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MODEL PARTICLE VELOCITY ON A VIBRATING SURFACE

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In mechanical processing of granular materials the basic process is screening, hence it is extremely important to have a well designed screen. A correctly determined real velocity at which material moves along the screen surface is a prerequisite of designing a screen. The authors made an attempt to develop a new method for determination of the velocity of a granular layer moving on the vibrating surfaces of sieves or oscillating conveyors. The method is based on an analytical description of phases of the model particle movement, and next on an empirical correction made for the layer. The particle velocity is determined on the edges of subsequent phases of the movement, hence the name "a phase method". The authors present preliminary results of investigations whose aim was to verify the existing models and to determine an empirical correction with respect to the impact of process variables.

Key words: particle material, screening, sieve, undersize, vibrating sieve, phase method

NOMENCLATURE

d - mean particle diameter [m],
g - acceleration of gravity [m/s^2],
H_p - initial height of the layer [m],
m - mass [g],
n - frequency of vibrations [1/s],
r - radius of the crank of the screen driving gear [m],
A_0 - amplitude of sieve vibrations [m],
t - time [s],
T - screen operating period [s],
K - dynamic factor [-],
u_m - mean velocity of material [m/s],
w - moisture content [%],

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α - angle of sieve inclination between the sieve plane and horizontal plane [1°],
β - angle of sieve trajectory between tangent of the initial sieve movement and sieve plane [1°],
φ - angle of crank rotation calculated from conventional position 0 [1°],
φ_0 - angle of crank rotation at which particles fall onto the sieve [1°],
μ_s - coefficient of static friction of particles against the sieve [-],
μ_z - coefficient of static friction of particle against a particle [-],
ω - angular frequency of the drive [1/s],
l - mesh size [m].

INTRODUCTION

Screening is the main process in granular material processing. The most frequent screening method is to feed material onto the sieve, and then to transport it along the sieve surface during which the feed is segregated into the oversize and undersize class.

The authors decided to investigate the movement of a granular layer moving along the screen surface, and the movement of the trough of an oscillating conveyor, because also here the mechanisms of material transport are the same. A basis for a description of the granular layer movement on the vibrating surface is a phase diagram of model particle movement. Such diagrams were proposed by several researchers, e.g. Dietrych (1962), Czubak (1964) and Olewski (1955) (cf. Fig. 1). When analysing diagrams presented in Fig. 1 we can observe some differences in particular phases of the movement. These differences were also noted by Błasiński and Wodziński who decided to verify experimentally which of the proposed diagrams were most accurate. The investigations were carried out using a multi-speed camera at 3000 frames/s. A screen with linear horizontal movement and a sieve inclined to the level were used. The screen reached the dynamic factor $K_{max} = 6$, and the investigations were carried out for the dynamic factor ranging from 1 to 4. A particle was a metal ball whose movement was filmed. From analysis of the image obtained it was concluded that the most realistic diagram appeared to be the one proposed by Dietrych. On this basis the velocity of model particle movement was determined for one cycle of screen operation. It was determined as a weighted average from mean velocities in subsequent phases of the particle movement, hence the name of this method is the "phase method".

Because so far the investigations had been carried out with a metal ball used as a model particle, the authors decided to repeat the experiments using balls produced from a different material as model particles and to analyse results again. An important element of almost all solutions is the assumption that particles fall onto the sieve surface in an inelastic way. So, it follows that a particle which is a model of a layer, should behave as a layer. During the fall of the layer onto the vibrating surface the energy transmitted by the surface to the layer is dispersed in the bed, so the layer does not rebound from the surface. This means that the coefficient of layer restitution is 0 and the coefficient of restitution of particles should be also equal to 0. The restitution coefficient of a metal ball is different than 0, hence it is not a reliable model of the layer. In our investigations either plasticine balls or a sand bag was used. It should be borne in mind that in real industrial conditions, the role of particles is played by the whole layer on the sieve which has the above mentioned properties.

