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AN ANALYSIS OF EFFECT OF PARTICLE SIZE ON BATCH FLOTATION OF COAL

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Abstract: This paper presents the analysis of the batch flotation with regard to the size of the floated particles with coal as an example. Empirical studies were conducted on samples of bituminous coal from the Jankowice coal mine (33 type in Polish classification). Experimental includes fractional flotation of coal samples with various particle size, the float and sink analysis of flotation products and determination the ash content in each flotation and densimetric fractions. The evaluation of flotational upgrade was based on the partition curve and value of heterogeneity index (standard deviation) with the division into concentrate and tailings as a measure of flotation efficiency. From the partition curve, partition size, which is assumed in flotation as the maximum size of floatable particles under the given physicochemical conditions, was calculated. On the basis of float and sink analyses, it was found that floated particles are particles with hydrophobic properties corresponding to the density (related to the mineral matter content) below 1.6 Mg/m^3. In this connection, the probability of detachment of a particle from a given particle size fractions, was calculated as a relation between the mass of particles with the density below 1.6 Mg/m3 in tailings and the mass of particles with this density in the feed. With the increase in particle size, the degree of heterogeneity increases and reaches the maximum value for particle size fractions 0.315–0.4 mm, then decreases for larger particles. The characteristics of the dependence of the degree of heterogeneity on particle size is analogous as in many flotation processes – the dependence of the flotation rate constant flotation on particle size.

Key words: *flotation, float and sink analysis, probability of detachment, indices flotation processing*

Introduction

Flotation is the most common and widely accepted method of enrichment of fine coal below 0.5 mm and even below 1 mm. This method has been used to enrich several million tones of coking and power coal in Poland.

Coal is a heterogeneous material of two kinds. The petrographic composition and content of mineral matter cause changes in particle hydrophobic properties, and actu-

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ally generate their distribution. The organic coal matter has hydrophobic properties, while the degree of hydrophobicity, measured by the contact angle, varies with the maceral composition. The components of the mineral matter, however, are hydrophilic. With the increase in the coverage rate of the particle by the mineral matter, the contact angle, the value of which depends also on the degree of surface roughness and the presence of microcracks, decreases. The relation between volume and surface properties for coal was proved and described with the use of deterministic and stochastic models of batch flotation (Brożek and Młynarczykowska, 2006, 2007, 2009).

The flotometric equation is also known which is based on the balances of forces acting in the three-phase particle-air bubble-liquid system, solved only numerically at the moment of particle detachment. This equation in the form of polynomial of 3rd degree has been presented as a trigonometric function with only one solution. The team of Drzymala (Watanabe et al., 2011) presented the analytical form of an equation, connecting the maximum size of floating particle and its hydrophobicity, expressed as a contact angle.

A significant aspect in flotation upgrading is constituted by the analysis of elementary phenomena occurring at phase boundaries, which have a decisive effect upon the process effectiveness. These problems have been widely described in the literature (Hukki, 1953; Kelsall, 1960; Bogdanov et al., 1964; Laskowski and Iskra, 1970; Woodburn et al., 1971; Bartlett and Mular, 1974; Jiang and Holtham, 1986; Flynn, 1987; Laskowski et al., 1991; Jiang. 1991; Laskowski, 2002; Vianna et al., 2003; Honaker and Ozsever, 2003).

A variable content of useful component, especially in the ores upgraded by flotation, requires deep milling for liberation of the minerals. Flotation comprises very fine particles (from a few to several tens microns) which, in given physicochemical and hydrodynamic conditions, either are able to make up a stable flotation aggregate or are taken by the rotating medium and are mechanically transported into the concentrate. In numerous works concerning both ores and coal the authors are still trying to describe the effect of the finest particles upon upgrading effectiveness, both theoretically and technologically. They are explaining the mechanisms which determine forming of flotation aggregates, simultaneously setting the limits of particle sizes which are subjected to classical flotation (De Bruyn and Modi, 1956; Collins and Jameson, 1976; Trahar and Warren, 1976; Scheludko et al., 1976; Bustamante and Warren, 1983; Crawford and Ralston, 1988; Drzymala, 1994a; 1994b; Malysa, 2000; Miettinen et al., 2010; Kowalczuk et al., 2011; Chipfunhu et al., 2011).

The analysis of the effect of particle size on flotation properties of coal in the batch flotation process is presented in this article.

The use of partition curves for the analysis of enrichment results is a common method of the separation accuracy evaluation and parameters calculated on the basis of those curves are the indicator of separation devices' quality.

