Physicochemical Problems of Mineral Processing, 42 (2008), 75-84
Fizykochemiczne Problemy Mineralurgii, 42 (2008), 75-84

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ATLAS OF UPGRADING CURVES USED IN SEPARATION AND IN MINERAL SCIENCE AND TECHNOLOGY PART III

Received April 27, 2008; reviewed; accepted July 31, 2008

The present Atlas (Part III) is the last from a series of articles on upgrading curves relating quality and quantity of products of separation. In this Atlas 12 additional upgrading curves were presented while 30 curves were presented in Parts I and II. In the previous papers the curves were grouped into categories: $A_/$ (α-insensitive curves with triangle or near triangle area accessible for plotting) A_o (α-insensitive curves with square area available for plotting) $B_/$ (α-sensitive curves with triangle plotting area, B_o (α-sensitive curves having square plotting area), $C_/$ (α-insensitive curves for $\beta > \alpha$ and triangle area for plotting), and C_o (α-insensitive curves for $\beta > \alpha$ or $\beta < \alpha$ and square area) where β stands for the content of a component in concentrate while α is the content of a component in the feed. In this Part III more complex upgrading curves were presented and an additional class of upgrading curves was introduced with irregular area for plotting. It was again stressed that all the upgrading curves contain the same information but in a different, specific for a given curve form. The use of upgrading curves depends on the needs and preferences of users. An appropriate matching of an upgrading plot with a set of separation results allows to approximate the curve with a suitable mathematical formula which can be used for characterizing separation. As previously, the readers are kindly asked to report unmentioned hitherto upgrading curves to jan.drzymala @pwr.wroc.pl for their future publication in the Internet http://www.ig.pwr.wroc.pl/minproc/krzywe%20wzbogacania.html

Key words: separation, upgrading, upgrading curves, separation efficiency

INTRODUCTION

Many aspects of separation and analysis of separation results have been discussed in detail in the previous parts of this series of papers. In Parts I and II 30 separation

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curves were presented. It was claimed there that there is unlimited number of separation curves. It will be shown here that it results from the principles of separation and considering the upgrading process as a relationship between quantity and quality of separation products.

During separation the feed is split into two products, that is concentrate and tailing. When more products are created, the system always can be treated as a two-product separation by considering certain product as the first product while the mixture of remaining products as the second product.

The principal equation, also called the separation mass balance, for a selected component (1) in the feed and in the products of separation is:

$$100\%\alpha_1 = \gamma_1\beta_1 + \gamma_2\vartheta_1 \quad (1)$$

There are other equations valid for the considered separation system:

$$\alpha_1 + \alpha_2 = 100\% \quad (2)$$

$$\beta_1 + \beta_2 = 100\% \quad (3)$$

$$\vartheta_1 + \vartheta_2 = 100\% \quad (4)$$

$$\gamma_1 + \gamma_2 = 100\% \quad (5)$$

where symbol 2 stands for the second component, that is everything except component 1. In the case of γ, symbol 1 means product 1 and symbol 2 means product 2. Equations 1-5, when combined, provide a modification of Eq.1, that is the mass balance for component (2):

$$100\%\alpha_2 = \gamma_1\beta_2 + \gamma_2\vartheta_2 \quad (1b)$$

In Eqs 1-5 α, β, ϑ stand for content of a component in the feed, concentrate and tailing, respectively, while γ_1 means yield of concentrate while γ_2 means yield of the tailing.

Thus, there are 8 unknowns and 5 equation in the separation systems considered as upgrading. Therefore, three values are needed for a complete solution of the balance of separation system. These unknowns can be taken from experimental data (for instance α_1, β_1, and ϑ_1 or γ_1, β_2, and ϑ_2, etc.).