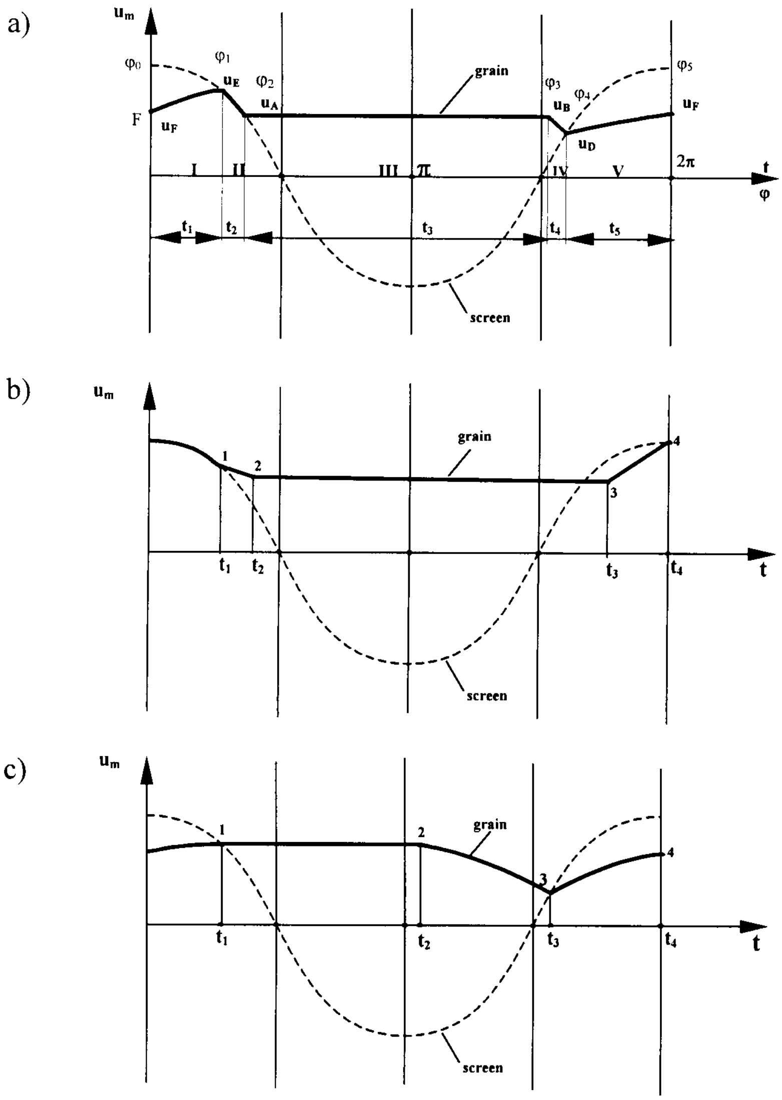

Fig. 1. Dependence of particle and sieve velocity on time
a) according to Dietrych, b) according to Czubak; c) according to Olewski

THE PHASE METHOD FOR DETERMINATION OF FEED VELOCITY ALONG THE SIEVE

A condition which must be satisfied to make the process of screening proceed, is the motion of particles against the sieve. This motion is induced by inertia forces from the machine drive which act on the layer (particles). Depending on the motion of a riddle, material on the sieve may slide on the surface, fly over it or move at a velocity equal to that of the surface, i.e. rest on it. The present screen designs force a resultant motion of the layer, i.e. such a motion where all mentioned types of motion occur.

As it has been mentioned already, many researchers (Dietrych, Czubak, Olewski) tried to describe the motion of a granular layer on a vibrating surface. Diagrams which they proposed (Fig. 1) illustrate particle motion describing all phases of the resultant motion, however equations proposed by them (equations (1), (2) and (3)) describe only the phase of flying.

