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Jan DRZYMAŁA*

ATLAS OF UPGRADING CURVES USED IN SEPARATION AND MINERAL SCIENCE AND TECHNOLOGY

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The atlas presents the existing in scientific and technical literature upgrading curves relating quality and quantity of products of separation for a given feed quality. The upgrading curves were classified into groups including $A_/$ (α-insensitive curves, triangle area accessible for plotting), A_o (α-insensitive curves, square area available for plotting), $B_/$ (α-sensitive curves with triangle plotting area, B_o (α-sensitive curves, having square plotting area), $C_/$ (α-insensitive curves, for $\beta > \alpha$ or $\beta < \alpha$ triangle area), and C_o (α-insensitive curves, for $\beta > \alpha$ or $\beta < \alpha$, square area). Other classifications are also possible. It was presented in the atlas that the shape of the upgrading curve depends on the upgrading parameters used for plotting but they contain and reflect the same information given in a specific, for each curve, way. The applicability of each upgrading curve depends on the needs of the user and personal preferences. An appropriate matching an upgrading curve with a set of separation results allows to approximate the curve with a simple mathematical formula which can be used in other applications. Since the possible number of separation curves is infinite, there is a need for collecting known upgrading curves and creating new ones. The readers are kindly asked to report, not mentioned in this atlas, upgrading curves to jan.drzymala @pwr.wroc.pl

Key words: separation, upgrading, enrichment, recovery, yield, effectiveness, efficiency

INTRODUCTION

Separation relies on splitting an initial material (feed) into two or more smaller portions in a real or virtual way. The final separation takes place due to splitting forces operation in the system. The separation can be real or virtual, selective or non-selective, etc. During the separation additional forces such as ordering, disordering, neutral, etc. can operate in the system (Fig. 1).

The separation systems may contain one or more components. The components of a separation system have numerous features such as size, density, hydrophobicity,

* Wrocław University of Technology, Wybrzeże Wyspianskiego 27, 50-370 Wrocław, Poland, jan.drzymala@pwr.wroc.pl.

magnetic susceptibility, etc. Certain features of the components are utilized for separation. The features and components of a separation system are interrelated and form a fractal-like structure (Fig.2).

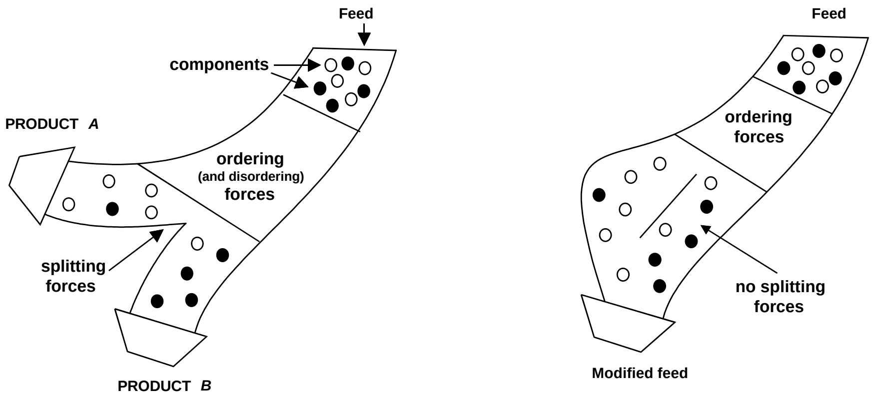


Fig. 1. Elements of separation process. Real separation (a) and virtual separation (b)

