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ATLAS OF UPGRADING CURVES USED IN SEPARATION AND MINERAL SCIENCE AND TECHNOLOGY
Part II

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The present Atlas (Part II) contains 12 new less known upgrading curves which relate quality and quantity of products of separation for a given feed quality α. Part II of the Atlas supplements the list of known upgrading curves presented in Part I. The classification of the upgrading curves used in this work is the same as previously. Group $A_/$ covers upgrading α-insensitive curves with triangle or near triangle area accessible for plotting, A_o: α- insensitive curves, square area available for plotting, $B_/$: α-sensitive curves with triangle plotting area, B_o: α-sensitive curves, having square plotting area, $C_/$: α-insensitive curves for β >α triangle area, and C_o: α-insensitive curves for β >α or β <α, square area, where β stands for content of a component in concentrate while α in the feed. It was emphasized in the paper that all upgrading curves contain the same information but in a different, specific for a given curve form. The use of upgrading curves depends on the needs and preferences of the user. An appropriate matching of upgrading plot with a set of separation results allows to approximate the curve with a suitable mathematical formula which can be used for characterizing separation. Additional curves will be present in Part III of the Atlas. The readers are kindly asked to report unmentioned upgrading curves to jan.drzymala @pwr.wroc.pl.

Key words: separation, upgrading, recovery, yield, efficiency

INTRODUCTION

Splitting an initial material (feed) into two or more portions in a real or virtual way is the essence of separation. The separation is possible due to ordering and splitting forces operating in the system. Depending on the character of forces, the separation can be real or virtual, selective or non-selective, etc.

The results of separation are usually presented in tabular and graphical forms. There is infinitive number of separation curves. When quantity of separation products

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and their quality are considered, such approach is called upgrading. There is infinitive number of upgrading curves. They are based on three principles parameters: quantity of products (γ), quality (usually expressed as content of a components in a product (β), and the content of a component in the feed (α). Different combinations of α, β, and γ provide new parameters which equally well, as the original ones, characterize the separation process as upgrading. New parameters created with α, β, γ are for instance recovery ($\varepsilon=\gamma\beta/\alpha$) or enrichment ratio $K=\beta/\alpha$. Pairs of upgrading parameters provide upgrading curves which represent the same data but in a different esthetical and graphical form. The usefulness of a given upgrading curve depends, to a great extent, on personal preferences.

Eighteen upgrading curves were presented and discussed in Part I of Atlas of Upgrading Curves (Drzymala, 2006). The present paper continues the effort to present the most important and useful upgrading curve existing in literature. Some upgrading curves have never been used before. It is easy to create new upgrading curves because there many selectivity indices in literature which can be combined into pairs and plotted as separation curves.

Complete plots with upgrading curves should contain lines (or points) of real, ideal, remixing, and no upgrading. When the shape of two curves is identical, they bear the same name but differ in Latin numerals.

UPGRADING BALANCE

For plotting upgrading curves, the same as in Atlas Part I (Drzymala, 2006) hypothetical results of separation were considered (Table 1). It was assumed that the feed contains only two components, that is component 1 and component 2 (rest of material). Only principal parameters, that is feed grade (α), yield of products (γ), content of component 1 (β_1), ($\beta_2=100\% - \beta_1$), and recoveries of both components are presented. Other parameters can be calculated using the formulas given in the axes of the upgrading curves. Details regarding calculation of parameters used for plotting the three MDTW upgrading curves are given in the appendix.

Table 1. Upgrading balance of a hypothetical separation. The data were used for calculation of upgrading curves

Product	Yield, γ (%)	Content of component 1 β_1, %*	Recovery of component 1 $\varepsilon=\gamma\beta/\alpha$, %	Recovery of component 2 $\varepsilon=\gamma\beta/\alpha$, %
K_1	12.06	81.70	64.00	2.00
K_1+K_2	20.14	60.40	79.01	9.43
$K_1+K_2+K_3$	42.27	32.44	89.07	33.71
$K_1+K_2+K_3+K_4$	70.14	21.73	98.99	63.92
Tailing	29.86	0.52	1.01	36.08
Feed	100.00	15.40=α	100.00	100.00