Dietrych formula:

$$u_m = \frac{g \cdot cos(\beta - \alpha)}{\omega \cdot sin\,\beta} \sqrt{\frac{K-1}{2}(K^2-1)} \tag{1}$$

where K is the dynamic factor determined by the equation:

$$K = \frac{A_0 \cdot \omega^2 \cdot sin\,\beta}{g \cdot cos\,\alpha} \tag{2}$$

Czubak formula:

$$u_m = \xi \cdot \frac{g \cdot k \cdot m}{p \cdot n \cdot 2} \cdot (cos\,\alpha \cdot ctg\,\beta - sin\,\alpha) \tag{3}$$

where ξ – refers to the effect of such phenomena as different velocity of a material on different depths of the layer; k is the ratio of the time of particle free flight to the trough vibration period; p is the prime integer greater than or equal to k, when the latter is an integer.

Olewski formula

$$u_m = K_\sigma \cdot N \cdot \frac{n^2 \cdot r}{g} \cdot \left(1 + 22 \cdot tg^{3/2}\,\alpha\right) \cdot \left(\frac{\alpha}{18°}\right) \tag{4}$$

In 1979 Błasiński and Wodziński developed a new method for determination of particle velocity on the sieve, which referred to all phases of motion. As it has been mentioned already, the investigations carried out by these researchers led to the conclusion that among the proposed diagrams which describe particle motion, the most realistic is the diagram proposed by Dietrych (Fig. 1. a). Five phases can be distinguished. A velocity in each phase of the motion is a logarithmic mean from two velocities at the extremes of each phase (formulae (5) through (8)).

$$u_F = u_D + \left(\frac{2 \cdot \pi \cdot g}{\omega} - \frac{\varphi_4 \cdot g}{\omega} \right) \cdot \left(\mu_s \cdot cos\, \alpha - sin\, \alpha \right) - \left(\mu_s \cdot sin\, \beta + cos\, \beta \right) \cdot r \cdot sin\, \varphi_4 \qquad (5)$$

$$u_E = \frac{\omega \cdot r \cdot sin\, \varphi_1 \cdot cos\, \beta}{cos\, \alpha} \qquad (6)$$

$$u_A = u_B = A_0 \cdot \omega \cdot \sqrt{1 - \frac{1}{K^2}} \qquad (7)$$

$$u_D = \omega \cdot r \cdot cos\, \varphi_4 \cdot cos\, \beta \cdot cos\, \alpha \qquad (8)$$

Having the mean velocities and duration of each phase, one can determine the mean particle velocity on a sieve as a weighted mean of these velocities; the weight is here the duration of each phase, formula (9):

$$u_m = \frac{\dfrac{u_E - u_F}{ln(u_E / u_F)} \cdot t_1 + \dfrac{u_E - u_A}{ln(u_E / u_A)} \cdot t_2 + u_B \cdot t_3 + \dfrac{u_A - u_D}{ln(u_A / u_D)} \cdot t_4 + \dfrac{u_D - u_F}{ln(u_D / u_F)} \cdot t_5}{T} \qquad (9)$$

where:

$$T = t_1 + t_2 + t_3 + t_4 + t_5 \qquad (10)$$

is the screen operation period.

The phase method of velocity determination was developed on the basis of the movement of a single model particle which in this case was a metal ball. The authors are of the opinion that the application of metal balls as model particles is dubious, so they decided to approach the subject again after some years using model particles prepared from a different material, whose restitution coefficient would be close to zero. Next part of this paper will refer to the experimental set-up and results obtained by the authors recently.

EXPERIMENTAL SET-UP AND METHOD

Experiments were carried out on a linearly vibrating screen (Fig. 2) with an electromagnetic vibrator of the frequency of vibrations equal to 50 Hz and smoothly adjustable amplitude in the range from 0 to 2 mm. It was possible to adjust the angle of inclination of the screen to the level from 0° to 20° (the investigations were carried out in the range from 0° to 10°) and to control the inclination of the sieve in the range from 0° to 40° in relation to the riddle (the investigations were carried out in the range from 0° to 15°). The screen can attain the dynamic factor $K_{max} = 20$ (experiments were made at K = 8 to 15). The main elements of the screen are as follows:

1. – screen base,
2. – spring suspension,
3. – supporting structure,
4. – tested sieve,
5. – electromagnetic vibrator,
6. – charging hopper,
7. – flow control valve,
8. – vessel for oversize fraction,
9. – vessel for undersize fraction.