When the separation involves many sets of separation data, it is very convenient to plot the separation result as upgrading curves. Still, for each set of data we need the values of three parameters from the list of 4 variables (α, β, ϑ, γ). In the case when all three selected parameters vary, a 3D plot is needed to represent the results of separation (Tumidajski et al., 2007). However, frequently one of the variables is constant. Usually, it is the content of the valuable component in the feed (α_1 = const) while we have all needed separation data for the remaining two variables. Then, it is convenient to plot the results of separation as a map in a x-y Cartesian system as a relationship between two variables (any other parameter can be calculated). Plotting the results for only two parameters of separation provide a map of results. Sometimes, if there is a law governing separation, we can get, instead of a cloud-shape scattered data, a linear relationship, but this case will be discussed a little further.

The results of separation cannot be represented by any number because there are limits of separation. The limits are: ideal separation, ideal mixing (when the 100% pure component under question, which forms concentrate, starts to be diluted with the unwanted component), and no separation. Then, equations 1-5 should be solved for:

$$\alpha_1 = \beta_1 \text{ (lack of separation)} \quad (6)$$

$$\beta_1 = 100\% \text{ (ideal separation)} \quad (7)$$

$$\gamma_1\beta_1/\alpha_1 \text{ (}=\varepsilon_l\text{=recovery,)} = 100\% \text{ (ideal mixing)} \quad (8)$$

For instance, for the plot relating yield (γ) and content (β), also know as the Henry curve, the limiting area is complex (Atlas, Part I) with its upper side represented by a hyperbolic $1/\beta$ curve.

When the separation results, drawn on a separation plot, form a line or curve which can indicate that there is a law governing the separation, the three "missing" equation are:

a) α_1 = const

b) for instance β_1 = certain value or values

c) law governing separation, that is equation relating any two variables (for instance $\gamma = a/\beta + b$ (Tumadajski *et al.*, 2007) or $\vartheta = a\beta + b$ (Stepinski, after Pudlo, 1971) and many other formulas (Drzymala and Hussin, 2005).

Since the law governing separation, as a rule, introduces new unknown or unknowns, in fact we have to use not three but four or more equations, depending on how many adjustable parameters are used by the law expressed as equation. The additional equations are:

d) constant(s) of the equation representing the law.

The separation governing law is a convenient tool for characterizing separation results provided that the equation contains only one adjustable parameter. Then, for separation systems obeying the law, a comparison of the separation results can be based on comparison of the one-adjustable parameter of separation.

As it was demonstrated, any separation system, considered as upgrading, consists of 8 parameters. In addition to that, these parameters can be combined into unlimited number of new parameters called indices, factors, numbers, ratios, efficiencies, etc. Each new parameter provides both a new equation and a new unknown. Thus, introducing new parameters does not provide any new information about the system. However, frequently the new parameters are very useful. For instance recovery in concentrate (ε) is defined as:

$$\varepsilon_1=\gamma_1\beta_1/\alpha_1 \quad (9)$$

where subscript 1 means component 1. Recovery indicates , usually in %, how much of component's mass was transferred to a given product. Parameters are the enrichment ratio (β/α) and the Hancock efficiency of separation $E=\varepsilon_{1,1}-\varepsilon_{2,1}$ or $E=\frac{10^4(\alpha-\vartheta)(\beta-\alpha)}{(\beta-\vartheta)(100-\alpha)\alpha}$ (see Fig. 1) (Taggart, 1943; Jowett, 1975). Numerous equivalent formulas for the Hancock efficiency parameter were given by Barskij and Rubinstein (1970). Many other separation parameters were presented in Parts I and II of the Atlas.

All the upgrading parameters can be grouped into pairs providing infinitive number of upgrading curves which represent the same data but in a different esthetical and graphical form. The usefulness of a given upgrading curve depends, to a great extent, on personal preferences.

