Physicochemical Problems of Mineral Processing, 38 (2004) 207-220
Fizykochemiczne Problemy Mineralurgii, 38 (2004) 207-220

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INFLUENCE OF GRAIN SIZE UPON ITS FALLING VELOCITY

Received May 14, 2004; reviewed; accepted June 21, 2004

The problem of determining the theoretically justified method of considering the natural shape of mineral grains in the calculations of their motion velocity in liquid media, especially their falling under the influence of the external mass force, in particular the force of gravity, has not been successfully solved so far. The presented paper shows a possibility of increasing the accuracy of calculations of this velocity with the use of the well-known formulas, discussed in the paper introduction, by means of the appropriate introduction of two different grain shape coefficients into them.

Key words: sedimentation, grain shape, falling velocity, shape coefficient

INTRODUCTION – FALLING VELOCITY OF SPHERICAL GRAINS

The motion velocity of mineral gains (v, m·s^{-1}) in the liquid medium (liquid or gas) under the effect of the acting external mass force is the principal criterion of their behaviour in the course of flow processes which play an important role in the technology of mineral processing[1]. This velocity is the function of the force exerted upon a grain by the mass force (most often gravity force or/and centrifugal force), evoking the grain motion (P_m, kg·m·s^{-2}) of surface forces (P_ψ, kg·m·s^{-2}), comprising the resistance against the grain moving through the medium, and also, in a significant degree, practically in the initial motion phase – initial velocity (v_p, m·s^{-1}), given to the grain in the moment of the motion start (Laščenko 1935; Budryk 1936; Barskij *et al.*

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[1] It concerns any technological processes into which the grained materials are subjected. Mineral processing was assumed to set the attention and also due to the significant position played by the flow operations in it.

1974 and others). In the present considerations the following idealized conditions were assumed: a spherical grain of a certain size (d, m) equal to the sphere diameter and density (ρ_z, kg·m^{-3}), it moves in the liquid (water) of density (ρ_c, kg·m^{-3}) and dynamic viscosity rate (μ_c, kg·m^{-1}·s^{-1}), occupying the unlimited spatial area, by means of free motion, (without possible actions of other objects), caused by gravity force of acceleration value (g, m·s^{-2}) which is constant in the entire area. The instant velocity of grain in relation to the medium (v, m·s^{-1}) is studied which is the function of motion duration time (t, s) – $v = v(t)$; the grain initial time is nought ($v_p = v(0) = 0$, m·s^{-1})[2]. Such conditions justify the assumption of the fact that the vectors of all forces acting upon the grain lie upon the straight line fixed by the vector of mass force whose sense determines a positive direction and the resultant force (P), acting on the grain, is constituted by the sum

$$P = P_m - P_\psi \tag{1}$$

Force P is the product of grain mass (m_z) – $m_z = V_z \cdot \rho_z$, V_z – grain volume, for sphere $V_z = {}^1/_6 \cdot \pi \cdot d^3$ – and the values of acceleration of grain motion (dv/dt) – $P = m_z \cdot \frac{dv}{dt} = V_z \cdot \rho_z \cdot \frac{dv}{dt}$ and it has properties of inertial force.

Mass force is the product of grain mass diminished by the value of uplift pressure and acceleration of mass force. In the field of gravity force it is

$$P_m = \frac{\pi \cdot d^3}{6} \cdot (\rho_s - \rho_c) \cdot g \tag{2}$$

Resistance force of the medium (P_ψ) has a general form

$$P_\psi = \Psi \cdot \rho_c \cdot v^2 \cdot d^2 \tag{3}$$

where Ψ – dimensionless resistance coefficient which is a complex function of many factors, especially Reynolds' number characterizing the motion of liquid around the grain in the considered system expressed by the formula

$$\mathrm{Re} = \frac{\rho_c \cdot v \cdot d}{\mu} \tag{4}$$

In general, it can be written that P_ψ is a certain function

$$P_\psi = f(d, v, \rho_c, \mathrm{Re}) \tag{3.1}$$

[2] accordingly the measurements of the quoted sizes are neglected, assumed in the entire work, according to the SI system of units

Difficulties in precise determining the general function form of the resistance coefficient, and firstly, practical requirements, led to working out, starting from the XVIIth century (Newton), numerous approximate forms of formulas for calculating the resistance force. They can be applied in limited ranges of grain motion conditions, most often defined by means of values of Reynolds' number, characteristic for their highlighted ranges. The advanced work upon the general description of the motion of grains in the described conditions was undertaken as late as in the first half of the XXth century (Laščenko 1935).

