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THE OPTIMAL BALL DIAMETER IN A MILL

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Abstract. This paper covers theoretical and experimental explorations for the sake of determining the optimal ball charge in mills. In the first part of the paper, on the basis of the theoretical analysis of the energy-geometric correlations, which are being established during the grain comminution by ball impact, as well as on the basis of the experiment carried out on grinding quartz and copper ore in a laboratory ball mill, there has been defined a general form of the equation for determining: the optimal ball diameter depending on the grain size being ground; and the parameter of the equation through which the influence of a mill is being demonstrated; then the parameter of the grinding conditions; and the parameter of the material characteristics being ground in relation to the ball size. In the second part of the paper, the process of making up the optimal ball charge has been defined providing the highest grinding efficiency, as well as its confirmation by the results of the experimental explorations.

keywords: mineral processing, optimal ball diameter, optimal ball charge, grinding

1. Introduction

The ball size in a mill has a significant influence on the mill throughput, power consumption and ground material size (Austin et al., 1976; Fuerstenau et al., 1999; Kotake et. al., 2004).

The basic condition, which must be met while grinding the material in a mill is that the ball, while breaking the material grain, causes in it stress which is higher than the grain hardness (Bond, 1962; Razumov, 1947; Supov, 1962). Therefore, for the biggest grain size, it is necessary to have a definite number of the biggest balls in the charge, and with the decreased grain size, the necessary ball size also decreases (Olejnik, 2010; 2011). For each grain size there is an optimal ball size (Trumic et. al., 2007).

The bigger ball in relation to the optimal one will have an excess energy, and consequently, the smaller ball mill has less energy necessary for grinding. In both cases, the specific power consumption increases and the grinding capacity decreases (Concha et al. 1992; Katubilwa and Moys, 2009; Erdem and Ergun, 2009).

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A great number of explorers were dealing with the questions of determining the maximal ball diameter depending on the material size being ground. A few empirical formulae have been proposed, out of which some are recommended for industrial mills (Bond, 1962; Razumov, 1947; Olevskij, 1948), but some of them have been defined on the basis of investigations carried out in laboratory mills and they have not been applied to industrial mills (Belecki, 1985; Supov, 1962).

All suggested formulae fit into the general form given by:

$$d_{b\max} = Kd^n \,, \tag{1}$$

where: $d_{b\,max}$ is the maximum ball diameter in a charge, d is the characteristic top limit of the material size which is being ground (d_{95} or d_{80} in the formulae is recommended for industrial mills); K and n are parameters, for which all authors say to be dependent on the mill characteristics, grinding conditions and characteristics of the material being ground. They are determined experimentally.

Even a simplified theoretical analysis clearly defines the influential factors on parameters K and n. The topic of this paper is, first of all, this kind of analysis and then the determination of the optimal ball charge model in a mill.

2. Theoretical background

Each grain size corresponds to a definite optimal ball size. The diameter of a ball is determined by the condition that, at the moment of breaking the grain, it has energy E, which is equal to energy E_0 necessary for grain comminution:

$$E = E_0 \,. \tag{2}$$

The ball impact energy on grain is proportional to the ball diameter to the third power:

$$E = K_1 d_b^3 \,. \tag{3}$$

The coefficient of proportionality K_1 directly depends on the mill diameter, ball mill loading, milling rate and the type of grinding (wet/dry). None of the characteristics of the material being ground have any influence on K_1.

The ball impact energy on the grain is turned into the action of comminution, which according to the Rittinger comminution law is directly proportional to the newly formed grain surface while being ground. We can suppose that the grain has got the form of a ball with diameter d and that the area of comminution occurs along the equator cross section. Then, the comminution energy E_0 is proportional to the surface of equator circle, that is, the grain diameter to the second power:

$$E_0 = K_2 d^2 \,. \tag{4}$$

It is clear that the coefficient of proportionality K_2 is not influenced by any of the characteristics of the material being ground.