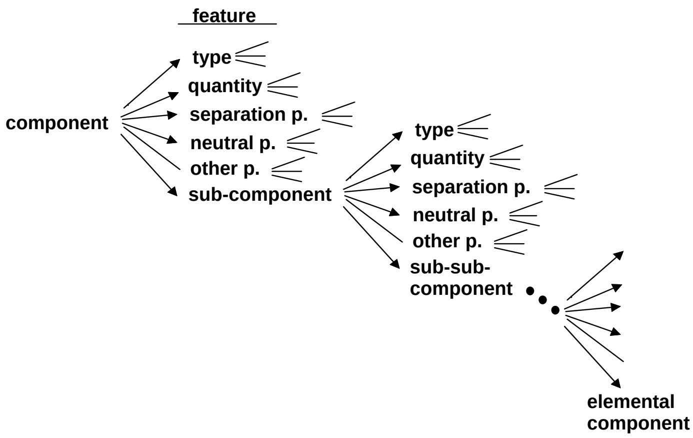


Fig. 2. Fractal-like structure of components and their features

Combining the features of a separation system into pairs provides different approaches that can be used for analyzing separation systems (Fig. 3) including for instance upgrading, classification, sorting, etc. The upgrading takes into account the quality and quantity of products. They can be considered either alone or combined together as well as combined with the feed quality. In other words the upgrading utilizes quantity of products (γ_j), and qualities expressed as content of components in products (β_{ij}), and the content of components in the feed (α_i) where *i* stands for component and *j* for product and they assume values 1,2,3…. Combinations of α, β, γ. can also be used. Thus, the starting parameters for analyzing separation as upgrading are α, β, γ. These parameters can be combined to form new parameters which equally well, as the original ones, characterize the process. New parameters created with α, β, γ are for instance recovery ($\varepsilon=\gamma\beta/\alpha$) or enrichment ratio $K=\beta/\alpha$. The number of parameters resulting from combinations of α, β, γ is infinite. Therefore, the number of possible upgrading curves is also infinite. They represent the same data but in a different esthetical and graphical form. Their usefulness depends to a great extent on personal preferences. Thus, there is a need to create an atlas of the upgrading curves and classify the upgrading curves. Such an attempt was undertaken and the results are presented in this work.

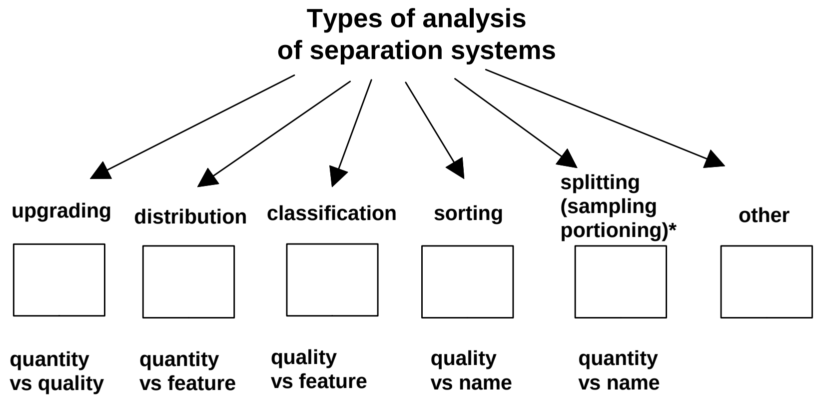


Fig. 3. Possible ways of analyzing separation results

Customarily, the upgrading curves are named after the person who first used the curve. In this atlas only the most known separation curves will be presented. Other curves will be collected in a second part of this atlas in the future. Their arrangement is based on a classification given below. It is recommended to plot in an upgrading curves not only the results of real separation as the real separation line but also the ideal separation (for a complete liberation), ideal mixing, and no upgrading lines (or points). Other lines, for instance the upgradeability that is the maximum possible upgrading for a given liberation, are also possible. The upgrading curve can be given either in a non-cumulative or cumulative way. In this atlas only cumulative upgrading curves, as being more universal, are considered. When the shape of the curves is identical, they bear the same with different Latin numerals.

CLASSIFICATION OF UPGRADING CURVES

Since there is an infinite number of upgrading parameters and curves, their classification can be accomplished in a great number of ways. Tentatively, until a more sophisticated way is designed, in this atlas we will used classification given in Table 1.

Table 1. Classification of upgrading curves utilized in this work

Group symbol	Description, sensitivity to variation of α and area available for plotting	Examples
$A_/$	α-insensitive curves, triangle area	Fuerstenau, Luszczkiewicz
A_o	α-insensitive curves, square area	not known
$B_/$	α-sensitive curves, triangle area	Henry I, II, III Mayer I, II, III (Dell) Holland-Batt (β) Holland-Batt H (Hancock) beta-beta
B_o	α-sensitive curves, square area	Halbich, Stępiński I, II, III, IV,
$C_/$	α-insensitive curves, for β >α, triangle area	not known
C_o	α-insensitive curves, for β >α, square area	Stępiński V, Hall