Newton's approximations are the oldest and most popular

$$P_d = \frac{1}{3} \cdot F_r \cdot v^2 \cdot \rho_c \tag{5}$$

where F_r – grain projective area – area of the perpendicular projection of the grain upon the plane perpendicular to the tangent to the grain motion track, fixed by its mass centre in the point of the grain instant position; for a sphere in its every position the projective area is equal to the area of the largest cross-section of the sphere: $F_r = \frac{\pi}{4} \cdot d^2$, thus

$$P_{d_{sphere}} = \frac{\pi}{12} \cdot d^2 \cdot v^2 \cdot \rho_c \tag{5.1}$$

and by Stokes

$$P_l = 3 \cdot \pi \cdot \mu \cdot v \cdot d\,. \tag{6}$$

The values of resistance force, expressed by formulas (5) and (6), are characteristic, respectively, to the conditions – turbulent (P_d) and laminar (P_l)[3] motions of the medium flowing around the grain. Actually, the resistance force contains always two "components" – a "dynamic" one - P_d - originating from the resistance of inertial liquid and dissipation of energy in turbulent swirls in the conditions and a "laminar" one - P_l - evoked by the friction resistance between liquid layers and the grain surface and liquid in the laminar flow.

The dependence of the resistance force from the grain motion velocity against the medium makes the absolute value of this force increase from the moment of the motion start and the acquisition of increasing velocity v under the effect of force P, and after some time (t_0) (Laščenko 1935 and others) its absolute value becomes so close to the value of mass force that their sum (1), causing the grain motion, aims at

[3] it is often assumed (often artificially) that the laminar motion is characterized by the value $\mathrm{Re} < \sim 1$ and turbulent one by $\mathrm{Re} > \sim 1000$

nought. Then the acceleration of grain motion disappears and it moves on with practically constant speed, called the boundary velocity ($v_0 = v(t \geq \sim t_0) = \sim$ const). Mineral grains of the most popular sizes and densities, subjected to flow operations (flow classification, sedimentation, gravitational enrichment), usually in the water medium, reach as a rule the boundary velocity after very short times t_0, which contributed to assuming the boundary velocity v_0, practically constant for given conditions, to be the value characterizing their behaviour in the discussed processes.

Substituting expression for P_ψ corresponding to the present flow conditions (P_d, P_l, $P_d + P_l$ or others) into formula (1) for the balance of forces acting upon the grain, and dividing this formula by two sides by the grain mass ($m_z = V_z \cdot \rho_z$) whereas $\frac{P}{m_z} = \frac{dv}{dt}$, we obtain the expression for acceleration balance in which the left side describes the acceleration of grain motion – dv/dt. The latter equals nought for the state of boundary velocity and after matching to nought the right side of the acceleration balance we receive an equation in which velocity v is only unknown, now already substituted by boundary velocity v_0. Solving this equation results in obtaining a formula to calculate the boundary velocity.

Assuming P_ψ according to (5.1) we receive an equation known as Newton-Rittinger's formula[4]

$$v_{0_{NR}} = \sqrt{2 \cdot g} \cdot \sqrt{\frac{(\rho_z - \rho_c) \cdot d}{\rho_c}} \tag{7}$$

which is useful when Reynolds' number assumes large values (large grains, at medium densities, $d \geq \sim 2 \div 5$ mm).

Assuming P_ψ according to (6) results in Stokes' formula

$$v_{0_S} = \frac{g \cdot (\rho_z - \rho_c) \cdot d^2}{18 \cdot \mu} \tag{8}$$

to calculate the boundary velocities of fine and very fine grains ($d \leq \sim 0{,}2$ mm).

The formal boundary of application of formulas (7) and (8) is determined by the value

$$\mathrm{Re} = \frac{\rho_c \cdot v_0 \cdot d}{\mu} = 36 \tag{9}$$

- v_{0_S} when $\mathrm{Re} \leq 36$
- $v_{0_{NR}}$ when $\mathrm{Re} \geq 36$

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[4] Rittinger derived this formula to be used in designing the operations of coal gravitational enrichment, assuming determining the resistance force according to Newton.

but assuming the permissible relative error of measurement of value *Δ* (for example, *Δ* = 0.05) changes the applicability ranges approximately:

– for Stokes' formula (8): into $0 \leq \mathrm{Re} \leq \sim 36 \cdot \Delta$, (for example $0 \leq \mathrm{Re} \leq 1{,}8$),
– for Newton-Rittinger's formula (7):into $36/\Delta \leq \mathrm{Re}$, (for example $720 \leq \mathrm{Re}$).