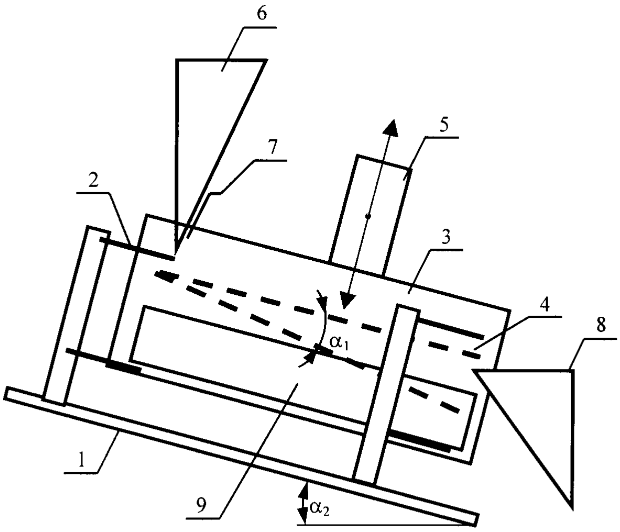

Fig. 2. Schematic diagram of the experimental screen

The model particles of spherical shape, 5 mm in diameter, were made from coloured plasticine. The choice of material was determined by material properties during rebound. It was justified to use a material with possibly small restitution coefficient because such particles as a model of the layer, should have the restitution coefficient equal to 0.

To record particle motion, a digital multi-speed camera with a PCI card and software supplied by Redlake Imaging Corporation was used. The camera made colour photos at the speed of 500 frames/s, shutter time 1/1500 s. The recorded image had the resolution 320×280 pixels. The investigation consisted in filming a fragment of the vibrating surface with particles moving on it. A schematic diagram of the experimental set-up is shown in Fig. 3.

Particles were supplied to the vibrating surface about 200 mm before the region of filming, which was 250 mm. Particles moved against the riddle edge with the scale 10 × 10 mm (cf. Fig. 4). Additionally, on the riddle frame two points at a distance of 178 mm from one another were marked.