*content of component 2 (β_2, %) is equal to: $\beta_2 = 100 - \beta_1$

UPGRADING CURVES

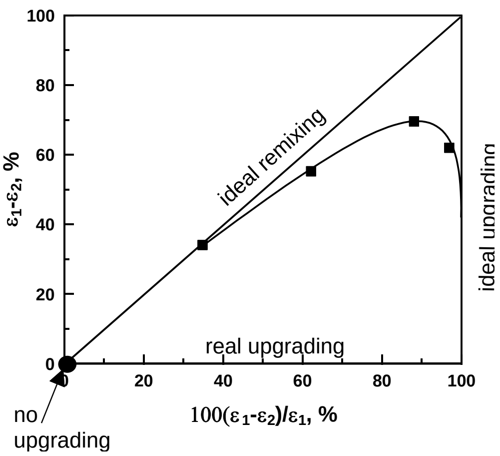
In the previous paper the upgrading curves were classified into three categories: A (α-insensitive), B (α-sensitive), C (α-insensitive but covering limited range of variables). The same classification is shown in Table 2 along with names of upgrading curves discussed in the Atlas (Part I and II). The upgrading curves are shown in Figs 1-3.

Table 2. Upgrading curves considered in the Atlas (Part I and this work Atlas Part II)

Group symbol	Description, sensitivity to variation of α and area available for plotting	Examples Atlas Part I	Examples Atlas Part II
$A_/$	α-insensitive curves, triangle area	Fuerstenau Luszczkiewicz	Correa et al. Fomienko 1 Fomienko 2
A_o	α-insensitive curves, square area	-	Correa Bieloglazov I, II
$B_/$	α-sensitive curves, triangle area	Henry I, II, III Mayer I, II, III (Dell) Holland-Batt β Holland-Batt H (Hancock) β-β	MDTW Hu Wei bai
B_o	α-sensitive curves, square area	Halbich, Stepinski I, II, III, IV,	Halbich II, III
$C_/$	α-insensitive curves, for β >α, triangle area	not known	MDTWc
C_o	α-insensitive curves, for β >α, square area	Drzymala (Stepinski V) Hall	MDTWr

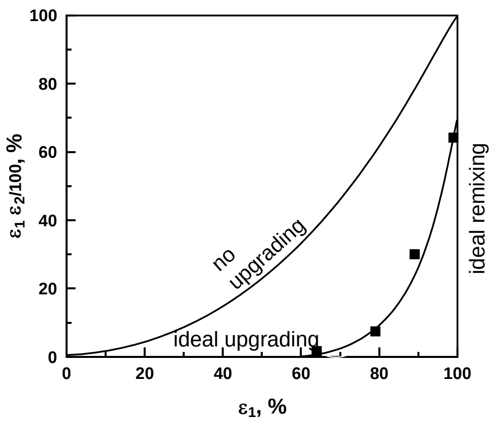
Fig. 1 A-F

Correa et al., 2007



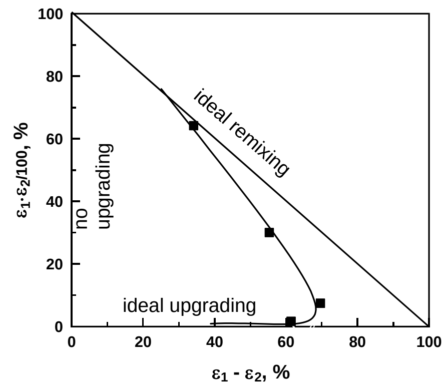
A

This work
based on Fomienko (1957) selectivity index $\varepsilon_1\varepsilon_2$



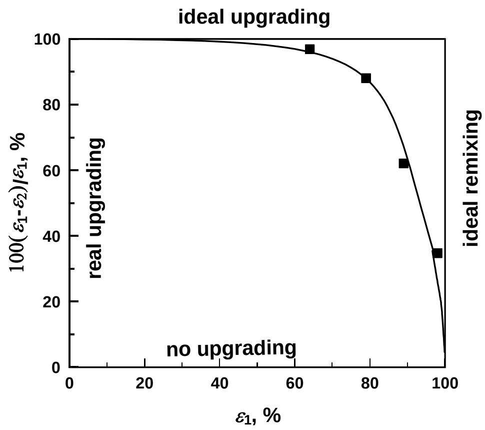
B

This work
based on Fomienko (1957) $\varepsilon_1\varepsilon_2$ and Hancock ε_1 - ε_2 selectivity index