In accordance with Eq. 2, the necessary condition for grain comminution is:

$$K_1 d_{bo}^3 = K_2 d^2 . \tag{5}$$

So, we have got the following: the optimal ball diameter d_{bo} is proportional to the grain diameter d to the exponent $n = 0.67$:

$$d_{bo} = \left(\frac{K_2}{K_1} \right)^{1/3} d^{2/3} = K_3 d^{0.67} . \tag{6}$$

The conclusion that can be drawn from Eq. 6 is quite clear and it shows that exponent n is neither influenced by mill's characteristics, grinding conditions, nor the characteristics of material being ground. All these influential factors are reflected only through the numerical value of parameter K, while the numerical value of exponent $n = 0.67$ has resulted from the theoretical energy–geometry relations shown by Eq. 2 to 6, which cannot be disputed.

In the formulae of the above mentioned authors, the numerical value of exponent n ranges from 0.2 up to 1.0 (Olevskij $n = 0.2$; Razumov $n = 0.3$; Bond $n = 0.5$; Baleski and Supov $n = 1.0$). Such great discrepancies in numerical values of exponent n are the result of the statistical data processing obtained from practice and investigations performed according to the model defined by Eq. 1, as well as the conviction that parameter n depends on mill's characteristics, grinding conditions and characteristics of the material being ground. If constant value $n = 0.67$ had been adopted in these result analysis, we would have obtained equally valid correlations, but only with the different values for proportionality coefficient K. Let us point out once again that the following issues have got an influence on the coefficient K: mill's characteristics, grinding conditions, and characteristics of the material being ground. Now, we have the right to put the following question: have not we attributed insufficient knowledge of the influence of the ball charge sliding in a mill under the different grinding conditions to the change of exponent n? There are some more similar justified questions.

This paper, describing a first part of investigation, aims at checking the hypothesis defined by Eq. 6 as well as its preference in relation to the hypothesis defined by Eq. 1.

Let us look back at the optimal ball charge in a mill. The necessary number of balls having the definite diameter N_b in a mill should be proportional to grain number N having the definite diameters which they can grind:

$$N_b \sim N . \tag{7}$$

The number of grains of the material with determined diameters depends on the grain size distribution. For a great number of materials the grain size distribution at the ball mill feed has been described by Gaudin-Schumann's equation:

$$d^{*} = \left(\frac{d}{d_{\max}} \right)^{m}, \tag{8}$$

where d^* is the filling load of grains less than d, d is the grain diameter, d_{max} is the maximum grain diameter, m is the exponent which characterizes the grain size distribution.

The number of balls, having the determined diameter in a mill, depends on the ball size distribution in the charge. Let us suppose that the ball size distribution in the charge can also be depicted by Gaudin-Schumann's equation:

$$Y = \left(\frac{d_b}{d_{b\max}} \right)^{c}, \qquad d_{b\min} < d_b < d_{b\max}, \tag{9}$$

where Y is the load of the balls having diameters less than d_b, d_b is the ball diameter, d_{bmax} is the maximum ball diameter in charge, d_{bmin} is the minimum ball diameter which can grind efficiently in a mill, c is the exponent which characterizes the ball size distribution.

The condition for efficient grinding, defined by Eq. 7, will be fulfilled when the grain size distribution and the ball size distribution are the same, which means that the parameters of both distributions are equal in Eqs. 7 and 9:

$$m = c\,. \tag{10}$$

In the second part of the paper, we will investigate the hypothesis defined by Eqs. 8, 9 and 10.

3. Experimental

Investigations were carried out in a laboratory ball mill having the size of DxL = 160x200 mm with a ribbed inside surface of the drum. The mill ball loading was 40% by volume, the rotation rate was equal to 85% of the critical speed.

Balls were made from steel: S4146, extra high quality, having hardness 62 ± 2 HRC according to Rockwell. Grinding tests were carried out with the samples of quartz having high purity of >99% SiO_2 as well as samples of copper ore consisting of 0.37% Cu, 67.48% SiO_2 and 15.02% Al_2O_3. Bond's working index for quartz is W_i = 14.2 kWh/t and for for copper ore W_i = 14.9 kWh/t.