Flotation enrichment in a machine with mechanical agitation of flotation pulp, occurs inherently in turbulent conditions (Mika and Fuerstenau, 1968).

Such a regime of separation leads to the phenomenon of particle detachment and transition the rules of useful components to tailings. In connection with this, the separation process is subject to the probability calculus. The statistical character of the separation process is expressed numerically by the partition function and its graphical representation – the partition curve (Tromp's curve), determining the probability of finding a particle with specific properties occurring in a given separation product.

The separation efficiency in industrial practice as said above, is measured by probable error. In flotation conditions the value of probable error depends on factors related to physical and physicochemical properties of particles, flotation machine characteristics and parameters related to the reagent regime in the flotation chamber.

The consequence of particle scattering is consist in their transition to improper products and this fact depends on the relation between forces acting on each particle. These comprise forces facilitating particle detachment from the air bubble and the particle adhesion force to an air bubble.

The partition number (recovery) for particles of the cell product (tailings) is expressed by the following formula:

$$T = \frac{n_k}{n_k + n_p} \tag{1}$$

where: n_k – the number of floatable particles of a narrow size fraction with given flotation properties occurring in the cell product, n_p – the number of particles with given flotation properties occurring in the froth product.

Formula (1) presents a fraction of the total number of floatable particles with given flotation properties which remained in a cell science they were subject to detachment. It may also determine the probability of a particle to occur in the cell product (tailings). This probability is equivalent to the probability of particle detachment from an air bubble, so in the case of flotation the partition number for tailings is the probability of particle detachment from the air bubble.

The analysis of the effect of particle sizes, on flotation properties of coal in the batch flotation process is presented in this article.

Empirical studies were conducted on samples of bituminous coal from the Jankowice coal mine (33 type in Polish classification). They include performing fractional flotation of the raw material samples with various particle size, the float and sink analysis of separation products and determination of the ash content in all density fractions.

The evaluation of the enrichment process was based on the partition curve and the value of heterogeneity index (standard deviation) with the splitting into the concentrate and tailings.

Experimental

Method of samples preparation

The coal sample was crushed in two stages, at first in a jaw crusher and then in a roll crusher to the particle size below 0.63 mm. The purpose of such a method of crushing was to limit the yield of the finest particles, below 0,1 mm, in the crushing product. The obtained crushing product was sieved to obtain the following size fractions: 0.1–0.2 mm, 0.2–0.315 mm, 0.315–0.4 mm, 0.4–0.5 mm and 0.5–0.63 mm. A sample with a wide range of particle sizes of 0.1–0.63 mm was made up from the narrow size fractions and with equal weight yield of respective size fractions in order to determine the degree of independence of the narrow particle size flotation process. The ash content was determined in all samples. Each sample was washed on a screen 0.063 mm before flotation in order to remove fine dust which could have affected the flotation results.

Fractional flotation

Flotation tests were carried out in a Denver type mechanical flotation machine with flotation cell volume of 1 dm^3. *n*-hexanol was used as a collecting-frothmaking reagent. It does not change pulp pH but only reduces the surface tension on the gas–liquid phases boundary, increasing at the same time air dispersion in the suspension. Also the adsorption of alcohol occurs on the surface of air bubbles which ensures their stabilization and prevents coalescence (Malysa, 2000; Krzan and Malysa, 2002). A solution with concentration of 0.08 g/dm^3 of hexanol was prepared. The solids content of 80 g in 1 dm^3 of the flotation suspension, is equivalent to the consumption of the reagent in the amount of 1000 g per 1 Mg of dry feed. Low concentration of flotation pulp was kept because of practical reasons because favourable coal flotation results can be achieved at low pulp density (Sablik, 1998).

A sample with the mass of 80 g was moistened in 1 dm^3 of solution in the flotation cell for 15 minutes, with the impeller off. Then the suspension was agitated for 5 minutes without the air inflow to the flotation cell, which facilitated better reagent adsorption on the surface of floated particles. After turning the air inflow on, fractional flotation was conducted, gathering subsequent fractions of the froth product for: 15 s, 15 s, 30 s, 60 s and 60 s. Total flotation time was equal to 180 s (up to white froth). The tailings product comprised the remains in the flotation cell.

Flotation of all six samples was conducted in the same conditions. The ash content was determined in flotation products.