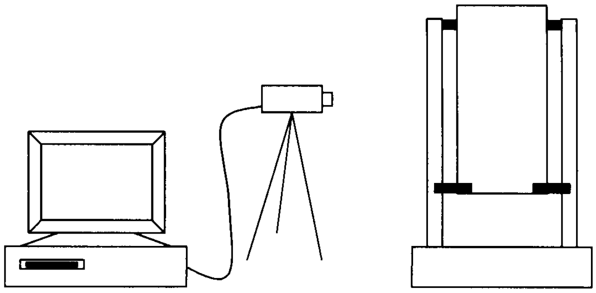

Fig. 3. Schematic diagram of the experimental set-up

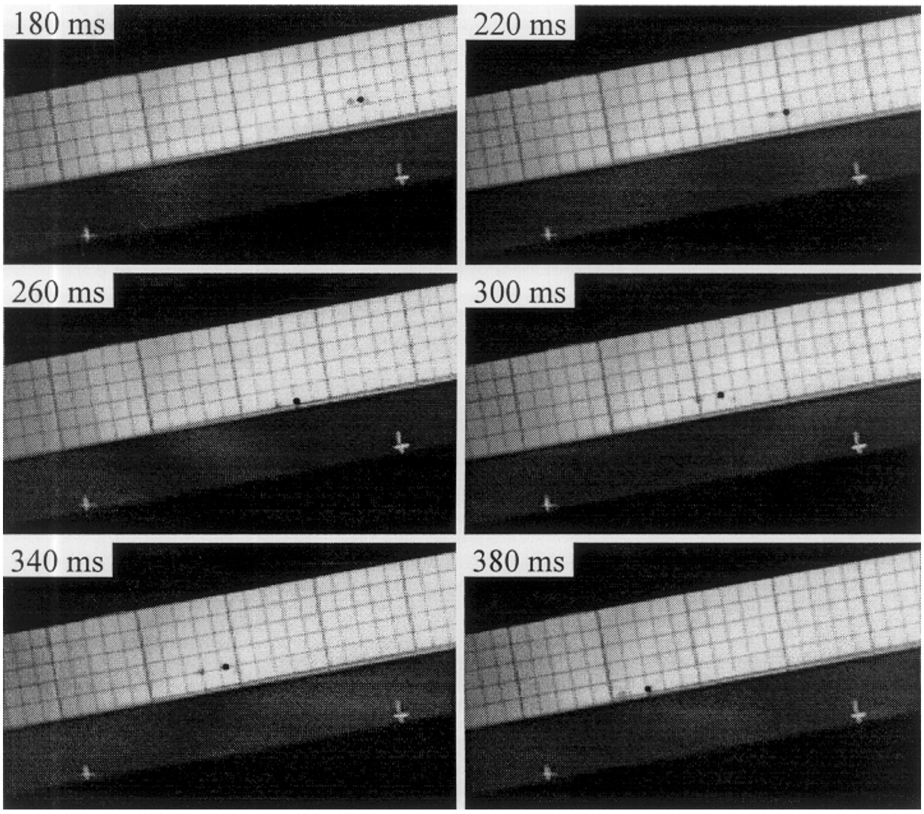

Fig. 4. Examples of the frames representing the system set-up: $\alpha_1 = 10°$; $\alpha_2 = 0°$; A = 1.15 mm

After having filmed the particle motion, the image was analysed using a special software provided along with the camera. Data concerning the position, transfer and velocity were read to a file and then analysed by generally available programs. Results are presented in a graphical form.

RESULTS AND DISCUSSION

When considering the process, the authors drew a conclusion that the most appropriate reference system would be such one in which the axis of abscissae would be in the plane of the vibrating surface and would be directed towards the layer motion, while the axis of ordinates would be perpendicular to the surface (cf. Fig. 5). The proposed reference system reflects material motion against the vibrating surface.

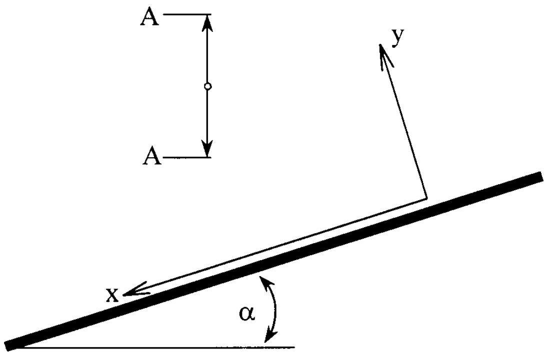

Fig. 5. Schematic diagram of the location of coordinate system vs. vibrating surface

Results presented in this study provide examples of a system with the following parameters: α_1 = 10°, α_2 = 0°, A = 1.15 mm (Figs. 6 and 7) at the dynamic factor K = 12.

