Physicochem. Probl. Miner. Process. 49(2), 2013, 443−451

www.minproc.pwr.wroc.pl/journal/

Physicochemical Problems of Mineral Processing

ISSN 1643-1049 (print)
ISSN 2084-4735 (online)

Received June 21, 2012; reviewed; accepted December 11, 2012

INTERRELATION OF THE FUERSTENAU UPGRADING CURVE PARAMETERS WITH KINETICS OF SEPARATION

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Abstract: It was shown in the paper that kinetic equations relating recoveries of two components in separation products and time, when combined together to eliminate the time parameter, provide mathematical equations which relate recoveries of the two considered components in concentrate. The obtained one-adjustable-parameter type equations are very useful for approximation of the separation results plotted as the so-called Fuerstenau upgrading curves. Most empirical mathematical formulas presently used for the Fuerstenau plots were derived using various kinetic equations while some are still awaiting for their kinetic derivation.

keywords: *separation, upgrading, kinetics, Fuerstenau curve, beneficiation, recovery, selectivity*

Introduction

Kinetics is one of the most important aspects of separation process (Arbiter, 1951; Arbiter and Harris, 1982; Laskowski, 1989; Wills and Napier-Munn, 2006). The rate of separation determines the time needed for removal of valuable components of ores (Schumann, 1942; Kelly and Spottiswood, 1989). The kinetics of separation is also a basic element of separation models relating components recovery, products grade and properties of the feed. These models can be based on probabilities, energies, forces and their combinations concepts (Drzymala, 2007b). The rate at which a separation process occurs depends on the specificity of the process and process parameters. As a result the kinetic equations relating recovery of an ore component with separation time can assume different mathematical forms (Schumann, 1942; Somasundaran and Lin, 1973; Yuan et al., 1996). Presently more than a dozen kinetic models are available and applied in separation science and technology (for instance Somasundaran and Lin, 1973; Agar et al., 1998; Xu, 1998; Polat and Chander, 2000; Cilek, 2003; Wills and

http://dx.doi.org/10.5277/ppmp130206

Napier-Munn, 2006; Brozek and Mlynarczykowska, 2007). The list of selected kinetic equations is presented in Table 1.

In this paper application of kinetic equations for approximation of separation results will be presented. It will be shown that the kinetics of two selected components of the ore provide an equation which can be directly used for construction and interpretation of the recovery–recovery Fuerstenau upgrading curves.

Table 1. Selected kinetic equations (ε – recovery of a component in separation product, ε_{max} – maximum recovery of the same component in separation product, k – rate constant of separation, t – separation time)

Model	Formula	
Zeroth-order model	$\varepsilon = k\,t$	(1)
First-order model	$\varepsilon = \varepsilon_{\max}\left(1-e^{-kt}\right)$	(2)
First-order with rectangular distribution of floatabilities	$\varepsilon = \varepsilon_{\max}\left[1-\frac{1}{k\,t}\left(1-e^{-k\,t}\right)\right]$	(3)
Fully mixed reactor model	$\varepsilon = \varepsilon_{\max}\left(1-\frac{1}{1+\frac{t}{k}}\right)$	(4)
Improved gas/solid adsorption model	$\varepsilon = \varepsilon_{\max}\left(\frac{k\,t}{1+k\,t}\right)$	(5)
$\frac{3}{2}$ -order model	$\varepsilon = \varepsilon_{\max}\left(1-\frac{1}{\left(1+\frac{1}{2}k\,t\sqrt{\varepsilon_{\max}}\right)^2}\right)$	(6)
Second-order model	$\varepsilon = \frac{\varepsilon_{\max}^2\,k\,t}{1+\varepsilon_{\max}\,k\,t}$	(7)
Second-order model with rectangular of floatabilites	$\varepsilon = \varepsilon_{\max}\left\{1-\frac{1}{k\,t}\left[\ln\left(1+k\,t\right)\right]\right\}$	(8)

Kinetic curves

In minerallurgy the kinetic curves relate recovery of a selected component and time of the process. According to Table 1, the kinetics of separation can assume different mathematical forms. Separation of components according to the first and second order kinetics is presented in Fig.1a. Figure 1a shows the kinetics of separation of one component and also the remaining components of the feed. It has to be emphasized that the sum of the considered and remaining components of the system provides the kinetics of concentrate formation (Fig. 1b).

