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Motion of a particle with stick-slip boundary conditions towards a flat interface: hard wall or free surface

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Abstract: Motion of a particle with stick-slip boundary conditions towards a hard wall or free surface is investigated in the range of Reynolds numbers much smaller than unity, based on the multipole expansion of the Stokes equations. The slip parameter can be interpreted as a measure of a solid particle roughness or as the effect of a surfactant on the motion of a small spherical non-deformable bubble. The particle friction coefficient is evaluated as a function of the distance from its center to the wall, based on the inverse power series expansion, and the results are used to derive explicit lubrication expressions for the friction coefficient, in a wide range of the slip parameters. It is pointed out that for a very small thickness of the fluid film, the lubrication expressions are more accurate than the series expansion. The drainage time is calculated and analyzed, and estimated in terms of explicit lubrication expressions.

Keywords: *particle, boundary conditions, hard wall, free surface*

1. Introduction

Motion of a spherical particle in a viscous fluid towards an interface, caused by external force, has been extensively investigated in the literature, with the emphasis on the effect of thin liquid films between the particle surface and the interface, and with a variety of particles: solid spheres with a rough surface, drops or bubbles (Scheludko, 1967; Jeffrey and Onishi, 1984; Małysa et al., 2005; Krasowska and Małysa, 2007). In particular, the influence of surfactants on the dynamics of drops and bubbles has been studied (Oguz and Sadhal, 1988), and the effect of the structure of the surface irregularities on the motion of rough solid particles (Lecoq et al., 2004). The processes of bubble attachment and coalescence have been investigated and the drainage time to reach the critical film thickness has been estimated and experimentally measured (Warszyński et al., 1996; Jachimska et al., 2001; Małysa et al., 2005; Krasowska and Małysa, 2007). The hydrodynamic force resisting the approach of two curved spherical hydrophobic (with a slip) and hydrophilic (without slip) surfaces have been analyzed (Goren, 1973; Vinogradova, 1995; Hocking, 1973), also in the context of a permeable surface (O'Neill and Bhatt, 1991).

For Reynolds number much smaller than unity (Batchelor, 2000; Kim and Karrila, 1991), particle moving towards an interface does not bounce (Gondret et al., 2002) – it slows down owing to lubrication forces (Jeffrey and Onishi, 1984). In the Stokes regime, bubbles and rough particles can be described by the stick-slip model (Felderhof, 1976; Felderhof, 1977), with a slip parameter ξ related to roughness of a solid particle or distribution of surfactant on the surface of a small spherical non-deformable bubble. In this paper, we apply this model to analyze rough particle and bubble dynamics, and the drainage time, in the range of a small thickness of the fluid film between the particle and the interface.

2. System and method

In this work, we study motion of a single spherical non-deformable particle of radius a in a fluid of viscosity η towards a flat non-deformable interface at z = 0. The regime of Reynolds numbers much

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smaller than unity is assumed (Batchelor, 2000; Kim and Karrila, 1991). The motion is caused by an external constant force $\tilde{F}$ acting on the particle in the direction perpendicular to the interface.

The fluid motion is given by the stationary Stokes equations (Batchelor, 2000; Kim and Karrila, 1991):

$$\eta \nabla^2 \mathbf{v}(\mathbf{r}) - \nabla p(\mathbf{r}) = 0 \tag{1}$$

$$\nabla \cdot \mathbf{v}(\mathbf{r}) = 0 \tag{2}$$

where $\mathbf{v}(\mathbf{r})$ and $p(\mathbf{r})$ are the velocity and pressure of the fluid, respectively. The particle velocity $\widetilde{U}$ is proportional to the external force $\tilde{F}$:

$$\widetilde{U} = \tilde{\mu}\tilde{F} \tag{3}$$

with the mobility coefficient $\tilde{\mu}$ which is a function of the distance $\tilde{z}$ from the particle center to the interface.This function depends on the boundary conditions at the particle surface and at the interface.