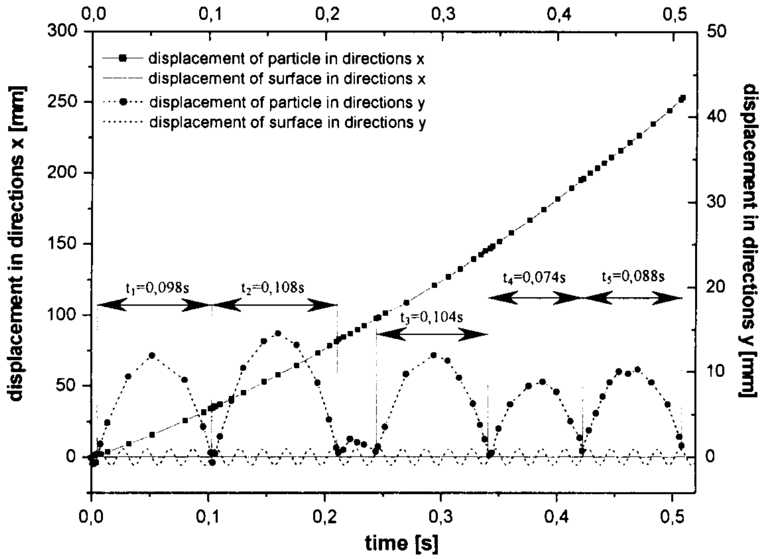

Fig. 6. Displacement of particles parallel to (x) and perpendicular to (y)

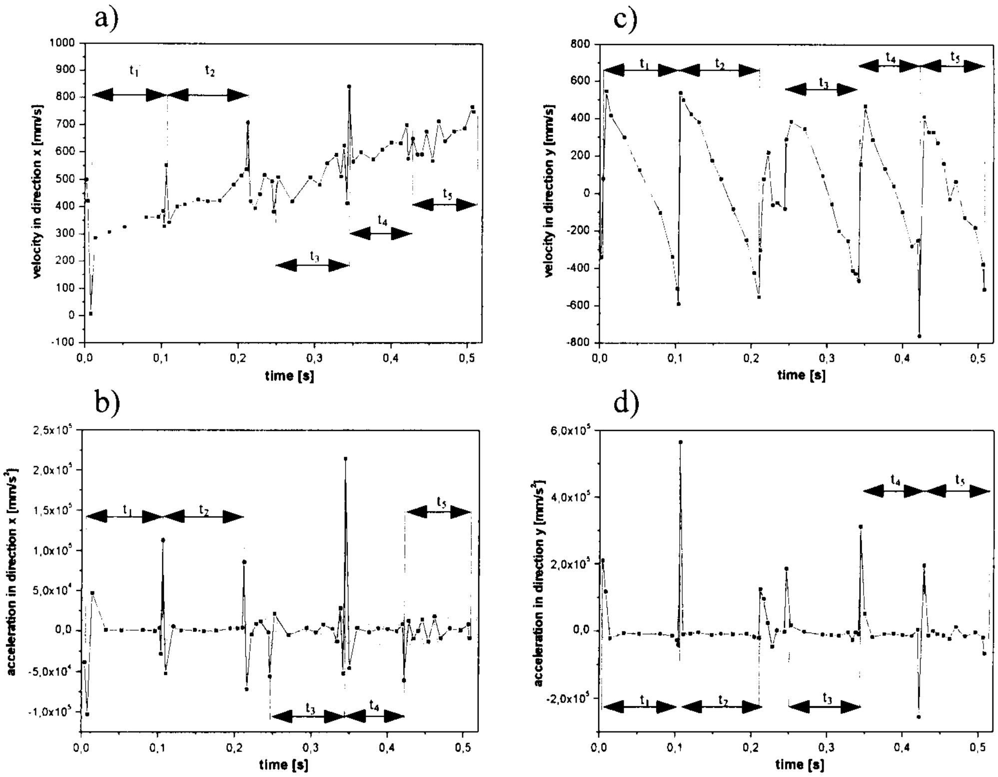

Fig. 7. Dependence of particle and surface velocity and acceleration parallel to (a, b) and perpendicular to (c, d)

Figure 6 shows a diagram of the displacement of particles and the surface in directions x and y. To better illustrate the particle trajectory in particular phases, both displacement compounds are mapped on one diagram. As the amplitude of the system vibrations in direction x is very small (in the example presented it is A_x= 0.17 mm) it is not possible to show in one diagram the dislocations of particles and the surface, so the dislocation of the surface in direction x is represented by a straight line in the diagram.