The first part of this investigation has been oriented towards testing the hypothesis defined by Eq. 5. For that purpose, there has been observed the grinding kinetics of narrow size fractions of quartz and copper ore sizes (-0.80/+0.63 mm; -0.63/+0.50 mm; -0.50/+0.40 mm and -0.40/+0.315 mm) with the ball charge of different diameters (Table 1). The dry mill grinding has been conducted. The volume of grinding samples was equal to the volume of the interspaces of balls and the interstitial gaps between the balls charge.

The grinding efficiency of the narrow particle size fractions with ball charge of various diameters has been observed through the constant of milling rate k in the equation of the grinding kinetics law for the first order grinding $R=R_0e^{-kt}$, where R is the unground remainder of the sample after grinding time t, and R_0 is the sample mass for grinding. The first order kinetics of grinding occurred for all ball charges and for all samples.

As the ball charges and narrow, the size fractions of the samples have different masses. In this research, the grinding efficiency has also been observed through the specific mill throughput per ground product, per unit mass of the ball charge Q_s (kg/h/kg). The specific throughput has been calculated at grinding time t = 3 min.

Tables 2 and 3 give the numeric values of constant grinding rate k of the narrow size fractions of quartz and copper ore as well as the specific mill throughput with the ball charge of various sizes.

Table 1. Characteristics of ball charge and sample mass of quartz and copper ore

Symbol of ball charge	Ball diameter in charge d_b (mm)	Charge mass (g)	Number of balls in charge N_b	Sample mass (g) quartz and copper ore			
				-0.80+0.63	-0.63+0.50	-0.50+0.40	-0.40+0.315
A	6	7171	8149	1080/827	1073/792	1100/770	1072/792
B	11	6920	1277	1116/869	1139/832	1148/809	1140/832
C	15	6729	482	1136/900	1140/862	1136/838	1153/861
D	20	6475	199	1155/991	1142/950	1124/923	1168/949

Table 2. Constants of milling rate k and specific throughput of mill Q_s for grinding narrow particle size fractions of quartz

Symbol of the ball charge	Ball diameter in charge d_b (mm)	Size fraction (mm)							
		-0.80+0.63		-0.63+0.50		-0.50+0.40		-0.40+0.315	
		$\boldsymbol{k}$	$\boldsymbol{Q_s}$	$\boldsymbol{k}$	$\boldsymbol{Q_s}$	$\boldsymbol{k}$	$\boldsymbol{Q_s}$	$\boldsymbol{k}$	$\boldsymbol{Q_s}$
A	6	0.098	0.726	0.132	0.828	0.137	0.889	0.156	0.930
B	11	0.222	1.224	0.216	1.220	0.180	1.142	0.149	0.989
C	15	0.281	1.381	0.216	1.229	0.167	1.101	0.133	0.943
D	20	0.233	1.418	0.159	1.169	0.122	0.979	0.084	0.780

Figures 1 and 2 show the dependence of milling rate constant k upon the ball diameter in the charge d_b while grinding different quartz and copper ore size fractions. In Figs. 1 and 2, one can see that for each size fraction there is a proper ball diameter, which provides the highest efficiency of grinding, in terms of the milling rate constant k and that it is the optimal ball diameter d_{bo} for grinding the given grain size.

By means of graphic interpolation, from Figs. 1 and 2, there has been determined optimal ball diameter d_{bo}, which provides the highest grinding efficiency of the corresponding size fraction and the results have been shown in Table 4. We can notice that the values for d_{bo} are very close in terms of both parameters for grinding efficiency.