UPGRADING BALANCE

For plotting the upgrading curves hypothetical results of separation were considered. The balance of upgrading in given in Table 2. Only the principal parameters, that is γ, β, α and some selected combined upgrading parameters (K=β/α, and ε= γβ/α) are included in the table. If no subscripts *i, j* at symbols γ, β, α in the table and in the figures are given, it means that the subscript is either 1 or 1,1 (*i*=1 means component 1; *j*=1 means product 1). Sometimes instead $\beta_{1,2}$ symbol ϑ is used which denotes ϑ_1 that is remaining (2) product (tailing).

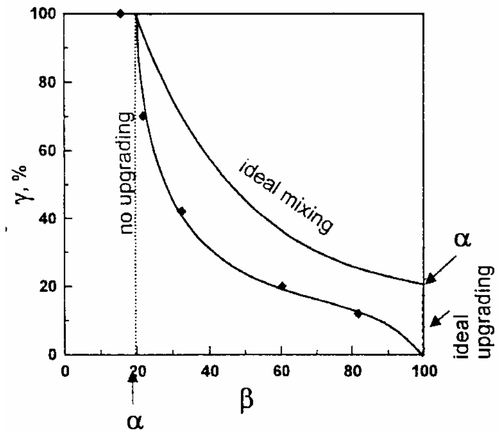
Table 2. Upgrading balance of a hypothetical separation

Product	Yield, γ(%)	Content, β, %	Upgrading ratio $K = \beta/\alpha$	Recovery $\varepsilon=\gamma\beta/\alpha$, %
Concentrate K_1	12.06	81.70	5.305	63.98
Concentrate K_2	20.14	60.40	3.922	79.01
Concentrate K_3	42.27	32.44	2.106	89.07
Concentrate K_4	70.14	21.73	1.411	98.93
Tailing	29.86	0.52	0.0338	1.01
Feed	100.00	15.40=α	1	100.00

ATLAS OF UPGRADING CURVES

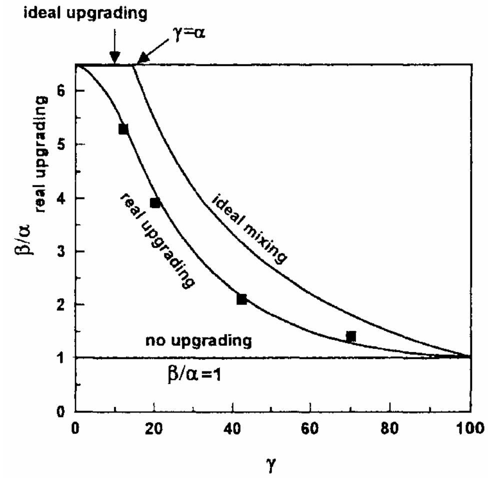
Upgrading curves that can be encountered in technical and scientific papers on separation are given in Figs 4-6. They are presented in groups according to the classification given in Table 1. The upgrading curves belonging to group B are presented in Fig. 4.

Henry curve
Henry, 1905; Reinhardt, 1911



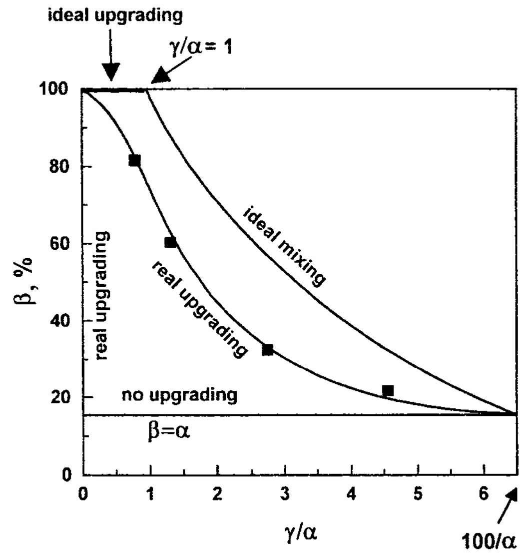
A

Henry II (enrichment ratio) curve
Holland–Batt, 1985



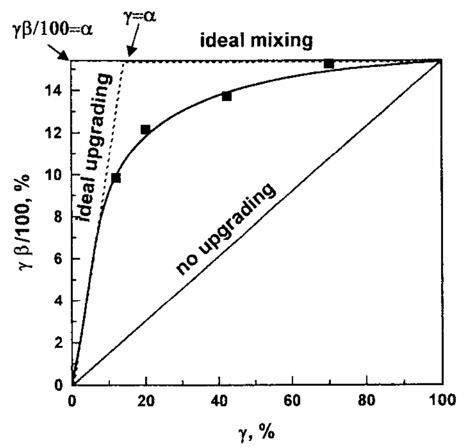
B

Henry III curve
(no references available)