Two types of interfaces at $z = 0$ are studied: a solid (hard) motionless wall (HW) with the fluid sticking to it:

$$\mathbf{v}(\mathbf{r})|_{z=0} = 0 \tag{4}$$

and a free surface (FS), with the perfect slip of the fluid:

$$v_z|_{z=0} = 0 \tag{5}$$

$$\sigma_{iz}|_{z=0} = 0 \quad \text{for} \quad i = x, y \tag{6}$$

where components of velocity and stress tensor are denoted as $\mathbf{v} = (v_x, v_y, v_z)$ and $\sigma_{ij} = \eta(\partial_i v_j + \partial_j v_i) - \delta_{ij} p$, respectively, with $i, j = x, y, z$.

The stick-slip boundary conditions at the particle surface S are assumed:

$$\mathbf{n} \cdot (\mathbf{v} - \widetilde{\mathbf{U}})\big|_S = 0 \tag{7}$$

$$\mathbf{t} \cdot (\mathbf{v} - \widetilde{\mathbf{U}})\big|_S = \frac{\lambda}{\eta} \mathbf{t} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \tag{8}$$

where $\mathbf{n}$ and $\mathbf{t}$ are the unit vectors normal and tangential to the particle surface, respectively, and $\widetilde{\mathbf{U}} = (0, 0, \widetilde{U})$. Here, λ is the slip length. It is useful to introduce the slip parameter (Felderhof, 1976; Felderhof, 1977):

$$\xi = \frac{\lambda}{a + 3\lambda} \tag{9}$$

which satisfies the relation:

$$0 \le \xi \le \frac{1}{3}. \tag{10}$$

The limiting case $\xi = 0$ corresponds to a hard sphere, or a non-deformable bubble covered with a surfactant in such a way that they behave in the same way as solid particle with the fluid sticking to its surface. The other limit of $\xi = 1/3$ describes a clean non-deformable spherical bubble with a free surface. We consider also several intermediate values of the slip parameter ξ = 0.001, 0.01, 0.05, 1/12, 0.12, 1/6, 0.21, 0.25, 0.29.

The model of stick-slip boundary conditions on the particle surface can be applied to a rough solid spherical particle or a non-deformable spherical bubble covered with a surfactant, with the slip parameter ξ interpreted, respectively, as a measure of a solid particle roughness or as the effect of a surfactant on the motion of a small spherical non-deformable bubble. The slip parameter ξ corresponds to the hydrodynamic radius $a_H = (1 - \xi)a$ (Felderhof, 1976; Felderhof, 1977; Cichocki et al., 1988; Cichocki et al., 2014).

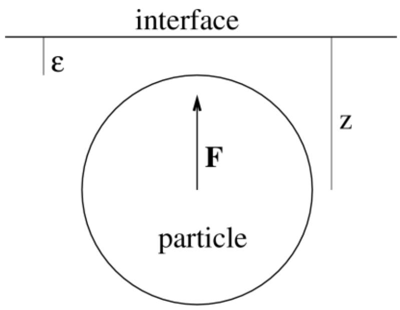

Fig. 1. System and notation

We now introduce dimensionless variables. Distances are normalized by the particle radius a:

$$z = \tilde{z}/a \tag{11}$$

velocities by the Stokes velocity $V_0 = \tilde{F}/(6\pi\eta a)$, and time by the Stokes time a/V_0. The dimensionless particle velocity:

$$U = \tilde{U}/V_0 \tag{12}$$

is equal to the dimensionless mobility coefficient:

$$\mu = 6\pi\eta a\tilde{\mu} \tag{13}$$

$$\mu = U \tag{14}$$

which is the inverse of the dimensionless friction coefficient:

$$\zeta = 1/\mu\ . \tag{15}$$

The friction coefficient $\zeta(z)$ as a function of the distance z from the particle center to the interface is evaluated numerically by the multipole expansion method (Cichocki et al., 1994; Cichocki et al., 1999) as the inverse power series:

$$\zeta(z) = \sum_{n=0}^{N} a_n t^n, \quad \text{with } t = \frac{1}{z}. \tag{16}$$

In this computation, large values of N are used, typically around 1000, in the range between 500 and 2000.

