and

$$\int_{x_1}^{x_2} f^{x_2} f(X)dX = m_c \int_{x_1}^{x_2} f^{x_2} f_c(X)dX + m_t \int_{x_1}^{x_2} f^{x_2} f_t(X)dX, \quad (5)$$

$$X_{\min} \leq X < X \leq X$$

where: $m_c = F(X)$, $m_t = 1 - F(X)$

Knowing the distribution and frequency functions, the average quality of the raw material and of the products can be calculated

$$\bar{X} = \int_{x_{\min}}^{x_{\max}} f^{x_{\max}} f(X)XdX = X + \int_{x_{\min}}^{x_{\max}} f^{x_{\max}} [1-F(X)]dX \quad (6)$$

$$\bar{X} = \int_{x_{\min}}^{x_{\max}} f^{x_{\min}} f(X)XdX = X_{\min} + \int_{x_{\min}}^{x_{\max}} f^{x_{\min}} [1-F(X)]dX \quad (7)$$

and

$$\bar{X} = \int_{x_a}^{x_f} f^{x_{\max}} f(X)XdX = X + a + \int_{x_a}^{x_f} f^{x_{\max}} [1-F_t(X)]dX \quad (8)$$

where X_{\min} and X_{\max} is the minimum and maximum content of the feed material- the range of the feed material. X_a and X_f is the lower and upper limit of separation process.

Thus, if the range of separation is Z

$$Z = X_f - X_a \quad (9)$$

Both the two products of the separation process contain particles having the property

$$X_a < X < X_f.$$

The characteristic of separation is the equalizing parameter $\eta(X)$ which can be determined from the following relationship (Pethő, 1982).

$$\eta(\bar{X}) = X = \sum_{i=1} T_i \Delta X_i \tag{10}$$

where $\eta(\bar{X})$ is the expected value of the partition function $T(X)$, i the number of products; $i = 1, 2, n$ and ΔX_i is the property interval (e.g. specific gravity, particle size, metal content, ash content, ..., etc). At this value of X , the error area with respect to the two products are equal. Thus, the mass of the particles m_{pl} with the property X larger than $\eta(X)$, in the fine product (m_f), and the mass of the particles (m_{ps}) with the property X smaller than $\eta(X)$ in the coarse product (m_c) are equal, i.e.

$$m_{ps} = m_{pl} = m_{ls} \tag{11}$$

These defective material fractions characterizes the sharpness of the separation and can be calculated from the following relationship:

$$m_{ls} = \overline{m_c} \int_{x_a}^{x_f} f_c(X) dX = m_t \int_{x_a}^{x_f} f_t(X) dX \tag{12}$$

Mathematically, the equalizing parameter $(X)=X$, is the first moment of the distribution $T(X)$. The absolute sum of the first moments of the defective mass fraction (the misrouted particles) with a property smaller than X (where $X_a < X < X_f$) in the product m_c , and that with a property greater than X ($X_a < X < X_f$) in the product m_t , is minimum at the equalizing parameter X . Consequently, the absolute value of the first moments of the defective material at X can be used as a measure of the sharpness of the separation process

$$\begin{aligned} M_{ls} &= m_c \int_{x_a}^{x_f} f_c(X)(X - \bar{X}) dX + m_t \int_{x_a}^{x_f} f_t(\bar{X})(X - \bar{X}) dX \\ &= m_c \int_{x_a}^{x_f} [1 - F(X)] dx + m_t \int_{x_a}^{x_f} F_t(X) dX \end{aligned} \tag{13}$$

CLASSIFICATION OF THE SEPARATION PROCESS

Theoretically, the separation process may have two extreme cases- in one case the separation is absolutely efficient (AES), in the other it is absolutely inefficient (AIS). With an absolutely efficient separation, the result (at the equalizing parameter) is mass yields equal to that of the real operation and the products of the separation process do not contain misrouted particles. Using the distribution and frequency function of the raw material, the average quality of the products \bar{X}_t^{AES} and \bar{X}_c^{AES} can be determined.