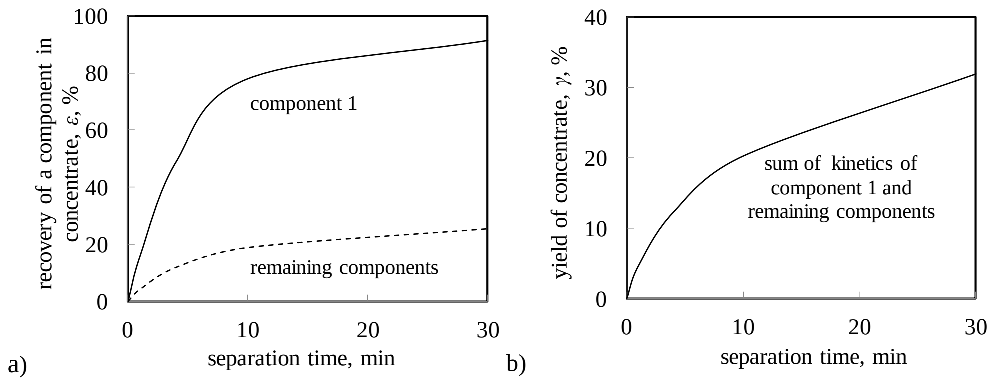

Fig. 1. Separation results plotted as a relationship between recovery of each component in concentrate and separation time (a), yield of components forming concentrate vs. separation time (b)

Separation and upgrading

The characterization of separation results can be accomplished in a number of ways. The most common is upgrading in which quality and quantity of separation products are analyzed (Drzymala et al., 2010; Kowalczuk and Drzymala, 2011). The results of upgrading can be presented either in a tabular or graphical form. Theoretically, there is an infinite number of upgrading curves while in literature about fifty of them can be

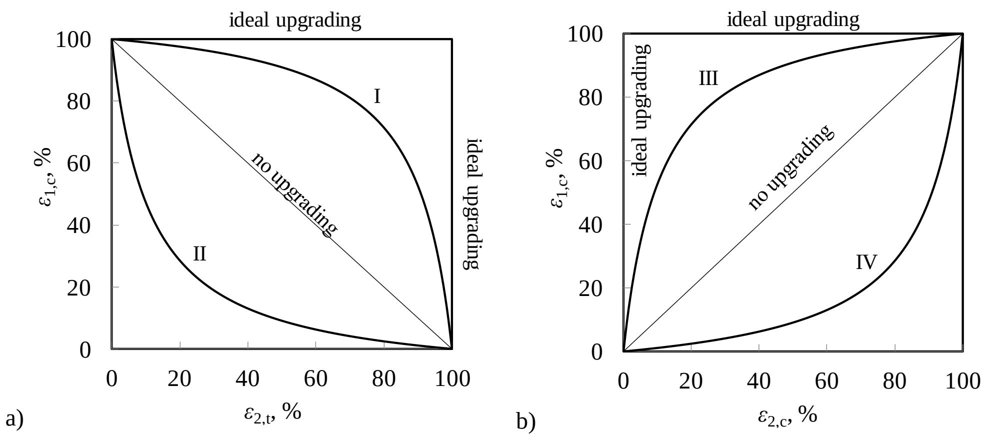

Fig. 2. The Fuerstenau upgrading curves: a) relationship between recovery of a component 1 in concentrate $\varepsilon_{1,c}$ and recovery of a second component in the tailing $\varepsilon_{2,t}$: I - recovery of component 1 in concentrate is greater than recovery of component 2 in concentrate, II - recovery of component 2 in concentrate is greater than recovery of component 1 in concentrate, b) relationship $\varepsilon_{1,c}$ vs. $\varepsilon_{2,c}$: III – recovery of component 1 in concentrate is greater than recovery of component 2 also in concentrate, IV – recovery of component 1 is smaller than recovery of component 2 in the same concentrate (after Drzymala and Ahmed, 2005)