Table 3. Constants of milling rate k and specific mill throughput Q_s for grinding narrow particle size fractions of copper ore

Symbol of the ball charge	Ball diameter in charge d_b (mm)	Size fraction (mm)							
		-0.80+0.63		-0.63+0.50		-0.50+0.40		-0.40+0.315	
		k	Q_s	***k***	Q_s	***k***	Q_s	***k***	Q_s
A	6	0.072	0.586	0.126	0.882	0.107	0.749	0.118	0.754
B	11	0.152	1.060	0.177	1.188	0.139	0.938	0.122	0.867
C	15	0.189	1.248	0.202	1.287	0.148	1.033	0.121	0.848
D	20	0.171	1.242	0.168	1.220	0.129	0.927	0.111	0.763

Table 4. Optimal ball diameter

Size fraction (mm)	Mean grain diameter, d (mm)	Optimal ball diameter d_{bo} (mm)				Mean value of the optimal ball diameter d_{bo} (mm)	
		In terms of constant milling rate, k		In terms of specific mill throughput, Q_s			
		Quartz	Copper ore	Quartz	Copper ore	Quartz	Copper ore
-0.80+0.63	0.715	16.0	16.5	16.5	17.5	16.25	17.0
-0.63+0.50	0.565	13.5	15.0	14.0	16.0	13.75	15.5
-0.50+0.40	0.450	12.5	14.2	13.0	15.0	12.75	14.6
-0.40+0.315	0.357	8.0	12.0	9.0	13.0	8.50	12.5

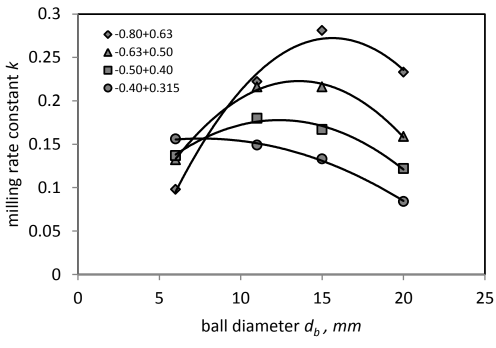

Fig. 1. Dependence of milling rate constant k while grinding particular size fractions of quartz upon ball diameter d_b in the charge

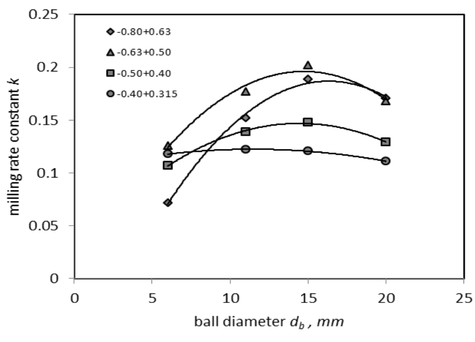

Fig. 2. Dependence of milling rate constant k while grinding particular size fractions of copper ore upon ball diameter d_b in the charge

Figure 3 shows the dependence of optimal ball diameter d_{bo} on mean grain quartz diameter d in a fraction, in the coordinate system [ln$d$; lnd_{bo}]. The linear arrangement of points in Fig. 3 points to the fact that there is a strong correlating connection of forms given by Eq. 1.

By the method of the least squares, it was possible to determine the numerical values of parameters *K* and *n* in Eq. 1, with a very high degree of correlation *r*, so that they, for the conditions of our experiment, would be as follows:

$$\text{quartz: } d_{bo} = 22.67 d^{0.87}, \ r = 0.95, \tag{11}$$

$$\text{copper ore: } d_{bo} = 19.77 d^{0.42}, \ r = 0.98. \tag{12}$$

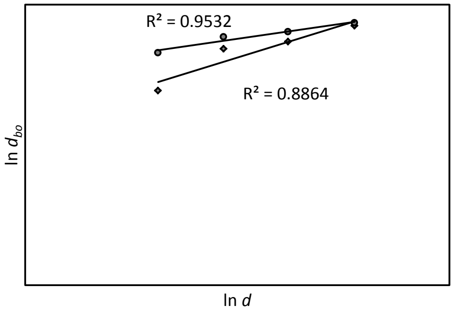

Fig. 3. Dependence of optimal ball diameter d_{bo} on mean grain diameter d in a fraction

The grinding tests on all samples have been performed under identical conditions. The only difference is in the characteristics of quartz and copper ore, in terms of Bond's working index. In other words, copper ore has got a higher Bond's working index and this should have been expressed in Eq. 12 in terms of a higher value for coefficient *K* compared to the value in Eq. 11 for quartz. However, this didn't happen. The value of *K* in Eq. 12 is less than the value in Eq. 11. This happened because of the incorrect hypothesis, which says that the exponent *n* depends on the characteristics of a mill, grinding conditions and raw material characteristics. It was incorrect to emphasize higher influence of Bond's working index on exponent *n* instead on parameter *K*.