$$\bar{X}_t^{AES} = 1 / m_t \int_{x_a}^{x_f} f_t(\bar{X}) dX = X_{min} + \int_{x_a}^{x_f} [1 - 1/m_t F(X)] dX \tag{14}$$

and

$$\bar{X}_c^{AES} = 1/m_c \int_{x_{\min}}^{x_{\max}} f(X) dX = \bar{X} + 1/m_c \int_{x_{\min}}^{x_{\max}} [1-F(X)] dX \quad (15)$$

For the absolutely efficient separation of the product m_t : $m_t f_t^{AES}(X) = f(X)$ if $X_{\min} < X < X$ and its $f_t^{AES}(X) = 0$ if $X > X$. For the other product m_c : $m_c f_c^{AES}(X) = f(X)$ if $X < X < X_{\max}$ and $f_c^{AES}(X) = 0$ if $X > X$. (Pethő, 1972).

Since the products do not contain misrouted particles, the values of the parameters which can be determined from Equations 12 and 13 are zero, i. e. $m_{ls} = 0$ and $M_{ls} = 0$.

If the separation zone ($z = X_f - X_a$) is equal to the range of the raw material ($X_{\max} - X_{\min}$), then, there is no separation according to the quality of the particles to be separated. Thus, the density and distribution functions of the separation products and also their average qualities are exactly the same as those of the raw material. Such a separation process is defined as absolutely inefficient (AIS).

The proportion of the misrouted particles in the separated products is given by the relationship:

$$m_{ls}^{AIS} = m_c \int_{x_{\min}}^{x_{\max}} f(X) dX = m_t \int_{x_{\min}}^{x_{\max}} f(X) dX \quad (16)$$

Taking into account that $\int_{x_{\min}}^{x_{\max}} f(X) dX = m_t$, the defective material fraction can be calculated directly from the yields.

$$m_{ls}^{AIS} = m_c m_t = m_t (1 - m_t) \quad (17)$$

The sum of the first moments with respect to the equalizing parameter for the absolutely inefficient separation process is

$$\begin{aligned} M_{ls}^{AIS} &= m_c (\bar{X}_{\max} - X) + \int_{x_{\min}}^{x_{\max}} f(X) F(X) dX - m_c \int_{x_{\min}}^{x_{\max}} f(X) F(X) dX \\ &= m_c m_t (\bar{X}_c^{AES} - X_t^{AES}) \end{aligned} \quad (18)$$

NEW INDEX OF THE SEPARATION PROCESS

The object of mineral processing, regardless of the methods used, is also the same, i.e. to separate the minerals into two or more products with the values in the concentrates, the gangue in the tailings, and the locked particles in the middlings. Such separations are, of course, never perfect, so that much of the middlings produced are, in fact, misplaced particles, i.e. those particles which ideally should have reported to the concentrate or the tailings (Pethő, 1972).

The index used to represent the efficiency of the separation process must comply with several requirements. It is of great importance that this efficiency index should (1) be applicable to any physical separation irrespective of the machinery used (2) result in 1 or 100% values for absolutely efficient separation and zero for absolutely

inefficient separation (3) be a function of the quantity and the composition of the raw material and the products and (4) have a physical meaning and its calculation should have no difficulties. The evaluation of the real separation process (actually occurring in practice) can now be determined from the parameters, previously discussed, concerning the absolutely efficient and inefficient separation, since the real separation lies between these two extremes (Figure 4).