found (Drzymala, 2006, 2007a, 2008). All of them are based on the same data but show the separation elements in different mathematical, graphical an esthetical forms. One of the upgrading curve is the Fuerstenau plot which relates recoveries of two components in the same or different products (Fig. 2). It has been recently quite frequently used because there are simple empirical equations which can be used for the approximation of separation results present as the Fuerstenau graphs (Drzymala and Ahmed, 2005).

The Fuerstenau plot and kinetics of separation

Since the kinetics of separation relates recovery of feed components in products with separation time while the Fuerstenau curve relates recoveries of components, it becomes obvious that elimination of time from the kinetic equations provides the Fuerstenau plot (Fig. 3).

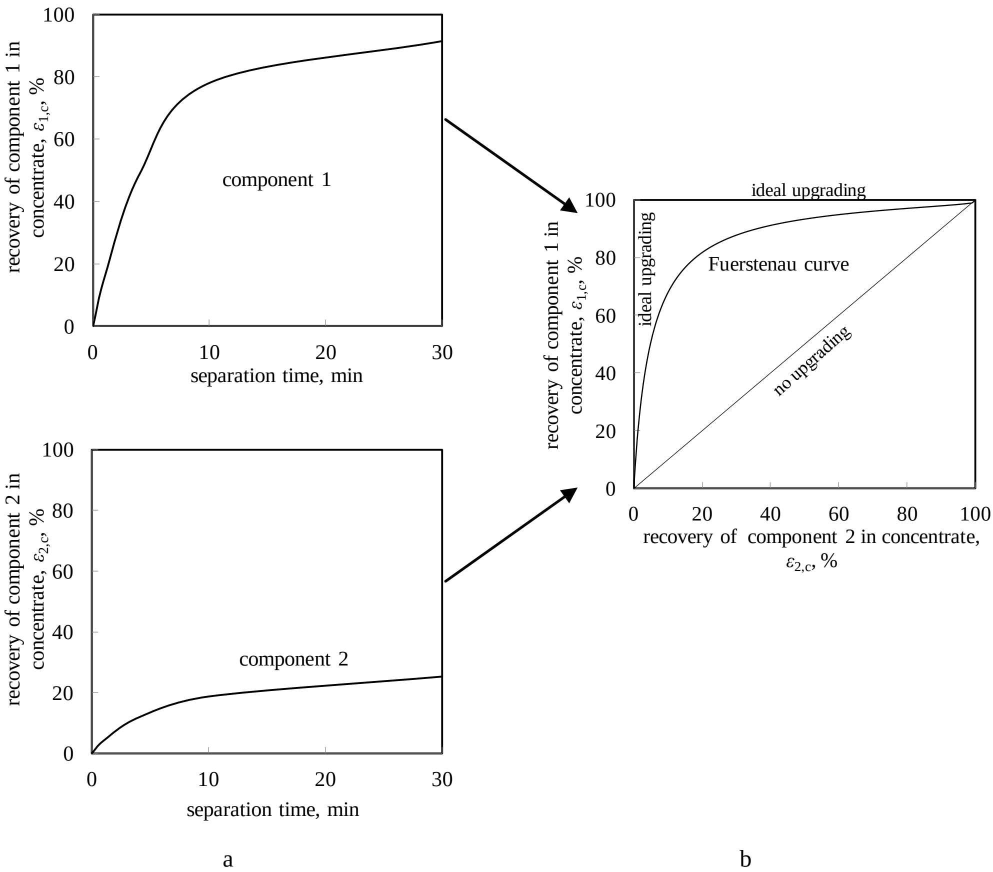

Fig. 3. The kinetics of separation of feed components (a) provide separation results in the form of the Fuerstenau upgrading curve (b)