The particle dynamics is given as:

$$-\frac{dz}{dt} = \frac{1}{\zeta(z)}\ . \tag{17}$$

3. Lubrication expression for the friction coefficient of stick-slip particle

It is instructive to use the numerical results (16) to derive simple asymptotic lubrication expressions for the friction coefficient ζ of the stick-slip particle. We define the dimensionless gap size:

$$\epsilon = z - 1. \tag{18}$$

For a small film thickness $\epsilon \ll 1$, the particle hydrodynamic friction is well-approximated by the following expression (Jeffrey and Onishi, 1984):

$$\zeta(\epsilon) = \frac{A}{\epsilon} - B\ln\epsilon + C - D\epsilon\ln\epsilon \tag{19}$$

with the constants A, B, C, D dependent on the slip parameter ξ and on the type of the flat interface bounding the fluid. For $\xi = 0$, values of these constants have been derived in (Jeffrey and Onishi, 1984; Cichocki and Jones, 1998). In this work, we evaluate them for several values of ξ, using the series expansion values of the friction coefficient $\zeta(z)$. Using the expression:

$$\epsilon = \frac{1-t}{t} \tag{20}$$

and comparing Eq. (16) with the series expansion of Eq. (19) in t, we obtain:

$$a_n = A + \frac{B}{n} - \frac{D}{n(n+1)}, \quad \text{for } n \to \infty. \tag{21}$$

We evaluate the coefficients A, B from the linear fit of a_n as a function of $1/n$ in the range of large n, and D from the linear fit of $(a_n - A)n$ as a function of $1/(n+1)$. Values of C are calculated from the expression (Jeffrey and Onishi, 1984; Cichocki and Jones, 1998):

$$C = a_0 - D + \sum_{n=1}^{N}\left(a_n - A - \frac{B}{n} + \frac{D}{n(n+1)}\right). \tag{22}$$

The resulting values of the coefficients A, B, C, D are listed in Tables 1-2. By interpolation, the coefficients can be easily obtained also for intermediate values of the slip parameter ξ.

In Figure 2, the lubrication expressions (19) for the friction matrix ζ are compared with the series expansion values evaluated from the series expansion (16). For ξ approaching zero, the range of validity of the lubrication expression is more and more limited to values of ϵ significantly smaller than ξ.

In Tables 1-2, the range of values of ϵ is explicitly indicated where the lubrication and the series expansion expressions agree with each other. For larger values of ϵ, the series expansion is more accurate. For smaller values of ϵ, on the contrary, the series expansion is less accurate.

For extremely small values of ϵ, additional effects of the slip on the solid wall need to be taken into account, as indicated in (Hocking, 1973). For such systems, the lubrication formulas from (Goren, 1973) could be applied, which, however, cannot be used for the limiting cases of a hard wall or a free interface, studied in this work. The lubrication expressions derived by (Vinogradova, 1995) for lubrication interactions between hydrophobic and hyfrophilic surfaces in principle could be used for our systems, but we checked that they do not reproduce the accurate results for the friction coefficient, shown in Fig. 2.