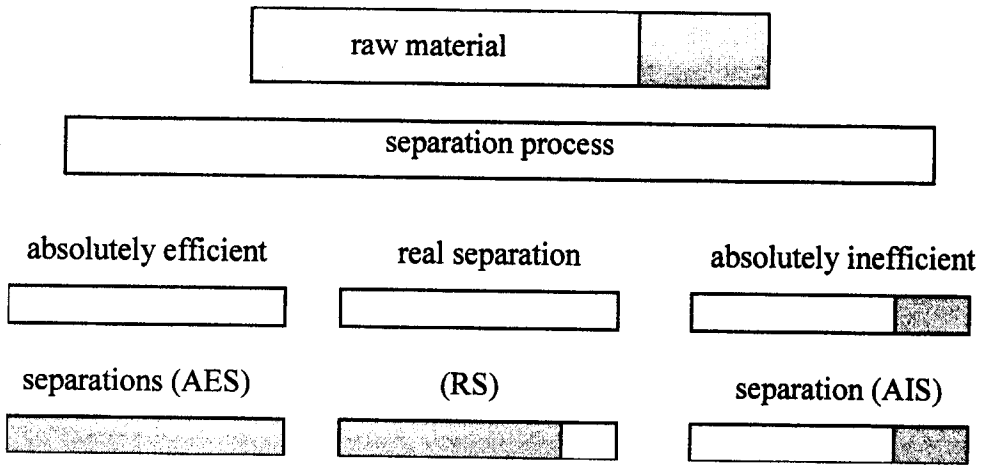


Fig. 4. Different possibilities of separation process

Rys. 4. Różne możliwości procesu separacji

To evaluate the separation operation under question, its efficiency parameter must be compared with those of an absolutely efficient and inefficient separation. The separation operation is the more efficient, the more it approaches the theoretical one, and is the less efficient, the nearer is to the absolutely inefficient. This definition can be formulated as follows.

$$\begin{aligned} \text{Efficiency index} &= \frac{[(\text{real separation parameter}) - (\text{parameters of absolutely inefficient separation})]}{[(\text{parameters of absolutely efficient separation}) - (\text{parameters of absolutely inefficient separation})]} \\ &= \frac{(RS - AIS)}{(AES - AIS)} \end{aligned} \quad (19)$$

Based on the above definition, the following efficiency indices (α) and (λ) can be calculated in terms of the proportion of the defective material and of the sum of the first moments with respect to the equalizing parameter, respectively.

$$\alpha = \frac{m_{1s} - m_{1s}^{AIS}}{m_{1s}^{AES} - m_{1s}^{AIS}} = 1 - \frac{m_{1s}}{m_{1s}^{AIS}} = 1 - \frac{m_{1s}}{m_1 m_s} \quad (20)$$

$$\lambda = \frac{m_{1s} - M_{1s}^{AIS}}{M_{1s}^{AES} - M_{1s}^{AIS}} = 1 - \frac{M_{1s}}{M_{1s}^{AIS}} \quad (21)$$

where: $m_{1s}^{AES} = 0$ and $M_{1s}^{AES} = 0$

Starting out from the proportion of the defective material of any product, the value of the index α can be calculated according to equation 12. For the product m_c , the perfect material fractions of the real, absolutely efficient and absolutely inefficient separations are $m_c - M_{1s}$, m_c and $m_t - M_{1s}^{AIS}$, respectively. Similarly those for the product m_t (in the above order) are $m_t - M_{1s}^{AES}$, m_t and $m_t - M_{1s}^{AIS}$.

The sum of the mass fractions of the two products are, in the former order, $1 - 2 M_{1s}^{AES}$, 1 and $1 - 2 M_{1s}^{AIS}$. Therefore, the index α is the complimentary yield of the defective material of the separation products referred to the absolutely inefficient separation and at the same time α is the yield of the real perfect mass fraction related to the absolutely efficient separation. In the case of an absolutely efficient separation the mass yield is in both cases, 1 .

A new interpretation of the sums of first moments can be obtained (Equations 22 and 23) by rearranging Equation 13. (Pethő, 1972).

$$\bar{M}_{1s} = (X_c - X_c) \bar{m}_c - (X_c^{AES} - X_i) m_t \quad (22)$$

$$M_{1s} = (\bar{X}_t^{AES} - X_i) m_t - (\bar{X} - X_t^{AES}) m_t \quad (23)$$

X_i is arbitrarily chosen, but after the choice is made, it remains constant.