Table 2. Kinetic formulas for different combinations of orders (0, 1, 3/2, 2) of separation kinetics
(k = k_1/k_2, $k' = k_2/k_1$, $\varepsilon_{1,c}$ = component 1 recovery in concentrate , $\varepsilon_{2,c}$ = component 2 recovery in concentrate)

$\varepsilon_{2,c}$ \ $\varepsilon_{1,c}$	0	1	$\frac{3}{2}$	2
0	$\varepsilon_{1,c} = k\,\varepsilon_{2,c}$ Eq. 9 (Fig. 4)	$\varepsilon_{1,c} = -k'\ln\left(\frac{\varepsilon_{2,c}}{100}\right)$ Eq. 10 (Fig. 5)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{(1+5k\varepsilon_{2,c})^2}\right]$ Eq. 11 (Fig. 8)	$\varepsilon_{1,c} = \frac{k'\varepsilon_{2,c}}{100(100-\varepsilon_{2,c})}$ Eq. 12 (Fig. 6)
1	$\varepsilon_{1,c} = -k\ln\left(\frac{\varepsilon_{2,c}}{100}\right)$ Eq. 13 (Fig. 5)	$\varepsilon_{1,c} = 100\cdot\left[1-\left(\frac{100-\varepsilon_{2,c}}{100}\right)^{k}\right]$ Eq. 14 (Fig. 7)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{\left(1-5k\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)\right)^2}\right]$ Eq. 15 (Fig. 9)	$\varepsilon_{1,c} = \frac{100^2\,k\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)}{100\,k\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)-1}$ Eq. 16 (Fig. 12)
$\frac{3}{2}$	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{(1+5\cdot k'\varepsilon_{2,c})^2}\right]$ Eq. 17 (Fig. 8)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{\left(1-5\cdot k'\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)\right)^2}\right]$ Eq. 18 (Fig. 9)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{\left(1+\frac{k\,(10-\sqrt{100-\varepsilon_{2,c}})}{\sqrt{100-\varepsilon_{2,c}}}\right)^2}\right]$ Eq. 19 (Fig. 10)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{\left(1+\frac{k'\varepsilon_{2,c}}{20(100-\varepsilon_{2,c})}\right)^2}\right]$ Eq. 20 (Fig. 11)
2	$\varepsilon_{1,c} = \frac{k\,\varepsilon_{2,c}}{100\cdot(100-\varepsilon_{2,c})}$ Eq. 21 (Fig. 6)	$\varepsilon_{1,c} = \frac{100^2\,k'\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)}{100k'\ln\left(\frac{100-\varepsilon_{2,c}}{100}\right)-1}$ Eq. 22 (Fig. 12)	$\varepsilon_{1,c} = 100\cdot\left[1-\frac{1}{\left(1+\frac{k\cdot\varepsilon_{2,c}}{20\cdot(100-\varepsilon_{2,c})}\right)^2}\right]$ Eq. 23 (Fig. 11)	$\varepsilon_{1,c} = \frac{100\,k\,\varepsilon_{2,c}}{\varepsilon_{2,c}\,(k-1)+100}$ Eq. 24 (Fig. 13)

Mathematical formulas of the Fuerstenau curves based on kinetic considerations

Since the kinetic equations applied to delineate separation results with time are well known (Table 1), their combinations provide mathematical formulas for approximation of the upgrading results plotted as the Fuerstenau curves. They are collected in Table 2. In all the equations, $k = k_1/k_2$, where k_1 and k_2 are rate constants of component 1 and component 2, respectively. The equations shown in Table 2 are also plotted as the Fuerstenau curves and presented in Table 3 as Figs. 4–13. The shape of each curve depends on the k value. The curves are asymmetrical, except the one shown in Fig. 13, which is symmetrical in relation to the diagonal line of the graph and results from separation of both components according to the second order kinetics.