A "universal" assumption of resistance force as a sum of $P_d + P_l$ leads to obtaining a formula, given for the first time by Budryk (1936), applicable in a wide range of values of Reynolds' number, characterizing the state of grain round flow

$$v_{0_B} = \frac{18 \cdot \mu}{d \cdot \rho_c} \cdot \left(\sqrt{1 + \frac{g \cdot (\rho_s - \rho_c) \cdot \rho_c}{162 \cdot \mu^2} \cdot d^3} - 1 \right) \tag{10}$$

The quoted formulas (7), (8) and (10) are, due to their simplicity, relatively often applied in cases which are unquestionable as far as a strong predominance of phenomena characteristic for turbulent round flow – Newton-Rittinger's formula (e.g. gravitational enrichment of large and medium grains) or laminar one – Stokes' formula (e.g. sedimentation processes). The most difficult concerns flow classification which concerns in majority of case the materials of grain size distribution 0.1 ÷ 1 mm or a little more, of the round flow characteristics through the classification medium about the boundary value Re = 36. Budryk's formula (10), well-fitted for such grains, makes problems resulting from the lack of possibilities of presenting it in the form $v_0 = S \cdot d^n$ where *S* – constant depending on grain and medium properties, invariable in a given process, *n* – power exponent at *d*, characteristic for the given formula (in Newton-Rittinger's formula *n* = ½, in Stokes' formula *n* = 2). For the so-called transition range, comprising such grains, many formulas were worked out, including many empirical ones, out of which the most useful is that given by Allen (Barskij *et al.*; Nowak, Sztaba 1986; *Zbiorowe* 1972 and others).

Assuming in the formerly described approach leading to deriving formulas for v_0, $P_\psi = \sqrt{P_d \cdot P_l}$ as the value of resistance, Allen's formula is obtained in the form:

$$v_{0_A} = \sqrt[3]{\frac{g^2}{9}} \cdot \sqrt[3]{\frac{(\rho_z - \rho_c)^2}{\rho_c \cdot \mu}} \cdot d \tag{11}$$

As opposed to Budryk's formula of a practically unlimited range of applications, Allen's formula can be used without a too large error, only for falling conditions in the transition range, called also Allen's range, between the ranges of applicability of Stokes' and Newton-Rittinger's formulas. A conclusion may be drawn that it is more relevant to assume P_ψ in the form of a sum, as in Budryk's formula, than as a geometric mean of P_d and P_l resistances, as in the case of Allen's formula.

Among the already mentioned works aiming at working out a uniform way of calculating the boundary velocity, a record generalizing the majority of quoted formulas is worth mentioning. After solving the general equation (3.1) by means of the dimension analysis and after introducing the resistance coefficient in the form $\Psi = K \cdot \mathrm{Re}^{n-2}$ where:

K – numerical constant

n – coefficient indicating the character of the round flow

$n = 1$	– laminar round flow:	$K = 3\cdot\pi$,
$1 < n < 2$	– round flow of mixed character (transition range):	$K = \pi/2$,
$n = 2$	– turbulent round flow:	$K = \pi/12$,

in the formerly given way the equation is obtained (*Zbiorowe* 1972)

$$v_0^n = \frac{4}{3} \cdot \frac{g}{K} \cdot \left(\frac{\rho_z - \rho_c}{\rho_c^{n-1}} \right) \cdot \mu^{n-2} \cdot d^{3-n} \tag{12}$$

as a general formula for the free falling velocity for any flow conditions. Changing value n in the given limits, the formulas for various types of grain motions are obtained, consistent with those given before.

SELECTED DETERMINATIONS OF GRAIN SHAPES APPLIED FOR FLOW PROCESSES

The discussed formulas, important for spherical grains, not occurring in mineral processing, cannot be directly applied in the industry. The problem of considering the shape of grains for calculating the boundary velocity of falling concerned the attention of researchers and engineers for a long time. The collected works and outlooks can be systematically arranged as follows (*Zbiorowe* 1972 and others).