Table 1. Lubrication constants for stick-slip particle approaching flat hard wall, as functions of the slip parameter ξ. Within the indicated ε range, the lubrication expression (19) differs from the series expansion (16) of ζ by at most 5%

ξ	A	B	C	D	ϵ range	N
0	1	1/5	0.97128	1/21	(0.0061, 0.78)	500
0.05	1/4	6.575	-15.743	33.	(0.0023, 0.04)	1000
1/12	1/4	3.5749	-6.2238	10.7	(0.0036, 0.20)	800
0.12	1/4	2.2000	-2.6414	4.55	(0.0029, 0.16)	1000
1/6	1/4	1.3250	-0.7890	1.92	(0.0030, 0.17)	1000
0.21	1/4	0.86073	-0.0109	0.95	(0.0030, 0.20)	1000
0.25	1/4	0.5750	0.36872	0.47	(0.0030, 0.24)	1000
0.29	1/4	0.3681	0.58214	0.21	(0.0030, 0.32)	1000
1/3	1/4	1/5	0.708214	0.033	(0.0030, 0.54)	1000

Table 2. Lubrication constants for stick-slip particle approaching flat free surface, as functions of the slip parameter ξ. Within the indicated ε range, the lubrication expression (19) differs from the series expansion (16) of ζ by at most 5%

ξ	A	B	C	D	ϵ range	N
0	1/4	9/20	1.03366	3/28	(0.003, 0.37)	2000
0.05	0	3.167	-6.973	18.9	(0.00045, 0.020)	2000
1/12	0	1.8333	-2.4624	6.07	(0.00088, 0.11)	1000
0.12	0	1.22222	-0.7840	2.57	(0.00038, 0.20)	2000
1/6	0	0.83333	0.05734	1.11	(0.00074, 0.17)	1000
0.21	0	0.62698	0.38761	0.574	(0.00033, 0.18)	2000
0.25	0	0.50000	0.53071	0.333	(0.00031, 0.19)	2000
0.29	0	0.40805	0.59516	0.197	(0.00030, 0.21)	1000
1/3	0	1/3	0.61586	1/9	(0.00062, 0.22)	1000

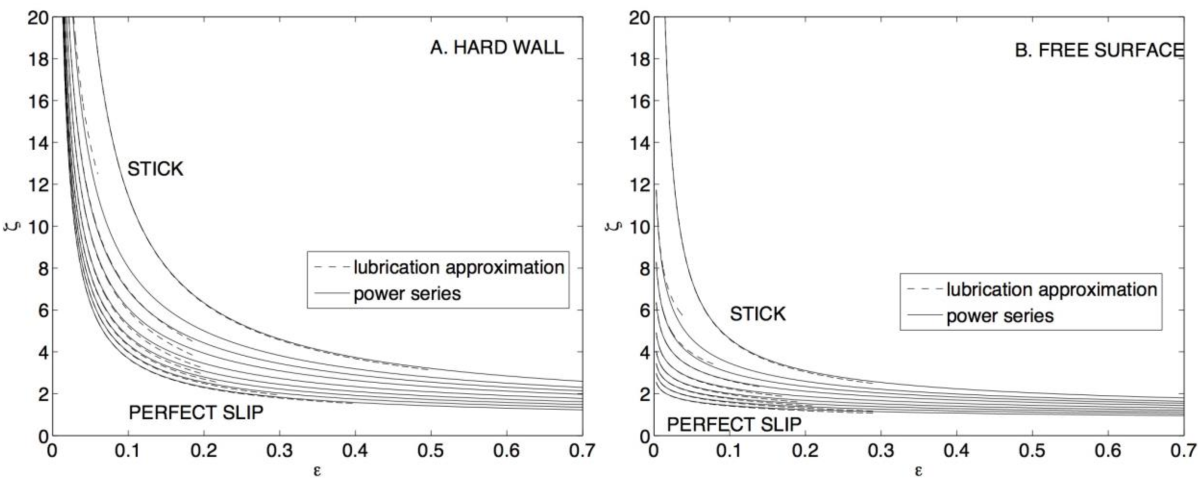

Fig. 2. Friction coefficient ζ of a stick-slip particle approaching (A) hard wall; (B) free surface. Top-down: ξ = 0 (stick), 0.05, 1/12, 0.12, 1/6, 0.21, 0.25, 0.29 and 1/3 (perfect slip)

4. Drainage time

In this section, we calculate how much time $T(\epsilon)$ does a stick-slip particle need to reduce the thickness of the film between its surface and the interface – from ϵ_0 until a smaller value $\epsilon < \epsilon_0$:

$$T(\epsilon) = \int_{\epsilon}^{\epsilon_0} \zeta(\epsilon')d\epsilon' \tag{23}$$

with the friction coefficient ζ evaluated as the series expansion (16). In Fig. 3 we plot T as a function of $\ln\epsilon$ for different values of the slip parameter ξ, separately for both types of the interfaces. It is clear that for a particle approaching a free surface, the drainage time is significantly smaller than for a particle moving towards a hard wall.