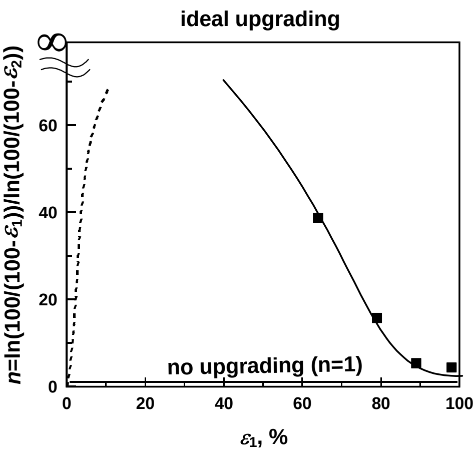
C

Correa, 2007, unpublished



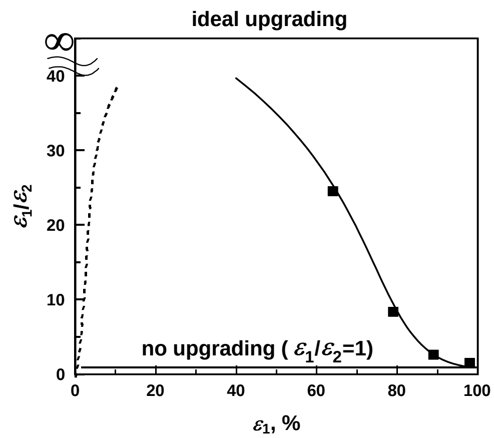
D

This work (Bieloglazov I)
based on selectivity index *n* (Bieloglazov, 1947; Petrova and Boteva, 2006)



E

This work (Bieloglazov II)
based on selectivity index $\varepsilon_1/\varepsilon_2$ used by Ulewicz et al., 2001



F

Fig. 1. Type A upgrading curves which are α-insensitive and offer either triangle-type (A, B, C) or square-type areas for plotting