When analysing the diagrams of dislocation velocity, the authors observed some regularities:

- in the moment when particles collided with the surface, in many cases the acceleration decreased below 0 (braking occurred);
- the peak of acceleration corresponded to the point of particle detachment in the displacement diagram;
- next, the acceleration decreased to zero and was maintained so until the next contact with the surface.

It can be concluded from the above that despite elasticity of the particle rebound, there are elements of phase motion which were discussed above.

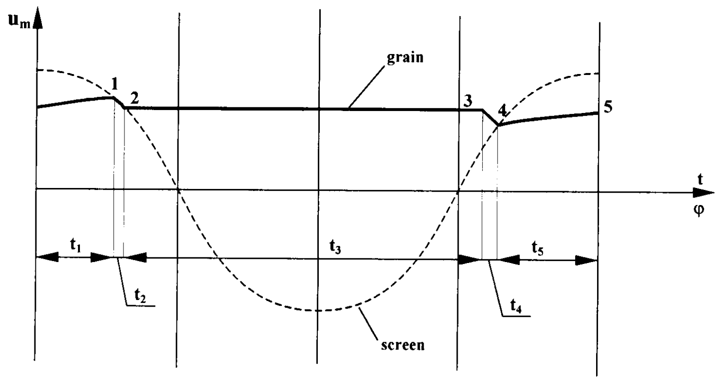

Fig. 8. Proposed diagram of particle motion along a vibrating surface

Figure 8 shows a diagram proposed by the authors to describe the particle trajectory during one of the periods of screen operation. This occurs when the screen works at a low value of the dynamic factor, i.e. K = 3 to 4.

The proposed diagram is based on the research carried out so far. It resembles a diagram proposed by Dietrych (Fig. 1a), however, the authors are of the opinion that the time of contact between the particles (t_2 and t_4 in Fig. 8) with the screen surface is much shorter than that proposed by Dietrych.

CONCLUSIONS

Results presented in this paper are an introduction to a research whose aim is to develop a new method for calculation of material velocity on a sieve – a phase method. The authors realise that model particles do not satisfy the assumption of non-elastic rebound and they will make experiments with particles of other materials.

It is true that the tested model particles are not characterised by a dynamic factor equal to 0 and the fall to the vibrating surface is elastic. However, such properties are characteristic of the particles of real granular materials which are screened in industry. Only thick granular beds encountered in thick-layer screening specific for fine- and very fine granular materials, can approach ideal conditions because only in such conditions the rebound does not occur.

Experiments will be also carried out with other configurations of the screen described in this paper, and on a circular motion screen. Results of investigation of model particle motion will be verified using the coefficients obtained in the course of process investigations performed on the same screens and for the same process conditions using different types of aggregates.

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W mechanicznym przerobie materiałów ziarnistych podstawowym procesem obróbki jest przesiewanie, zatem niezwykle ważne jest aby przesiewacz na którym odbywa się proces był poprawnie zaprojektowany. Właściwe określenie rzeczywistej prędkości z jaką porusza się materiał na powierzchni sitowej jest koniecznym warunkiem do zaprojektowania przesiewacza. Autorzy niniejszej pracy podjęli próbę opracowania nowej metody wyznaczania prędkości poruszającej się warstwy ziarnistej na powierzchni drgającej przesiewaczy lub przenośników wibracyjnych. Metoda ta opiera się na analitycznym opisie faz ruchu modelowego ziarna, a następnie empirycznej korekcji dla warstwy. Prędkość ziarna będzie wyznaczana na krańcach poszczególnych przedziałów (faz) ruchu, stąd nazwa „metoda fazowa". Omawiana praca prezentuje pierwsze wyniki badań mających na celu weryfikacje istniejących modeli i określenie empirycznej korekty uwzględniającej wpływ zmiennych procesu.