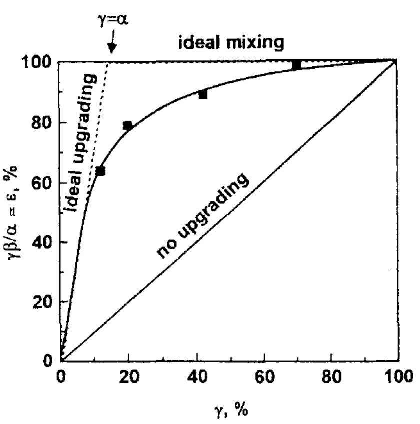
C

Mayer I (original) curve
(Mayer, 1950, 1951, 1952a, 1952b; Stępiński, 1952, 1964, 1965; Tarjan 1981)



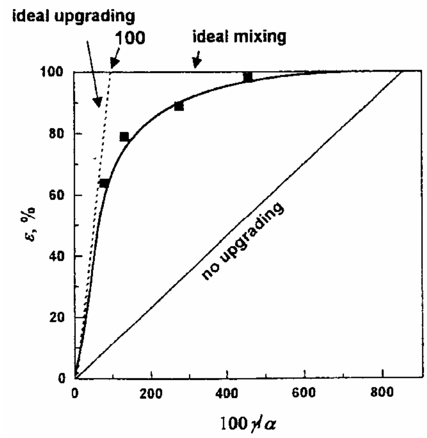
D

Mayer II curve
(Stępiński, 1952, 1964, 1965,
Nixon and Moir, 1956/7)