1. Empirical methods consisting in experimental determining for respective materials (usually minerals) numerical coefficients introduced into the formulas important for spherical grains. These are the oldest methods and are only valuable for the materials tested experimentally, cannot be broadened or generalized, do not use the values directly concerning the shape of grains.
2. Other experimental methods, using coefficients fixed for solids of shapes defined by means of geometric descriptions. In this group special attention should be put upon empirical corrections introduced into formulas for spherical grains and fixed not for the grains of directly defined shapes but of different values of the spherity coefficient (C_k), discussed further in point 3.b. There are also other coefficients (*Zbiorowe* 1972), determined for solids of regular shapes, falling at the values of Reynolds' number, characterizing the round flow of the grain by the liquid – classification medium – in ranges 0 ÷ 0.05 (correction to Stokes' formula) and over 2000 (correction to Newton-Rittinger's equation). There is no information to prove these empirical dependences for regular grains and no concept to broaden the method into the transition area ~ 0,05 < Re< ~ 2000.

3. Calculation methods based upon introducing the values taking into consideration the non-spherity of real grains into formulas for spherical grains. Real grains can be possibly divided into certain groups (Barskij *et al.* 1974). Out of these the following methods can be differentiated.
   3.a. Assuming as the grain size the substitute grain diameter (d_z) – a sphere diameter of the volume equal to that of the grain

$$d_z = \sqrt[3]{\frac{6 \cdot V_z}{\pi}} \approx 1.2407 \cdot \sqrt[3]{V_z} \tag{13}$$

For practical purposes d_z is usually fixed as an average value D_z, e.g. for a certain grain class, deducting at random a certain number (N – usually not less than a few hundred) of grains and determining their mass m_N. Knowing the material density ρ_s, it can be calculated:

$$D_{z_{(N)}} = \sqrt[3]{\frac{6 \cdot m_N}{\pi \cdot N \cdot \rho_s}} \approx 1.2407 \cdot \sqrt[3]{\frac{m_N}{N \cdot \rho_s}} \tag{13.1}$$

The application of a substitute diameter has been more and more generally used for a long time in any calculations of flow processes. This principle can be also applied in this paper.

3.b. Introducing the coefficients into formulas, which are directly connected with grain geometry (without empirical corrections, as in point 2). The most important ones are given below:

– spherity coefficient (C_k)[5] – relation of sphere surface (F_k) of the volume equal to the grain (sphere of d_z diameter) to the grain surface (F_z)

$$C_k = \frac{F_k}{F_z} = \frac{\pi \cdot d_z^2}{\pi \cdot d_k^2} = \frac{d_z^2}{d_k^2} \tag{14}$$

where: d_k – diameter of the sphere of the surface equal to the grain (F_z),

– circularity coefficient[5] (C_c) – relation of the length of outline (c_z) of the perpendicular projection of the grain on the plane on which it is placed in the position in which the centre of the grain mass is situated at the closest of this plane (“the most stable position”) to the length of the circle perimeter (c_k) of the area equal to the area of grain projection (F_p)

$$C_c = \frac{c_z}{c_k} = \frac{c_z}{2 \cdot \sqrt{\pi \cdot F_p}} \tag{15}$$

[5] Proposed by Wadell in 1934.

the diameter of a circle of the area F_p is the grain projection value (d_p)

$$d_p = 2 \cdot \sqrt{\frac{F_p}{\pi}} \approx 1.1284 \cdot \sqrt{F_p} \tag{16}$$

– grain shape coefficient, given by Nowak (1980)

$$C_{AN} = \frac{d_z^2}{d_p^2} = \sqrt[3]{\frac{9 \cdot \pi}{16}} \cdot \frac{V_z^{2/3}}{F_p} \approx 1.2090 \cdot \frac{V_z^{2/3}}{F_p} \tag{17}$$

3.c. Introducing a coefficient similar to those described in point 3.b. but indirectly considering the change of conditions of the grain round flow depending on its shape in the form of "dynamic shape coefficients" (Barskij *et al.* 1974). This paper does not present examples of such propositions.

Only certain shape coefficients, discussed in point 3.b., are used in the paper. In this domain several significant propositions and elaborations were noted in Poland in the second half of the previous century. Apart from the quoted work by Nowak (1980) there are important investigations by Sysło (1964) on considering the grain shape in calculating the velocity of grains falling for which Stokes' formula can be applied. Another reason why these works are important is that their final result in the form of corrections to formulas was obtained in both cases by means of heuristic considerations, obtaining then very good results of experimental verification.