UPGRADING BALANCE

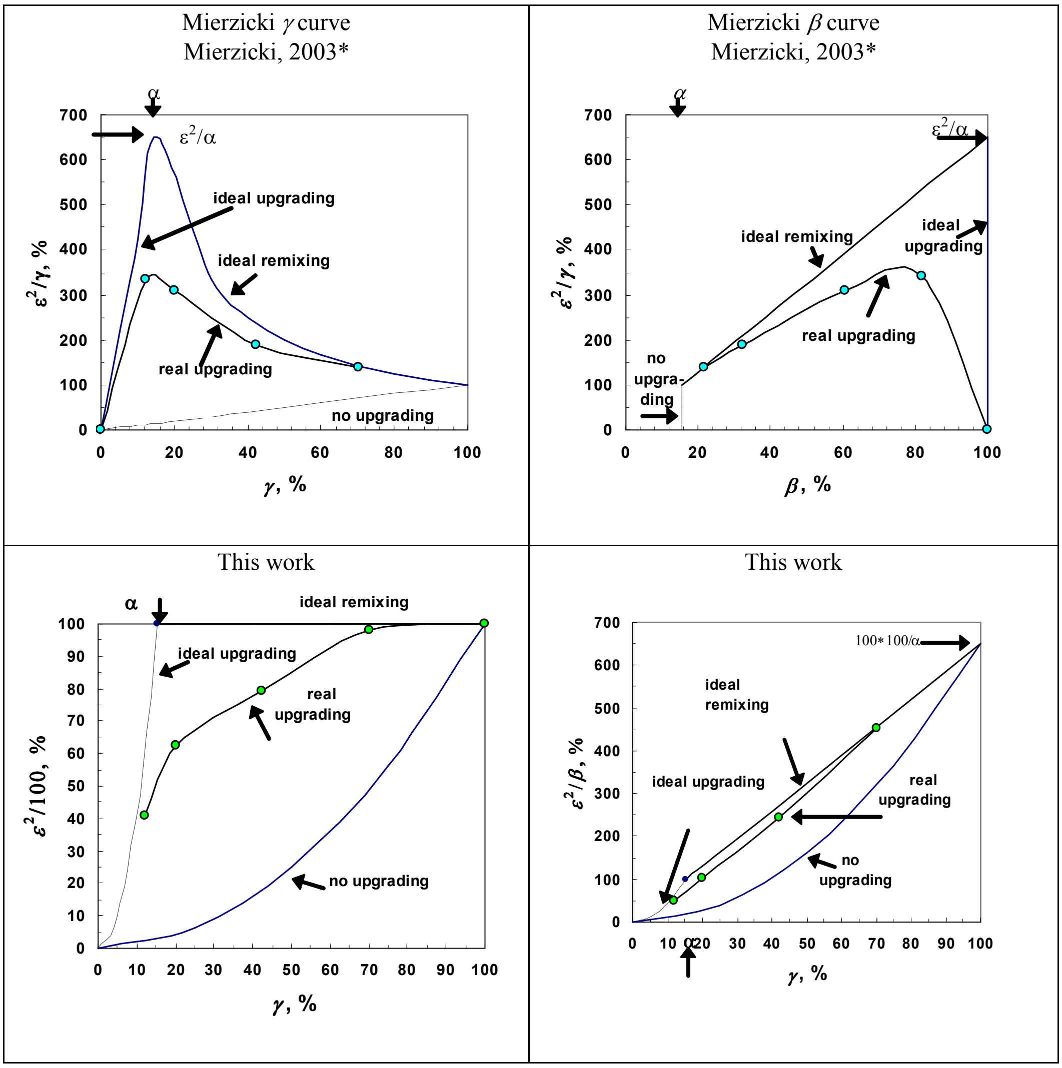
For plotting upgrading curves, the same hypothetical results of separation (Table 1) were considered as in Atlas Parts I and Part II (Drzymala, 2006, 2007). It was assumed that the feed contains only two components, that is component 1 and component 2 (rest of material). Only principal parameters, that is feed grade (α), yield of products (γ), content of component 1 (β), and recoveries ε of both components are presented. Other parameters can be calculated using the formulas given in the axes of the upgrading curves.