Table 3. The Fuerstenau plots for different combinations of kinetics of separation (ε_{max} = 100%). Kinetic and separation curves equations are presented in Table 2

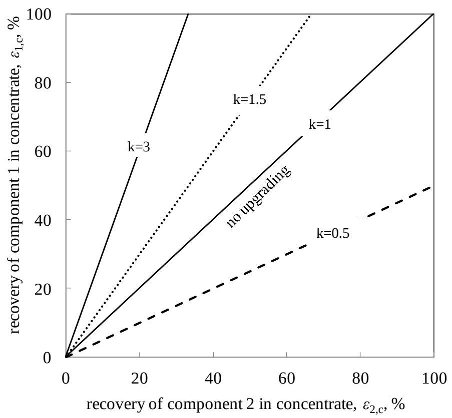

Fig. 4.

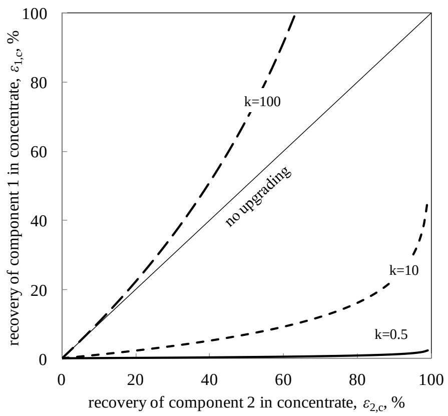

Fig. 5.