Float an sink analysis of flotation products

All products of fraction flotation were subject to the float and sink analysis in carbon tetrachloride solutions were performed. The liquid maximum density was equal to the density of pure CCl_4, that is 1.60 Mg/m^3. Lower values of liquid density; 1.35 and 1.225 Mg/m^3, were obtained by dilution with anhydrous ethyl alcohol. The choice of liquid density was conditioned by the mass of the obtained densimetric fractions in each flotation product, which should be large enough to enable the ash content determination. The lowest particle density was equal to 1.20 Mg/m^3, because all particles a in the liquid to occur such density. In the fraction with density higher than 1.60 Mg/m^3, the average density of particles was determined by the pycnometer method, the values of which were given in the brackets. The ash content was determined in all densimetric fractions.

Results and discussion

Flotation kinetics

The fractional flotation results are presented in Table 1, in which the cumulative yields, the ash contents in the fractions and coordinates of the upgrading curve are concentrate grade θ, tailings grade β, recovery combustible and volatile matter in the concentrate ε and ash recovery in tailings ε' (Figs 1–6).

Flotation kinetic curves of the tested size fractions are illustrated in Fig. 7. Their analysis indicates that for all size fractions, apart from the largest size fraction (0.5–0.63 mm), in the time interval from 0 to about 20 s, the flotation kinetics proceeds according to the ½ order kinetics equation (Bogdanov 1959, Brozek and Mlynarczykowska, 2006). With the increase in flotation time, the order of flotation kinetics increases (Pogorelyj, 1962). It can be the sum of orders of kinetics ½ and 1 or ½ and 3/2. For the size fraction 0.5–0.63 mm, most probably it is a sum of orders of kinetics ½ and 3/2 in the whole flotation time interval. The ½ order kinetic curve is represented by a sector of parabola. The limit of the derivative of the function of the concentrate yield dependence to time $\gamma_k(t) = R(t)$, for the ½ order kinetics is equal to (Brozek and Mlynarczykowska, 2007):

$$\lim_{t \to 0} \frac{dR}{dt} = \frac{k}{\sqrt{C_o}} \tag{2}$$

where: R – recovery of flotation, k – flotation rate constant of the ½ order, C_0 – the initial concentration of particles under flotation in the flotation cell, t – time of flotation.

The limit of the derivative, given by equation (2), presents the slope of the flotation kinetics curve at the point $t = 0$. As it can be seen from Fig. 7, the slope of the flotation kinetics of the initial time of flotation decreases with the increase of particle size. Respectively for the constant value of the solids concentration in suspension C_o = const. (80 g/dcm^3 = 0.074), with a decreasing slope of the function $R(t)$ at the point $t = 0$, the flotation rate constant for the ½ order kinetics decreases with the increase of particle size.

The exact determination of the kinetics equations, including the flotation of order changes during the process, will be subject to separate discussions and publication.