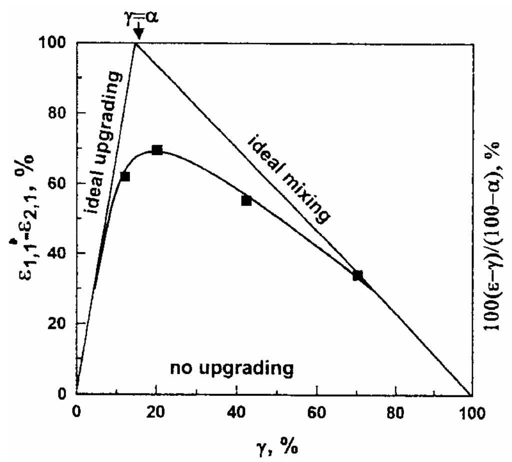
E

Mayer III (Dell curve)
Dell, 1953, 1961, 1969, 1972



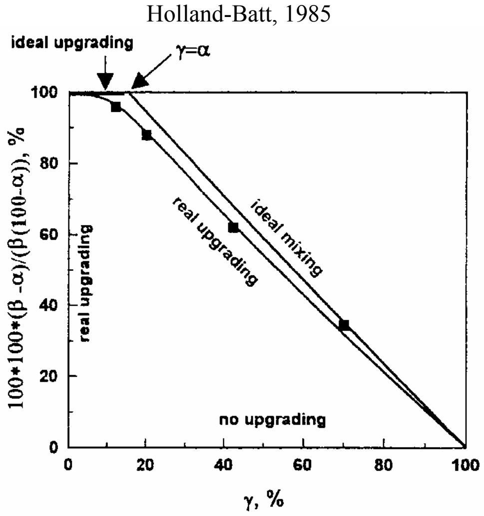
F

Holland-Batt (Hancock parameter) curve
Holland-Batt, 1985



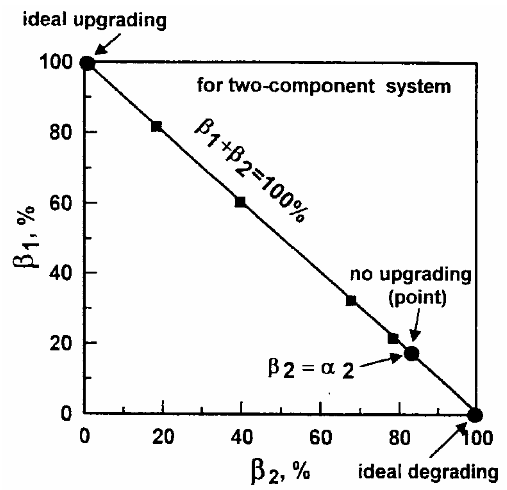
G

Holland-Batt (β) curve
Holland-Batt, 1985



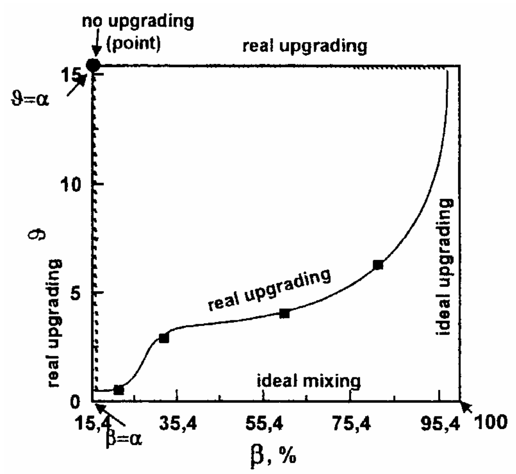
H

beta-beta curve
Hall, 1971



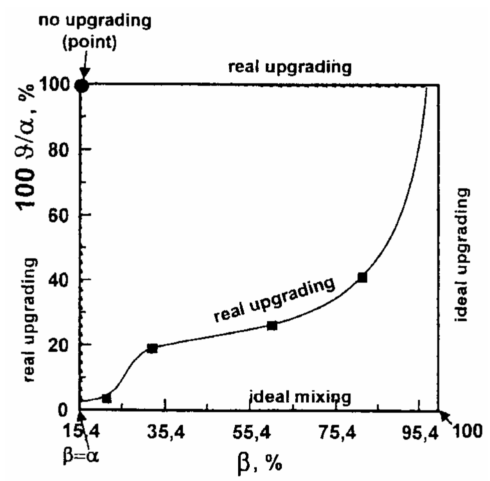
I

Stępiński I curve
Stępiński, 1955, 1958; Pudło, 1957



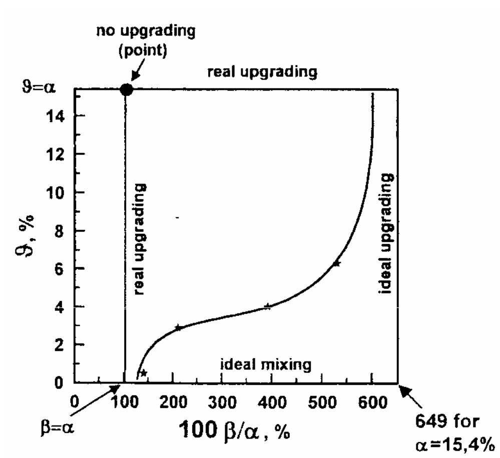
J

Stępiński II curve
(this work, based on Pudło I curve)



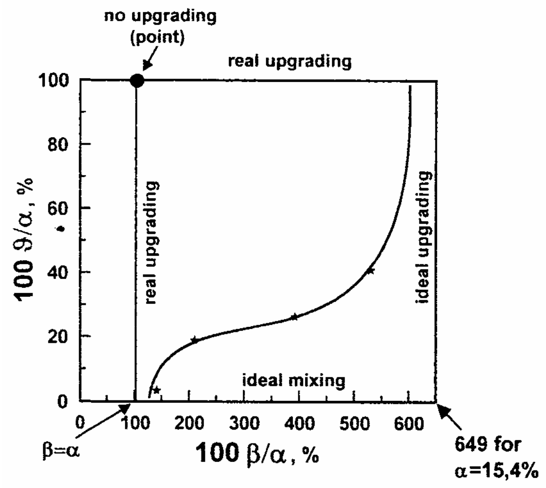
K

Stępiński III curve
(this work, based on Stępiński I)