In this paper, in our theoretical analysis, we have come to the concision, that parameter *n* has got the constant value: $n = 0.67$ and that it does not depend on the characteristics of a mill, the grinding conditions and raw material characteristics. Thus, consequently, the optimal ball size is defined by Eq. 6.

By means of the least squares, we have determined the numerical values of parameters *K* in Eq. 6, so that they, for the conditions of our experiment and with a very high degree of correlation *r*, are as follows:

$$\text{quartz: } d_{bo} = 19.69 d^{0.67}, \ r = 0.94, \tag{13}$$

$$\text{copper ore: } d_{bo} = 23.45 d^{0.67}, \ r = 0.97. \tag{14}$$

Equations 13 and 14 confirm our theoretical hypothesis that exponent *n* has got the constant value $n = 0.67$ and that the influence of the mill characteristics, grinding conditions and raw material characteristics has been demonstrated only in terms of the numerical value of parameter K. The higher value of the Bond's working index for copper ore has led to the higher value of parameter K in Eq. 14 in relation to the value of the same one in Eq. 13 for quartz, and that is the theoretically expected issue.

The second part of this paper refers to the testing of the hypothesis for modeling the optimal ball charge in a mill, defined by Eqs. 8 to 10. The optimal ball charge in a mill has been formed in the following way.

1. We define the value of the exponent in Eq. 8 for the material to be ground
2. We define maximum ball diameter d_{bmax}, according to one of the known formulae
3. The ball load with different diameters in the charge ranging from d_{bmax} to d_{bmin} is to be calculated according to Eq. 9, where the exponent m=c.

The grinding tests were carried out on artificially formed samples of quartz and copper ore having the grain size of -0.80/+0.315 mm, whose particle size distribution is described by Eq. 8 so that, for both samples, it is as follows

$$d^* = \left(\frac{d}{0.8}\right)^2 . \tag{15}$$

The maximum and minimum ball diameters in the charge, in accordance with Eqs. 14 and 15 are as follows:

quartz:
$$d_{b\max} = 19.69 d^{0.67} = 19.69 \cdot 0.8^{0.67} = 17.0\,\text{mm}, \tag{16}$$

$$d_{b\min} = 19.69 d^{0.67} = 19.69 \cdot 0.315^{0.67} = 9.1\,\text{mm}, \tag{17}$$

copper ore:
$$d_{b\max} = 23.45 d^{0.67} = 23.45 \cdot 0.8^{0.67} = 20.2\,\text{mm}, \tag{18}$$

$$d_{b\min} = 23.45 d^{0.67} = 23.45 \cdot 0.315^{0.67} = 10.8\,\text{mm}. \tag{19}$$

In accordance with the available ball load used or making the charge, the balls with the following diameters have been used: 10.3 mm; 12.7 mm; 15 mm; and 19 mm. There have been formed there different charges: E, F and G. The charge E was made up according to the hypothesis of this paper according to the equation:

$$Y_E = \left(\frac{d_b}{19}\right)^2, \qquad 10.3 < d_b < 19.0. \tag{20}$$

In charge F, the larger balls prevail and their size distribution follows the equation:

$$Y_F = \left(\frac{d_b}{19}\right)^4, \qquad 10.3 < d_b < 19.0. \tag{21}$$

In charge G, the smaller balls prevail and their size distribution follows the equation:

$$Y_G = \left(\frac{d_b}{19}\right)^{1.5}, \quad 10.3 < d_b < 19.0. \tag{22}$$

In Table 5 we have given the composition of samples in the charge according to the size distribution. The ball mill loading is 40% by volume. The quartz sample mass for grinding is 915 g and for copper ore 787 g.