FALLING VELOCITY OF NON-SPHERICAL FINE GRAINS

In simplification, it can be stated that the resistance of medium exerted on very fine grains, falling at their purely laminar round flow through the medium, originates as the resistance of friction on the entire surface of grains. Consequently, it should be assumed the value d_k, determined by the grain surface, which can be appropriately assumed to be a representative grain size. Its value can be determined but it is hard to reach. It is, however, connected with the value of substitute diameter d_z, value of spherity coefficient C_k, assumed for calculations. It results from formula (14):

$$d_k = \frac{d_z}{\sqrt{C_k}} \tag{18}$$

Taking into account this dependence, the force of laminar resistance is written as

$$P_l' = 3 \cdot \pi \cdot \mu \cdot v \cdot \frac{d_z}{\sqrt{C_k}} \tag{6.1}$$

Here and in the subsequent sequence of values, in which the grain shapes were taken into account, they are marked with the ′ sign.

Acting in the same way as in deriving Stokes' formula, we obtain the expression, which takes into account the shape of grains:

$$v'_{0_S} = \frac{g \cdot \sqrt{C_k} \cdot (\rho_z - \rho_c) \cdot d_z^2}{18 \cdot \mu} \tag{8.1}$$

FALLING VELOCITY OF NON-SPHERICAL LARGE GRAINS

Performing an analogical consideration as in chapter 3. for large grains which are subject mainly to the force of dynamic resistance, it should be observed that the character of this resistance is different from laminar. Especially its value depends upon the front surface area of the grain projective surface F_r – determined in formula (5). This value cannot be measured in relation to the grain falling in the liquid. As assistance we can use a certain phenomenon occurring during the falling of grains on which the dynamic resistance acts significantly. This is a so-called principle of maximum projection, mentioned in some works concerning the motion of solids in liquid media. This phenomenon was analysed in detail also by Sysło (1964).

Descriptively, it consists in the fact that the equilibrium of forces of resistance acting upon the grain leads to setting such spatial position of the falling grain that its projection surface is the largest among all the projections of this grain. Due to that it can be assumed that the projection value d_p (16) is a proper denotation of the grain size. It results from the definition of the shape coefficient C_{AN} (17) that it is connected with a disposable, assumed to be basic, substitute value by the expression

$$d_p = \frac{d_z}{\sqrt{C_{AN}}} \tag{19}$$

Acting as in deriving the modified Stokes, formula and substituting (19) into (5.1) we obtain its modified form

$$P'_d = \frac{\pi}{12 \cdot \sqrt{C_{AN}}} \cdot d_z^2 \cdot v^2 \cdot \rho_c \tag{5.2}$$

which, applied to derive Newton-Rittinger's formula, leads to its form:

$$v'_{0_{NR}} = \sqrt{2 \cdot g \cdot C_{AN}} \cdot \sqrt{\frac{(\rho_z - \rho_c) \cdot d_z}{\rho_c}} \tag{7.1}$$

GENERALIZED FORMULA FOR FALLING VELOCITY OF NON-SPHERICAL GRAINS

The transformed formulas (7.1) and (8.1) do not solve the problem of considering the grain shape in calculating the boundary falling velocity in a complete way. They do not cover the intermediate range, which is the most important for the needs of flow classification and in which the round flow of grains by the medium occurs at the values of Reynolds' number in the range (0.05) 0.1 ÷ 1000 (2000)[6].

Allen's formula (11), which is applied in this case, complicates greatly a possible repetition of the applied procedure to obtain the formula analogical to (7.1) and (8.1). In this situation this procedure was applied to derive a formula corresponding to Budryk's equation (10). After accepting formulas (5.) and (6.1) to be both components of resistance force and initial assumptions as in Budryk's formula, we achieve its form

$$v'_{0_B} = \frac{18 \cdot \mu}{\rho_c \cdot d_z} \cdot \frac{C_{AN}}{\sqrt{C_k}} \cdot \left(\sqrt{\frac{g}{162} \cdot \frac{C_k}{C_{AN}} \cdot \frac{(\rho_s - \rho_c) \cdot \rho_c}{\mu^2} \cdot d_z^3 + 1} - 1 \right) \tag{10.1}$$

In the works (Nowak, Sztaba 1986, Sztaba 1992) detailed remarks were given about the results of experiments performed to verify the usefulness of formula (10.1) To do this, the range of values of Reynolds' number 1 ÷ 1000 was selected. Quartzite of density 2640 kg·m^{-3} and grain size distribution 0.06÷5 mm was the tested material, scattered carefully into 20 narrow grain classes.