For ϵ_0 within the lubrication regime, i.e. for $\epsilon < \epsilon_0 \ll 1$, the drainage time can be also evaluated with the use of the lubrication expressions from Eqs. (19) and (23):

$$T(\epsilon) = Aln\epsilon_0 + B\epsilon_0(1 - \ln\epsilon_0) + C\epsilon_0 - \frac{D\epsilon_0^2}{2}(\ln\epsilon_0 - \tfrac{1}{2}) - A\ln\epsilon - B\epsilon(1 - \ln\epsilon) - C\epsilon + \frac{D\epsilon^2}{2}(\ln\epsilon - \tfrac{1}{2}). \tag{24}$$

For $\epsilon_0 = 0.05$, and $\xi = 0$ or $\xi \geq 0.05$, these lubrication expressions are shown in Fig. 3 and compared with the results of the series expansion. The lubrication expressions explain the same slope of all the curves with $\xi > 0$ in panel A of Fig. 3, visible for small values of ϵ.

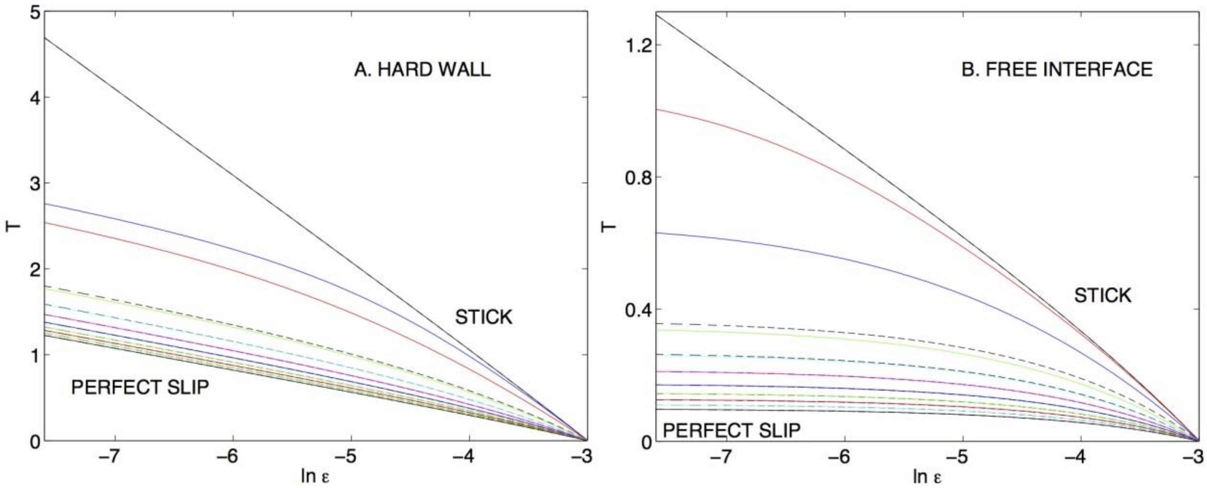

Fig. 3. Time T of drainage to reach the film thickness ϵ, starting from the film thickness $\epsilon_0 = 0.05$. The particle moves towards (A) hard wall and (B) free interface. Top-down: $\xi = 0$ (stick), 0.001, 0.01, 0.05, 1/12, 0.12, 1/6, 0.21, 0.25, 0.29 and 1/3 (perfect slip). Solid lines: series expansion (16); dashed lines: lubrication expression (19)