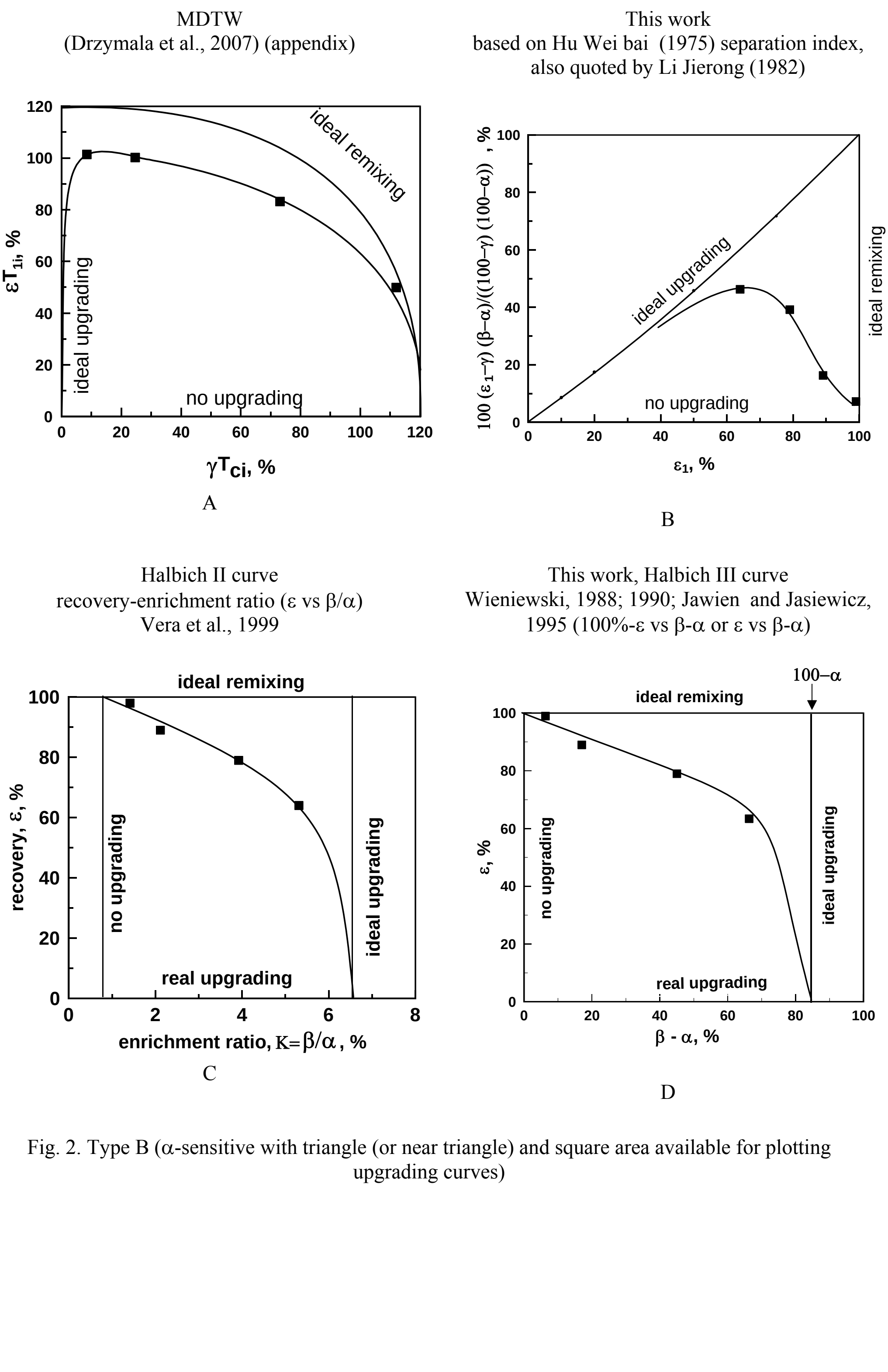


Fig. 2. Type B (α-sensitive with triangle (or near triangle) and square area available for plotting upgrading curves)

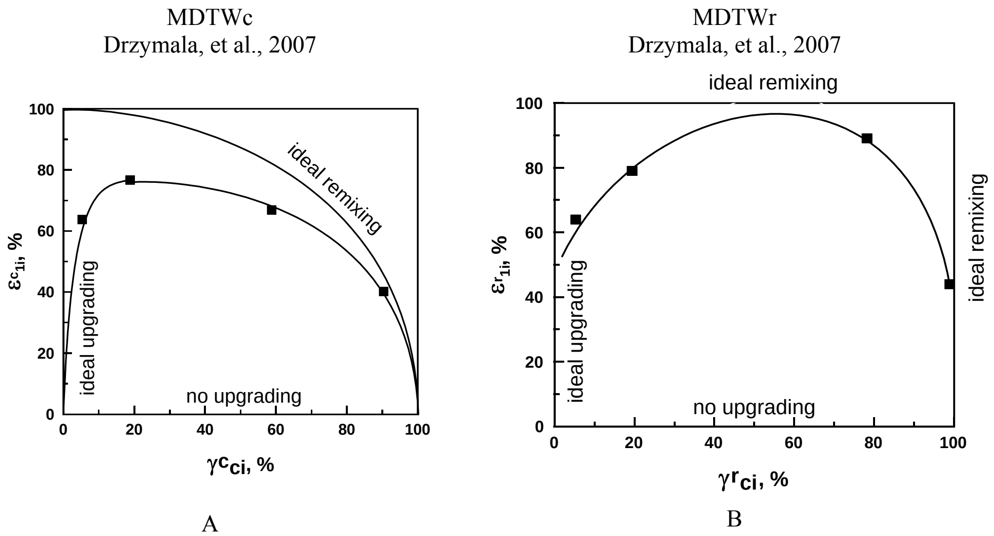


Fig. 3. C type of upgrading curve (α-insensitive curves, for $\beta > \alpha$, square or triangle area available for plotting)

CONCLUSIONS

Present Atlas of upgrading curves still represent a small number of possible upgrading curves. Therefore Part III of the Atlas will be prepared for publication in 2008. It will contain complex upgrading curves as well as other which exist in literature but have been so far mentioned, for many reasons, in the Atlas.