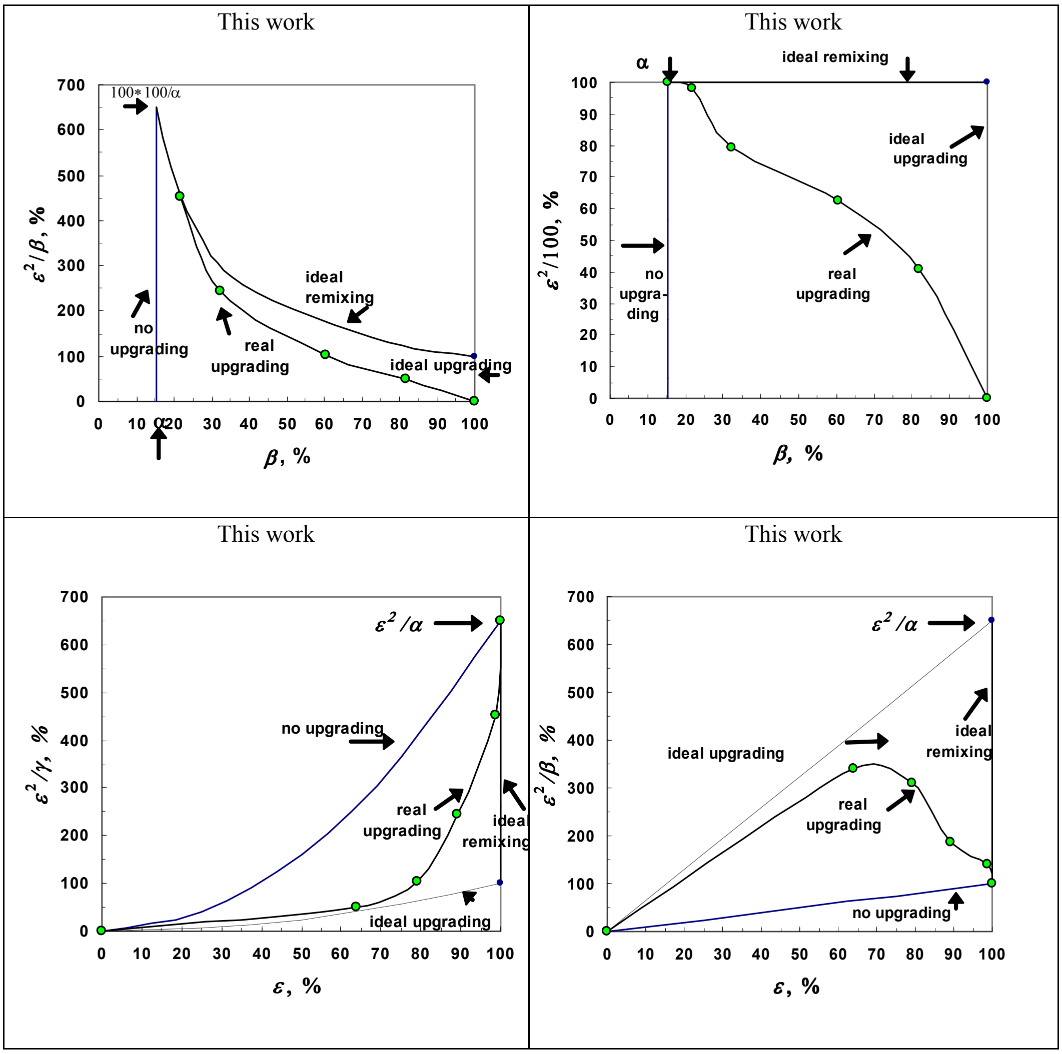
Table 1. Upgrading balance of a hypothetical separation. The data were used for calculation of upgrading curves

Product	yield, γ (%)	content of component 1, β, %	recovery of component 1 $\varepsilon=\gamma\beta/\alpha$, %	recovery of component 2 $\varepsilon=\gamma\beta/\alpha$, %
K_1	12.06	81.70	64.00	2.00
K_1+K_2	20.14	60.40	79.01	9.43
$K_1+K_2+K_3$	42.27	32.44	89.07	33.71
$K_1+K_2+K_3+K_4$	70.14	21.73	98.99	63.92
Tailing	29.86	0.52	1.01	36.08
Feed	100.00	15.40=α	100.00	100.00

UPGRADING CURVES

In the previous paper the upgrading curves were classified into three categories: A (α-insensitive), B (α-sensitive), C (α-insensitive but covering limited range of variables). There were additional symbols for the shape of the area available for plotting (/ for triangle area and *o* for square area). The same classification was used in this Atlas III. A new class of separation curves B_u was added for α-sensitive curves with irregular shape available for plotting. The upgrading curves considered in this Atlas are shown in Fig. 1.