Average values of the following items were set for each class in precisely determined and controlled conditions:

- D_z – substitute value d_z,
- D_p – projection value d_p,
- $\overline{F}_z$ – surface of grains F_z
- $\overline{v}_r$ – real boundary velocity, marked here as v_r to be clearly distinguished, of falling in water at strictly controlled temperature and determining with the corresponding values ρ_c and μ from the tables.

On the basis of these data for each class the average values were calculated:

- $\overline{C}_k$ – sphericity C_k,
- $\overline{C}_{AN}$ – coefficient C_{AN}
- $\overline{v}_{0_B}$ – value of boundary falling velocity (for grains of size D_z) according to Budryk's formula for spherical grains (10),
- $\overline{v}_{r_B}$ – values of boundary falling velocities (for $D_z, \overline{C}_k, \overline{C}_{AN}$) according to formula (10.1),

[6] according to various authors and also the required calculation accuracy (numbers in brackets concern higher accuracy)

and also, respectively for $\bar{v}_{0_B}$ and $\bar{v}_{r_B}$, average values of Reynolds' number, not analysed in the present paper.

Figure 1 (Nowak, Sztaba 1976; Sztaba 1992) presents the results of performed investigations.

The figure shows the course of dependence between the mentioned values and values D_z. The use of logarithmic scale results from a large differentiation of grouping of measurement points for fine grains in relation to coarse ones. There is a clear improvement of consistency of calculation results of value $\bar{v}_{r_B}$ according to formula (10.1) as compared to $\bar{v}_{0_B}$, formula (10), without considering the grain shapes, with the real values of velocity $\bar{v}_r$. The presented graphs allow us only to formulate quantitative (descriptive) conclusions. To make them precise, the coefficients of linear regression (ρ_k) were calculated of the form:

$$Y = a \cdot X + b \tag{20}$$

obtaining

- for $Y = \bar{v}_{0_B}$, $X = \bar{v}_r$: a = 0.60555; b = -0.00178; ρ_k = 0.99638,
- for $Y = \bar{v}_{r_B}$, $X = \bar{v}_r$: a = 0.84961; b = 0.00234; ρ_k = 0.99944.

Supposing that in case of an ideal compatibility X and Y the presented values would be: a = 1, b = 0 (ρ_k = 1), we can speak about a significant improvement of accuracy of calculations (estimated mostly on the basis of the value of the coefficient a) when we take into account the shape coefficients as compared to the results obtained without taking them into consideration. In another, parallel, series of experiments, the following results were obtained, respectively:

- for $\bar{v}_0$ ($\bar{v}_{0_B}$): a = 0.53025; b = 0.00175,
- for $\bar{v}_r$ ($\bar{v}_{r_B}$): a = 0.82095; b = 0.00185

and other similar ones.

Fig. 1 presents also the courses of dependences of coefficients of shape (linear scale) on grain sizes, which reveal certain regularities indicating the need of further research on the methods of determining their values.

Coefficient C_k, decreasing very regularly with the growth of grains up to very small values (below 0.05), indicates that with the growth of grain sizes there are significant, undoubtedly overestimated, results of the measurements of specific surface resulting from the fact that the developed elements of the surface were included in it. In spite of applying the flow methods for the measurements of specific surface (in Tovaroff's apparatus) little sizes of the surface relief elements (in relation to the sizes of large grains) cause significant overestimations of measurement results and, respectively, the decrease of the value of C_k. On the other hand, a regular course of the discussed dependence is a proof of high accuracy of measurements as such, regardless the discussed limited adequacy of the real measured value to the assumed one.

Respectively, the course of the dependence C_{AN} on D_z reveals the presence of an interesting minimum but, at the same time, a large scatter of points confirms the necessity of significant improvement of the measurement accuracy to be obtained, including the projective diameter D_p.