In this case, M_{1s} is the difference of the first moments with respect to X_i . Thus, it takes into account the average qualities of the real and of the absolutely efficient separation. Therefore, M_{1s} is not only the error moment of separation characterizing the given separation (Mayer 1967 and Mayer 1971) but permits the comparison with the absolutely efficient separation for any separation product. In connection with the efficiency index (Equation 21) it is useful to rewrite equations 13, 22 and 23 as follows:

$$M_{1s} = m_c (\bar{X}_c - \bar{X}_c^{AES}) \quad (24a)$$

$$M_{1s} = m_t (\bar{X}_t^{AES} - \bar{X}_t) \quad (24b)$$

$$M_{1s}^{AIS} = m_c (\bar{X} - \bar{X}_c^{AES}) \quad (25a)$$

$$M_{1s}^{AIS} = m_t (\bar{X}_t^{AES} - \bar{X}) \quad (25b)$$

Introducing the former moment sums into Equation 13, the efficiency index can be directly calculated as follows:

$$\lambda = \frac{\bar{X} - \bar{X}_t}{\bar{X} - \bar{X}_t^{\text{AES}}} = \frac{\bar{X} - \bar{X}_c}{\bar{X} - \bar{X}_c^{\text{AES}}} \quad (26)$$

Starting from this efficiency index the average quality of the raw material can be given by the relationship

$$\bar{X} = \frac{\bar{X}_t \bar{X}_c^{\text{AES}} - \bar{X}_c \bar{X}_t^{\text{AES}}}{(\bar{X}_t + \bar{X}_c^{\text{AES}}) - (\bar{X}_c + \bar{X}_t^{\text{AES}})}$$

Equation 26 indicates that the efficiency index λ also describes the ratio of the change of the average qualities. Equations 20 and 26 also show that the sums of the first moments of the defective mass fractions result in the same efficiency parameters as those calculated from the qualities of the products.

CONCLUSIONS

The separation process can be evaluated by comparing the sharpness factor of the actual, the absolutely efficient, and the absolutely inefficient operations.

The new index can be used to calculate the average qualities of the raw material. It also describes the change of the average qualities.

The sums of the first moments of the defective mass fractions result in the same efficiency parameters as those calculated from the qualities of the products.

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Pethó Sz., Tarshan M. M., Nowy indeks sprawności dla oszacowania pracy separatora, *Fizykochemiczne Problemy Mineralurgii*, 33 (1999), 163-172 (w jęz. angielskim)

Praca separatora jest charakteryzowana przez współczynnik określający sprawność separacji. Koncepcja współczynnika sprawności operacji przerobczych powinna być interpretowana z dużą ostrożnością. Chociaż, użycie tego indeksu w sposób jakościowy jest ważne i użyteczne, trudności rodzą się w przedstawieniu tego indeksu w formie ilościowej. Przyczyną tego jest konieczność uwzględnienia dwóch przeciwstawnych wielkości takich jak wychód i uzysk. Indeksu sprawności procesu separacji nie należy przedstawiać jako pojedynczą wielkość, chociaż w praktyce przemysłowej jest to często robione, istnieje bowiem nieskończona ilość kombinacji wychodu i uzysku, które mogą dawać pojedynczą wartość. Zła efektywność procesów separacji może mieć swoje źródło w:

- ograniczeniach surowcowych reprezentowane przez krzywą separacji
- ograniczeniach w urządzeniu, które uniemożliwiają osiągnięcie maksymalnej separacji jaka wskazują krzywe separacji
- czynnik ludzki minimalizujący powyższe dwa ograniczenia.

W artykule badano indeks sprawności separacji dwóch procesów separacji. Procesy separacji zostały opisane przez porównanie współczynnika sprawności danej operacji z absolutną sprawnością i absolutną niesprawnością te ze operacji.