Indeed, in case of the hard wall and $\varepsilon \ll 1$, the drainage time for solid particles (with $\xi = 0$) is well-approximated by $-\ln\varepsilon$ plus a constant, and for particles with all the other values of the slip parameter by $-\ln\varepsilon/4$ plus a constant, which depends on ξ. The same linear relation $-\ln\varepsilon/4$ plus a constant holds in the lubrication regime for a particle with stick boundary conditions approaching a free interface, as shown in panel B of Fig. 3. These scalings are in agreement with the corresponding values of A listed in Tables 1 and 2.

From Tables 1-2 it is clear that the range of the lubrication approximation has an upper limit, which for $\xi = 0.05$ is smaller than $\epsilon_0 = 0.05$, and rapidly drops down for $\xi < 0.05$. This effect is more pronounced in case of free surface. Therefore, for $0 < \xi < 0.05$, the lubrication expressions cannot be used to estimate the drainage time in Fig. 3.

For a particle with $\xi > 0$ approaching a free interface, $A = 0$. From Eq. (24) it is clear that if $A = 0$, then the particle comes into contact with the free surface in a finite time:

$$T_0 = B\epsilon_0(1 - \ln\epsilon_0) + C\epsilon_0 - \frac{D\epsilon_0^2}{2}(\ln\epsilon_0 - \tfrac{1}{2}). \tag{25}$$

This property is illustrated in Fig. 4, where the drainage time $T(\epsilon)$ close to a free interface is evaluated from the series expansion (16) and shown as a function of ϵ, for different values of the particle slip parameter ξ.

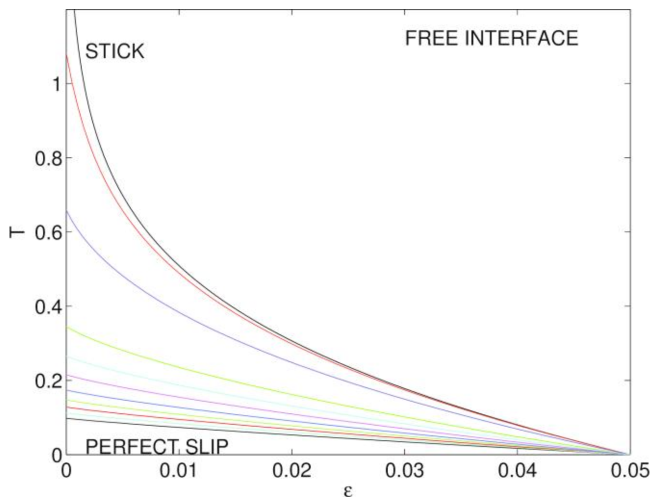

Fig. 4. Time $T(\epsilon)$ of drainage to reach the film thickness ε, starting from the film thickness $\epsilon_0 = 0.05$. The particle with ξ = 0, 0.001, 0.01, 0.05, 1/12, 0.12, 1/6, 0.21, 0.25, 0.29 and 1/3 moves towards a free interface. For $\xi > 0$, $T(0)$ is finite

5. Conclusions

In this work, friction coefficient ζ of a stick-slip particle which moves towards a flat interface, hard wall or free surface, has been evaluated by performing series expansion in the inverse distance from the particle center to the interface. The results have been used to derive lubrication expressions for ζ, and their range of validity has been determined, by specifying the upper limit of the film thickness ϵ. The advantage is that in this way useful simple expressions for ζ are provided. They can be applied also for values of ε so small that the series expansion with N = 1000 or 2000 is not accurate.

We demonstrated that the lubrication expressions can be used to estimate the drainage time, in the range of ϵ depending the slip parameter ξ, and excluding very small values of ξ. The results can be applied to micro bubbles covered with surfactant or rough micro particles in the limit of Reynolds numbers much smaller than unity.

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