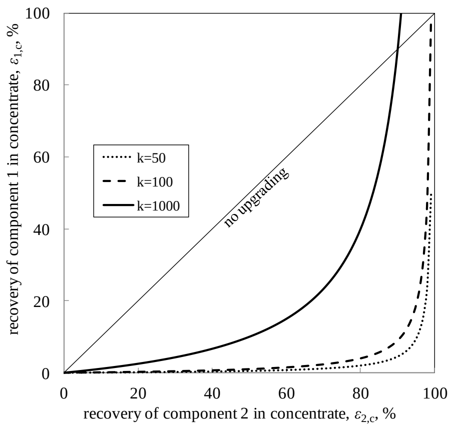

Fig. 6

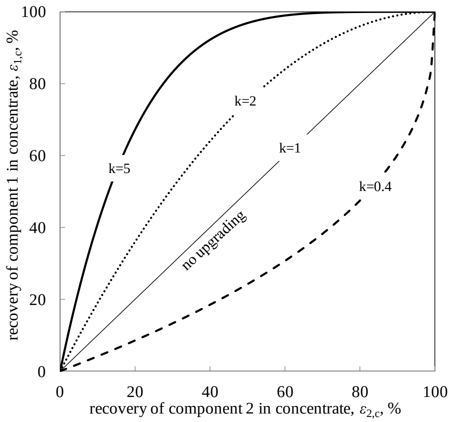

Fig. 7

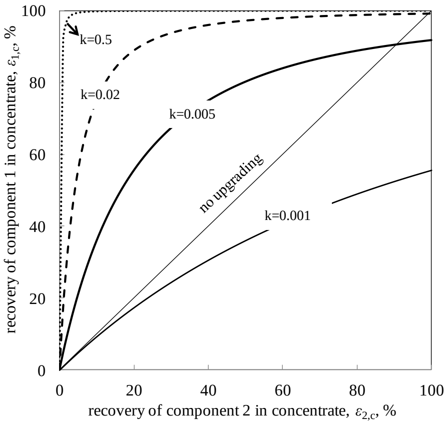

Fig. 8

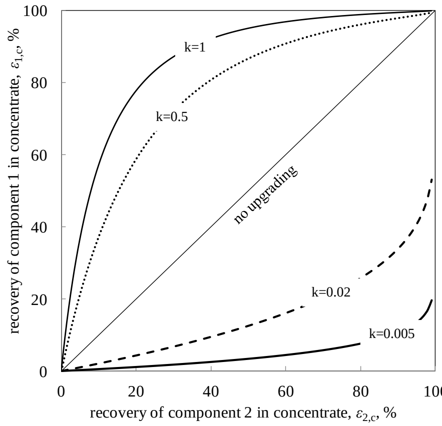

Fig. 9

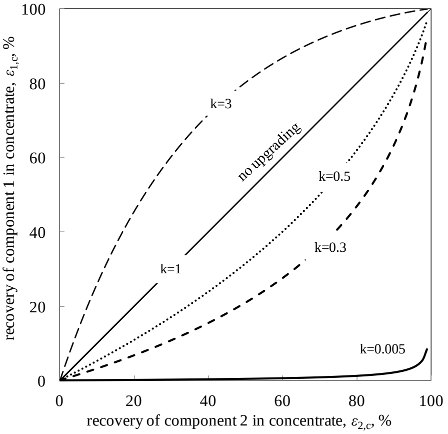

Fig. 10

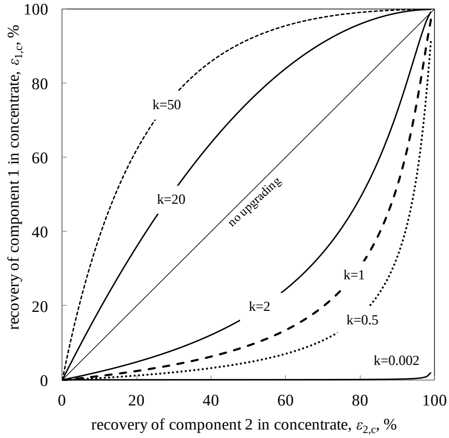

Fig. 11

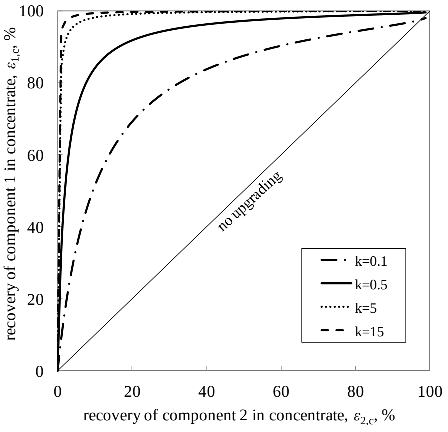

Fig. 12

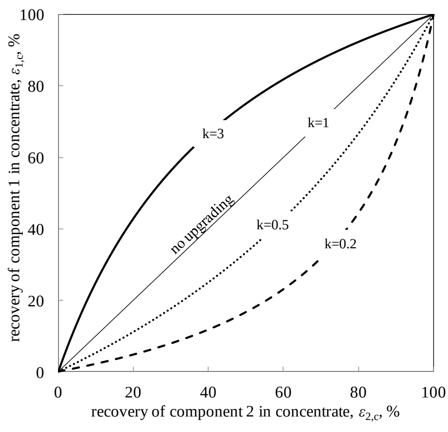

Fig. 13

A comparison of the derived here equations indicates that most empirical equations proposed by Ahmed and Drzymala (2005) can be predicted from the kinetic considerations except equation:

$$\varepsilon_{1,c} = \frac{\varepsilon_{2,c}^{b}}{100^{(b-1)}} \tag{19}$$

which is awaiting for kinetic justification.

Conclusions

The Fuerstenau upgrading plots relate recovery of components in products. It results from this paper that most mathematical equations that are used for approximation of separation results using the Fuerstenau upgrading curves can be derived from kinetic considerations. Since kinetics relates recovery of a component in a product and time, elimination of time in the kinetic equations for two components provide mathematical equations characterizing separation, which can be plotted as the Fuerstenau curves. The separation curves, including the Fuerstenau upgrading plot, are very useful for approximation, characterization and interpretation of laboratory and industrial results of separation.

Acknowledgements

Financial support of Statutory Research Grant S20119 and B20073 is greatly acknowledged.

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