The grinding efficiency with ball charges *E*, *F* and *G*, has been observed in terms of the constant milling rate of the first-order k and the specific throughput of mill Q_s per ground product per unit mass of the charge, on the controlling screen with the mesh size of d = 0.315 mm at the grinding time of t = 3 min. The results of grinding are shown in Table 6 and they confirm our hypothesis that the highest grinding efficiency is provided by charge *E*, where the ball size distribution is identical with the one with the grain size distribution of the material being ground.

The accomplishment of this principal in industrial mills is possible by loading the mills with the balls of different diameters, in the proper correlation, where the value of exponent c in Eq. 9 is brought closer to the value of the exponent m in Eq. 8.

Table 5: Composition of samples and charges according to the size distribution

Particle size distribution, d (mm)	Weight W (%)	Ball diameter d_b (mm)	Charge E		Charge F		Charge G	
			W (%)	M (g)	W (%)	M (g)	W (%)	M (g)
-0.80+0.63	38	19	38	2732	58	4157	29	2090
-0.63+0.50	23	15	23	1653	21	1480	16	1159
-0.5+0.40	14	12.7	14	1006	12	853	15	1089
-0.40+0.315	25	10.3	25	1797	9	667	40	2898
	100		100	7188	100	7147	100	7236

Table 6: Milling rate constant k and specific throughput of mill Q_s, with different ball charges

Indicator for the grinding efficiency	Charge					
	E		F		G	
	Quartz	Copper ore	Quartz	Copper ore	Quartz	Copper ore
k	0.119	0.078	0.108	0.072	0.105	0.073
Q_s	0.890	1.084	0.821	1.075	0.796	1.071

4. Conclusion

All proposed formulae for determining the ball diameter, depending on the diameter of the grain size material being ground, fit into the general form given in the equation:

$$d_b = Kd^n,$$

where d_b is the ball diameter, d is the diameter of the grain size material being ground, K and n are parameters, for which all authors say, depend on mill characteristics, grinding conditions and characteristics of the material being ground and which are consequently determined by experiments.

By means of theoretical analysis of energy–geometry correlations, which are being established during the process of grain comminution by ball impact, it has been clearly proved that exponent n, by which grain diameter d has been raised to a power, does not have any influence on the characteristics of mill characteristics, grinding conditions and characteristics of the material being ground. All these influential factors have only been reflected by numerical value of parameter K, while the numerical value of exponent n is constant and amounts to 0.67. This result has been proved by the results of our investigation in this paper. Thus, the general form of the formula for determining the ball diameter, depending on the diameter of the grain size material being ground is:

$$d_b = Kd^{0.67}.$$

Starting from the physical fact that the required number of balls of determined diameter N_b in a mill should be proportional to number of grains N of determined diameters which will be ground, we have come to the theoretical hypothesis that, in order to achieve effective grinding, the ball size distribution in the charge should be the same with the material grain size distribution being ground.

In a great many cases, the grain size distribution of a ball mill feed is well described by Gaudin-Schumann's equation:

$$d^* = \left(\frac{d}{d_{\max}} \right)^m ,$$

where d^* is the grain fill level less than d, d is the grain diameter, d_{max} is the maximum grain diameter, m is the exponent which characterizes the grain size distribution.

The optimal ball charge in a mill should be made up in such a way that the ball size distribution of a charge should be in accordance with Gaudin-Schumann's equation:

$$Y_E = \left(\frac{d_b}{d_{b\max}} \right)^c , \; d_{b\min} < d_b < d_{b\max},$$

where Y is the ball fill level having the diameter less than d_b, d_b is the ball diameter, d_{bmax} is the maximum ball diameter in the charge, d_{bmin} is the minimum ball diameter which can grind efficiently in a mill, c is the exponent which characterizes the ball size distribution so that the most efficient grinding can be achieved if the condition $m=c$ is fulfilled.

The results of this experiment in this paper have completely proved the given hypothesis.

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