Appendix. Calculations of parameters needed for plotting MDTW (Mayer-Drzymala-Tyson-Wheelock) upgrading curves

All details regarding calculations of selectivity indices $\varepsilon^{T}_{1i}, \varepsilon^{T}_{1i}, \varepsilon^{r}_{1i}, \gamma^{T}_{ci}, \gamma^{c}_{ci}$, and γ^{r}_{ci} can be found in the original paper by Drzymala et al. (2007). The needed equations are given in Table 3. Starting numbers are yield γ and recovery ε.

1. Use equation 4 to calculate θ_o for each test.
2. Use $\tan\theta_i = \sum\varepsilon_{1i} / \sum\gamma_{ci}$ to calculate θ_i for each experimental point.
3. Use equation 3 to calculate θ^{T}_{i} for each experimental point.
4. Use equations 1 and 2 to determine $\sum\gamma^{T}_{ci}$ and $\sum\varepsilon^{T}_{1i}$, respectively.
5. Given $\sum\varepsilon^{e}_{1i} = 100$, calculate $\sum\varepsilon^{eT}_{1i}$ by using equation 9.

6. Calculate $\sum \gamma_{ci}^{eT}$ by using the following combination of equations 5, 7 and 8: $\sum \gamma_{ci}^{eT} = 100 \cos\theta_i^T / \sin\theta_i$.
7. For circular normalization apply equations 10 and 11 to determine $\sum \varepsilon_{1i}^{c}$ and $\sum \gamma_{ci}^{c}$, respectively.
8. For rectangular normalization and $\theta_i^T > 45^o$, use equations 12 and 13 to determine $\sum \varepsilon_{1i}^{r}$ and $\sum \gamma_{ci}^{r}$, respectively.
9. For rectangular normalization and $\theta_i^T < 45^o$, use equations 14 and 15 to determine $\sum \gamma_{ci}^{r}$ and $\sum \varepsilon_{1i}^{r}$, respectively.

Table 3. Equation needed for calculations of parameters for the MDTW, MDTWr and MDTWc upgrading curves. After Drzymala et al., 2007

$$\gamma_{ci}^{T} = \gamma_{ci} \frac{\cos\theta_i^T}{\cos\theta_i} \quad (1)$$

$$\varepsilon_{1i}^{T} = \varepsilon_{1i} \frac{\sin\theta_i^T}{\sin\theta_i} \quad (2)$$

$$\theta_i^T = \frac{90^o(\theta_i - 45^o)}{\theta_o} \quad (3)$$

$$\tan(\theta_o + 45^o) = \frac{100}{\alpha_1} \quad (4)$$

$$\gamma_{ci}^{e} = R_i^e \cos\theta_i \quad (5)$$

$$\varepsilon_{1i}^{e} = R_i^e \sin\theta_i \quad (6)$$

$$R_i^e = \varepsilon_{1i}^{e} / \sin\theta_i = 100 / \sin\theta_i \quad (7)$$

$$\gamma_{ci}^{eT} = \frac{\gamma_{ci}^{e} \cos\theta_i^T}{\cos\theta_i} \quad (8)$$

$$\varepsilon_{1i}^{eT} = \frac{\varepsilon_{1i}^{e} \sin\theta_i^T}{\sin\theta_i} \quad (9)$$

$$\varepsilon_{1i}^{c} = \frac{R_i^c \sin\theta_i^T \varepsilon_{1i}^T}{\varepsilon_{1i}^{eT}} = \frac{100\sin\theta_i^T \varepsilon_{1i}^T}{\varepsilon_{1i}^{eT}} \tag{10}$$

$$\gamma_{ci}^{c} = \frac{R_i^c \cos\theta_i^T \gamma_{ci}^T}{\gamma_{ci}^{eT}} = \frac{100\cos\theta_i^T \gamma_{ci}^T}{\gamma_{ci}^{eT}} \tag{11}$$

$$\varepsilon_{1i}^{r} = \varepsilon_{1i}^{er} \frac{\varepsilon_{1i}^T}{\varepsilon_{1i}^{eT}} = 100 \frac{\varepsilon_{1i}^T}{\varepsilon_{1i}^{eT}} \tag{12}$$

$$\gamma_{ci}^{r} = \frac{\varepsilon_{1i}^{er} (\gamma_{ci}^T)^2}{\varepsilon_{1i}^T (\gamma_{ci}^{eT})} = \frac{100(\gamma_{ci}^T)^2}{\varepsilon_{1i}^T (\gamma_{ci}^{eT})} \tag{13}$$