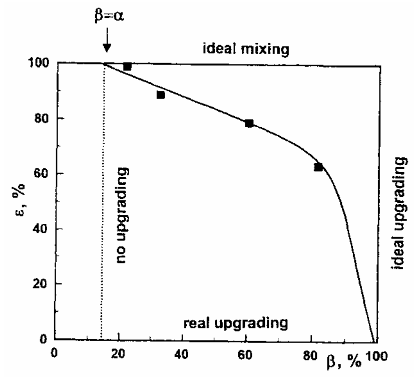
L

Stępiński IV curve
Pudło, 1971



M

Halbich curve
Halbich, 1934

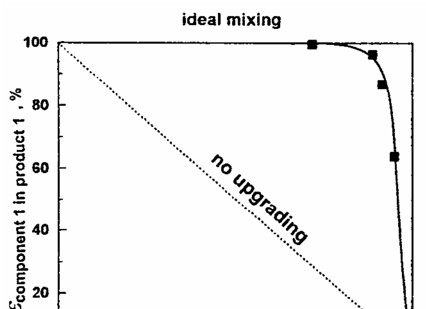


N

Fig. 4. Type $B_{/}$ (α-sensitive with triangle or near triangle area available for plotting) upgrading curves (A-I and type B_0 upgrading curve which are α-sensitive curves, square area, J-N)

The upgrading curves belonging to group A are presented in Fig. 4.

Fuerstenau curve
Fuerstenau et al., 1988/92



Luszczkiewicz curve
Luszczkiewicz, 2002

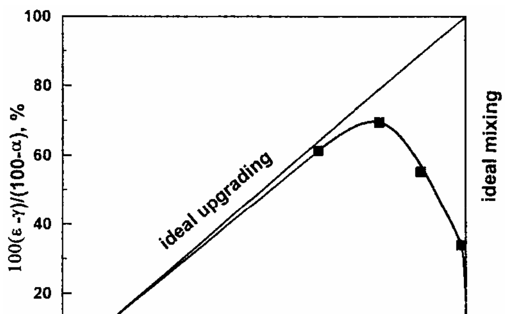
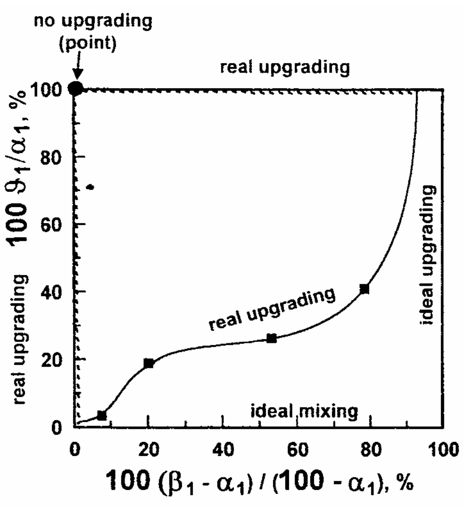


Fig. 5. Type $A_{/}$ upgrading curves which are α-insensitive curves and offer a triangle area for plotting

The upgrading curves belonging to group C are presented in Fig. 6.

Stępiński V curve
Drzymała, 2005



Hall curve
Hall, 1971 (a similar curve shape is for 100 (100-β)/(100-α)

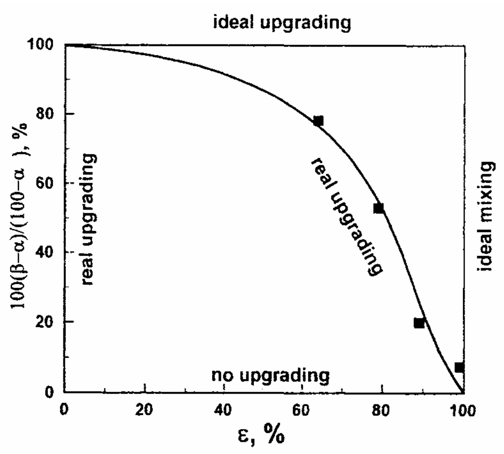


Fig. 6. C_o type of upgrading curve (α-insensitive curves, for $\beta > \alpha$, square area)

The presented in this atlas upgrading curves represent a small number of all possible plots. In a next publication additional upgrading curves created to extend the list of available plots will be offered.