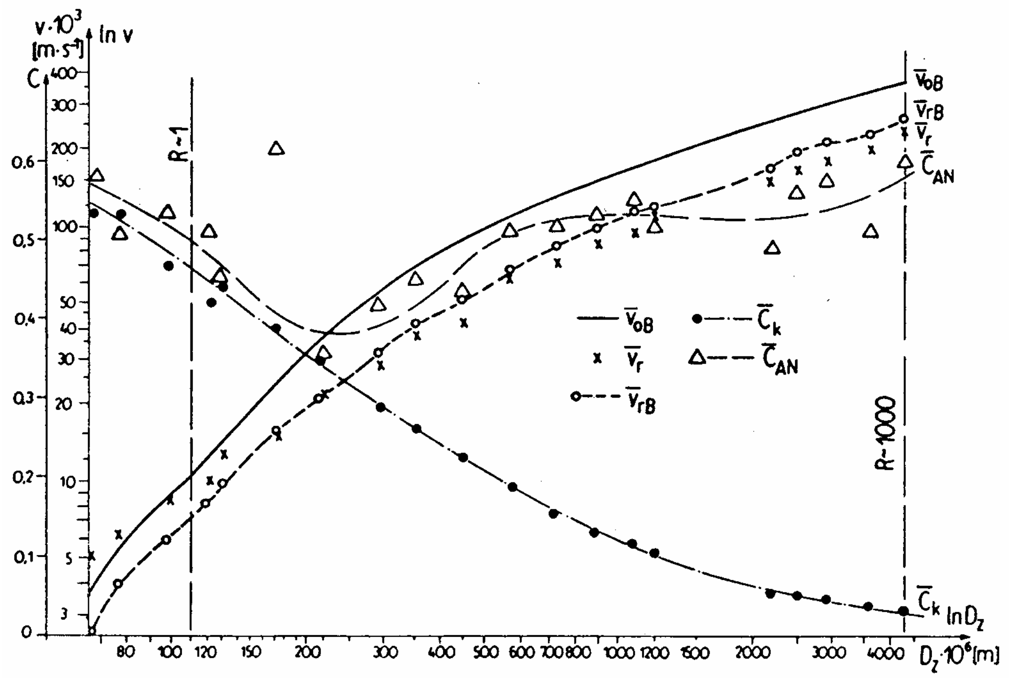

Fig. 1. The calculated and real boundary velocities of quartzite grains falling freely in water (explanations in the text; R ≡ Re), according to (Nowak, Sztaba 1986; Sztaba 1992)

In spite of this, the course of the discussed dependence is so much univocal that the final results, already discussed, can be treated in a half-quantitative way. It should be noted that the attempts of improvement of the results of calculations of boundary velocities, with taking into account the coefficients of shape and with the same experimental data, gave worse results or results showing a large scatter than the presented ones.

CONCLUSIONS

1. The investigations confirmed the correctness of the concept of applying several coefficients of shape in the calculations of boundary velocities of non-spherical grains. It seems to be inevitable, especially for the intermediate range (~1 < Re< ~1000).
2. When the initial assumptions were simplified and formalized, the improvement of the results of calculations of $\bar{v}_0$, experimentally confirmed, was obtained. This fact confirms the correctness of the assumed research, though obtaining a possibly total consistence of calculation results and experimental results need further investigations, which are being continued.
3. The methods of measurements determining the values of the coefficients of shape of grains should be perfected (average values of these coefficients in grain sets).
4. It was not planned to provide a general presentation of calculating the boundary velocity which is a special case of grain characteristic velocity ($v_c = v_c(t)$; $v_0 = v_c(\infty)$) in relation to the medium and which is not discussed. The aim of this paper was to present the title possibility of improving the results of calculating the boundary velocity. The author assumed the oldest and most popular methods of calculating this velocity by means of formulas based on the formally considered balance of basic forces acting upon the grain in the liquid medium to constitute the presentation object. The development of the method as well as its applications for the method of calculating the grain velocity in the medium, based upon the most contemporary knowledge about the principles of flow processes are the subject-matter of further investigations.

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Problem określenia teoretycznie uzasadnionej metody uwzględniania naturalnego kształtu ziarn mineralnych w obliczeniach prędkości ich ruchu w ośrodkach płynnych – zwłaszcza ich opadania pod wpływem zewnętrznej siły masowej, w szczególności siły ciężkości – nie znalazł dotychczas w pełni zadowalającego rozwiązania ogólnego. Prezentowane opracowanie przedstawia możliwość zwiększenia dokładności obliczeń tej prędkości z zastosowaniem znanych wzorów – omówionych we wstępie opracowania – drogą odpowiedniego wprowadzenia do nich dwóch różnych współczynników kształtu ziarna.