$$\gamma_{ci}^{r} = \gamma_{ci}^{er} \frac{\gamma_{ci}^T}{\gamma_{ci}^{eT}} = 100 \frac{\gamma_{ci}^T}{\gamma_{ci}^{eT}} \tag{14}$$

$$\varepsilon_{1i}^{r} = \frac{\gamma_{ci}^{er} (\varepsilon_{1i}^T)^2}{\gamma_{ci}^T (\varepsilon_{1i}^{eT})} = \frac{100(\varepsilon_{li}^T)^2}{\gamma_{ci}^T (\varepsilon_{1i}^{eT})} \tag{15}$$

Results of transformation and normalization for results of separation from Table 1are given in Table 4. The data were used for plotting three ugrading curves: MDTW (Fig. 2A), circular MDTWc (Fig. 3A) and rectangular MDTWr (Fig. 3B)

Table 4. Results of transforming the coordinate system followed by either circular or rectangular normalization. α=15.40%

Pro-duct	$\Sigma\varepsilon_{1i}$ (%)	$\Sigma\gamma_{ci}$ (%)	θ_i deg.	θ_i^T deg.	$\Sigma\varepsilon_{1i}^T$ (%)	$\Sigma\gamma_{ci}^T$ (%)	$\Sigma\varepsilon_{1i}^{eT}$ (%)	$\Sigma\gamma_{ci}^{eT}$ (%)	$\Sigma\varepsilon_{1i}^c$ (%)	$\Sigma\gamma_{ci}^c$ (%)	$\Sigma\varepsilon_{1i}^r$ (%)	$\Sigma\gamma_{ci}^r$ (%)
K_1	64.0	12.1	79.3	85.2	64.9	5.4	101.4	8.4	63.8	5.3	64.0	5.3
$K_{1,2}$	79.0	20.1	75.7	76.2	79.2	19.4	100.2	24.6	76.7	18.8	79.0	19.4
$K_{1,2,3}$	89.1	42.3	64.6	48.7	74.1	65.0	83.2	73.1	66.9	58.8	89.1	78.2
$K_{1,2,3,4}$	98.9	70.1	54.7	24.0	49.3	110.7	49.9	112.0	40.2	90.4	44.0	98.9

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Atlas przedstawia 12 różnych mniej znanych krzywych wzbogacania. Wiążą one jakość produktów separacji od ich ilości. Cześć II Atlasu uzupełnia listę znanych krzywych wzbogacania zawartą w części I Atlasu. Utrzymano klasyfikację krzywych wzbogacania na grupy: $A_/$ (nieczułe na zawartość składników w nadawie z trójkątnym obszarem dostępnym do kreślenia krzywych), A_o (nieczułe na zawartość składników w nadawie z kwadratowym obszarem dostępnym do kreślenia), $B_/$ (czułe na zawartość składników w nadawie z trójkątnym obszarem dostępnym do kreślenia), B_o (czułe na zawartość składników w nadawie z kwadratowym obszarem dostępnym do kreślenia), $C_/$ (nieczułe na zawartość składników w nadawie dla $\beta > \alpha$ oraz $\beta < \alpha$, obszar trójkątny), oraz C_o (nieczułe na zawartość składników w nadawie α dla $\beta > \alpha$ oraz $\beta < \alpha$, obszar kwadratowy), gdzie β oznacza zawartość składnika w koncentracie a α w nadawie. W pracy podkreślono ze wszystkie krzywe wzbogacania zawierają te same informacje lecz SO_4 podane w innej, specyficznej dla danej krzywej, formie graficznej. Ich stosowalność zależy od potrzeb użytkownika i osobistych preferencji. Odpowiednie skojarzenie krzywej wzbogacania z danymi pomiarowymi pozwala na aproksymacje krzywych odpowiednimi równaniami matematycznymi, użyteczne do opisu separacji. Dalsze krzywe będą podane w Części III Atlasu następnego wydania tego czasopisma. Autor prosi o nadsyłanie nieopisanych dotąd krzywych wzbogacania pod adres: jan.drzymala @pwr.wroc.pl