This work
100*100/α
ε²/β, %
no upgrading
real upgrading
ideal remixing
ideal upgrading
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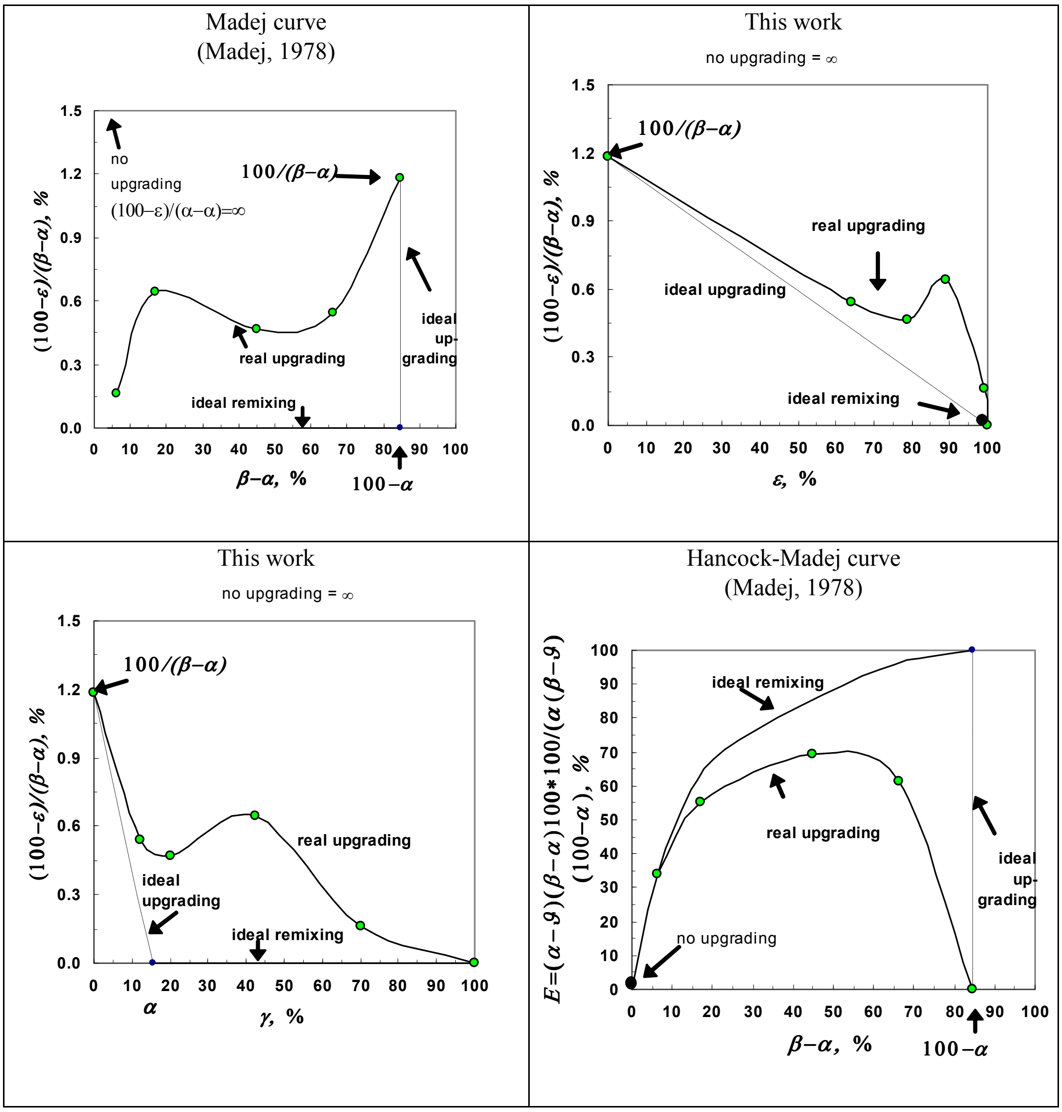