The author of this atlas asks all interested in a further development of the atlas to submit new and omitted upgrading curves to make the list more complete. The curves will be collected and published as a next part of this publication and later on the internet. The upgrading curves should meet the following standards: be cumulative and contain lines (or points) of real, no, and ideal separation lines, and if possible, the ideal mixing line. The curve will be named after the author of the curve. The propositions can be submitted either by e-mail to jan.drzymala@pwr.wroc.pl or sent by post service to Jan Drzymala, Wroclaw University of Technology, Mining Engineering Department, 50-370 Wroclaw, Poland.

CONCLUSIONS

There are many available upgrading curves. Their shape depends on the upgrading parameters used for plotting but they contain and reflect the same information, though in a specific for each curve way. Their applicability seems to depend on the needs of the user and personal preferences. The possible number of separation curves is infinitive. They characterize well separation process in contrast to single separation parameters which do not fulfill that role.

LIST OF SYMBOLS

γ- yield of a product; yield of product 1; yield of concentrate, %,
γ_j – yield of product *j*, where *j* is 1, 2, 3 …, % ,
β - content of a component in a product; content of component 1 in product 1, content of useful component in concentrate, %,
$\beta_{i,j}$ - content of component *i* in product *j*, where *i* is 1, 2, 3 …and *j* is 1, 2, 3 … , %,
α - content of a component in the feed; content of component 1 in the feed, content of useful component in the feed, %,
α_i - content of component *i* in the feed, where *i* is 1, 2, 3 …, %,
ϑ - content of component 1 in product 2 (tailing or rest of material), same as $\beta_{1,2}$, %,
K - enrichment (upgrading) ratio ($K=\beta/\alpha$), 100K gives enrichment ratio in %,
ε - recovery ($\varepsilon=\gamma\beta/\alpha$) of a component in a product, recovery of component 1 in product 1, recovery of component 1 in concentrate, %,
$\varepsilon_{i,j}$ – recovery of component *i* in product *j*, where *i* is 1, 2, 3 …and *j* is 1, 2, 3 … , %

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Atlas zawiera znane z literatury naukowej i technologicznej krzywe wzbogacania przedstawiające zależność jakości produktów separacji od ich ilości dla danej jakości nadawy. Krzywe wzbogacania zostały sklasyfikowane na grupy: $A_{/}$ (nieczułe na zawartość składników w nadawie z trójkątnym obszarem dostępnym do kreślenia krzywych), A_o (nieczułe na zawartość składników w nadawie z kwadratowym obszarem dostępnym do kreślenia), $B_{/}$ (czułe na zawartość składników w nadawie z trójkątnym obszarem dostępnym do kreślenia), B_o (czułe na zawartość składników w nadawie z kwadratowym obszarem dostępnym do kreślenia), $C_{/}$ (nieczułe na zawartość składników w nadawie dla $\beta > \alpha$ oraz $\beta < \alpha$, obszar trójkątny), oraz C_o (nieczułe na zawartość składników w nadawie α dla $\beta > \alpha$ oraz $\beta < \alpha$, obszar kwadratowy), gdzie β oznacza zawartość składnika w koncentracie a α w nadawie. Istnieją jeszcze inne możliwe podziały krzywych wzbogacania. W pracy pokazano, że kształt krzywych wzbogacania zależy od parametrów wzbogacania użytych do ich kreślenia i zawierają one te same informacje lecz w innej specyficznej dla danej krzywej formie graficznej, a stosowalność wybranej krzywej wzbogacania zależy od potrzeb użytkownika i osobistych preferencji. Odpowiednie skojarzenie krzywej wzbogacania z danymi pomiarowymi pozwala na aproksymacje krzywych odpowiednimi równaniami matematycznymi, które mogą być użyteczne do innych aplikacji. Ponieważ liczba możliwych krzywych separacji jest nieskończona, istnieje potrzeba zebrania znanych krzywych wzbogacania i scharakteryzowania nowych. Czytelnicy tej publikacji proszeni są o nadsyłanie nieopisanych dotąd krzywych wzbogacania pod adres: jan.drzymala @pwr.wroc.pl