*Mierzicki (2003) considered the following upgrading curves: $\gamma^2\beta^3 = f(\gamma)$, $\gamma\beta^2 = f(\gamma)$, $\gamma\ \beta^2/\alpha^2 = f(\gamma)$, $\frac{\alpha^3}{\gamma^2\beta^2} = f(\gamma)$, $\frac{\alpha^3}{\gamma^2\beta^2} = f(\beta)$, $\gamma^2\beta^3 = f(\beta)$, and $\gamma\beta^2 = f(\beta)$.

Fig.1. Different upgrading separation curves considered in this Atlas (Part III).
All the curves are B_u type (α-sensitive and irregular shape of area available for plotting), expect the $\varepsilon^2/100$ vs β curve which is of B_o type

CONCLUSIONS

The present Atlas (Part III) is a final part in a series of papers providing different upgrading curves relating quality and quantity of the products of separation. In all three parts of the Atlas 42 curves were shown and characterized. This number of curves is still not great in comparison to unlimited number of possible upgrading curves. The new curves presented in this paper and the ones which will be generated in the future, as a rule, are very complex and can be applied only for very special purposes.

ACKNOWLEDGEMENTS

Financial support from the Ministry of Science and Higher Education (zlec. 342-865) is gratefully acknowledged.

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Drzymala J., *Atlas krzywych wzbogacania do opisu separacji stosowanych w nauce i w przemyśle mineralnym*. Część III, Physicochemical Problems of Mineral Processing, 42 (2008), 75-84 (w jęz. ang)

Obecny Atlas (cześć III) jest ostatnią z serii prac o krzywych wzbogacania wiążących ilość i jakość produktów separacji. W obecnym atlasie przedstawiono 12 dodatkowych krzywych wzbogacania, podczas gdy poprzednio w częściach I i II pokazano 30 krzywych wzbogacania. W poprzednich artykułach krzywe były pogrupowane na kategorie: $A_{/}$ (α-nieczułe z trójkątnym lub blisko trójkątnym obszarem dostępnym do rysowania) A_{o} (α- nieczułe krzywe z kwadratowym obszarem dostępnym do rysowania), $B_{/}$ (α-czułe krzywe z trójkątnym lub blisko trójkątnym obszarem rysowania, B_{o} (α-czułe krzywe z kwadratowym obszarem dostępnym do rysowania), $C_{/}$ (α-nieczułe krzywe dla $\beta > \alpha$ i trójkątnym obszarem rysowania), C_{o} (α-nieczułe krzywe dla $\beta > \alpha$ lub $\beta < \alpha$ i kwadratowym obszarem rysowania), gdzie β oznacza zawartość składnika w koncentracie podczas gdy α oznacza zawartość rozpatrywanego składnika w nadawie. W części III Atlasu zaprezentowano bardziej skomplikowane krzywe wzbogacania tworzące nowa klasę krzywych, które zostały określone symbolem B_{u} oznaczającym czułość na α oraz nieregularny obszar dostępny do rysowania. Podkreślono ponownie, że wszystkie krzywe wzbogacania zawierają te same informacje, ale w innej, specyficznej dla danej krzywej formie. Zastosowanie danej krzywej wzbogacania powinno zależeć od preferencji użytkownika. Wykazano, że odpowiednie połączenie krzywej wzbogacania z rezultatami separacji pozwolić może na wyznaczenie równania matematycznego, które może pełnić rolę prawa rządzącego separacją. Podobnie jak poprzednio, czytelnicy proszeni są o nadsyłanie nieopisanych dotąd krzywych wzbogacania pod adres jan.drzymala@pwr.wroc.pl dla ich opublikowania w Internecie pod adresem http://www.ig.pwr.wroc.pl/ /minproc/krzywe%20wzbogacania.html.

słowa kluczowe: separacja, wzbogacanie, krzywa wzbogacania, efektywność wzbogacania