Table 1. The results of fractional flotation and indices of flotation upgrading

Particle size [mm]	Produkt name	Flotation time [s]	Ash A [%]	$\sum\gamma\downarrow$ [%]	θ [%]	$\sum\gamma\uparrow$ [%]	β [%]	ε' [%]	ε [%]	σ_0 [–]	σ_{th} [–]	σ_0/σ_{th} [–]
0.1–0.63	K1	15	12.40	32.06	12.4	100	37.93	100	45.24	0.237	0.604	0.39
	K2	30	17.01	41.39	13.44	67.94	49.97	89.52	57.72			
	K3	60	19.85	49.90	14.53	58.61	55.22	85.33	68.70			
	K4	120	28.81	57.03	16.32	50.10	61.22	80.88	76.87			
	K5	180	53.66	59.93	18.13	42.97	66.60	75.46	79.04			
	Tailings		67.53	100	37.93	40.07	67.53	71.35	100			
	Feed			100								
0.5–0.63	K1	15	7.99	11.37	7.99	100	33.51	100	15.73	0.161	0.472	0.34
	K2	30	6.91	17.61	7.61	88.63	36.79	97.29	24.47			
	K3	60	10.71	24.36	8.47	82.39	39.05	96.00	33.53			
	K4	120	11.08	31.78	9.08	75.64	41.58	93.85	43.46			
	K5	180	12.22	36.45	9.48	68.22	44.90	91.39	49.62			
	Tailings		47.3	100	33.51	63.55	47.30	89.69	100			
	Feed			100								
0.4–0.5	K1	15	8.17	23.3	8.17	100	42.16	100	36.99	0.273	0.635	0.43
	K2	30	13.01	30.93	9.36	76.70	52.48	95.49	48.47			
	K3	60	16.49	38.97	10.83	69.07	56.84	93.13	60.07			
	K4	120	22.73	45.33	12.50	61.03	62.16	89.99	68.57			
	K5	180	36.25	49.20	14.37	54.67	66.74	86.56	72.84			
	Tailings		69.07	100	42.16	50.80	69.07	83.23	100			
	Feed			100								
0.315–0.4	K1	15	10.47	32.69	10.47	100	40.14	100	48.89	0.287	0.632	0.45
	K2	30	17.67	42.62	12.14	67.31	54.56	91.48	62.55			
	K3	60	20.00	51.64	13.52	57.38	60.94	87.11	74.62			
	K4	120	32.08	58.49	15.69	48.36	68.58	82.61	82.39			
	K5	180	50.58	61.60	17.45	41.51	74.61	77.14	84.96			
	Tailings		76.55	100	40.14	38.40	76.55	73.22	100			
	Feed			100								
0.2–0.315	K1	15	12.60	45.47	12.60	100	37.36	100	63.43	0.260	0.484	0.54
	K2	30	23.84	56.56	14.81	54.53	57.99	84.66	76.92			
	K3	60	31.92	65.85	17.22	43.44	66.71	77.58	87.02			
	K4	120	56.59	72.42	20.79	34.15	76.18	69.64	91.57			
	K5	180	63.86	75.43	22.51	27.58	80.85	59.69	93.30			
	Tailings		82.93	100	37.36	24.57	82.93	54.55	100			
	Feed			100								
0.1–0.2	K1	15	16.46	50.47	16.46	100	36.89	100	66.80	0.245	0.482	0.51
	K2	30	23.38	62.05	17.75	49.53	57.71	77.48	80.87			
	K3	60	31.39	70.46	19.38	37.95	68.18	70.14	90.01			
	K4	120	59.60	77.16	22.87	29.54	78.65	62.99	94.30			
	K5	180	71.51	80.03	24.62	22.84	84.24	52.16	95.59			
	Tailings		86.07	100	36.89	19.97	86.07	46.60	100			
	Feed			100								

To determine the degree of independence of flotation of particles of different sizes, the flotation of the wide size fraction 0.1–0.63 mm was conducted and compared with the kinetic curve for this particle size reproduced from the sum of curves for narrow size fractions with the weight of 0.2 for each of them. Figure 8 presents both kinetics curves. As it can be seen, the kinetics course is similar and these curves practically overlap. On this basis a conclusion can be drawn that particular particle size fractions float independently from each other.

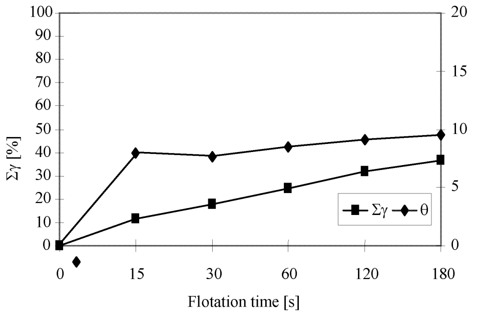

Fig. 1. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.5–0.63 mm

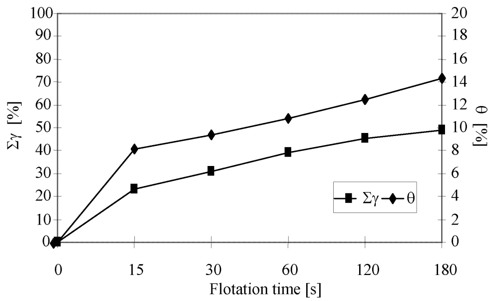

Fig. 2. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.4–0.5 mm

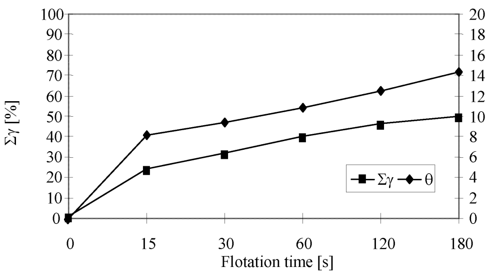

Fig. 3. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.4–0.315 mm

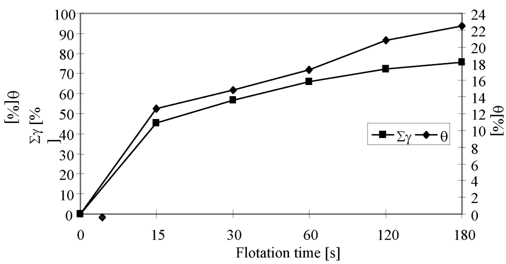

Fig. 4. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.2–0.315 mm

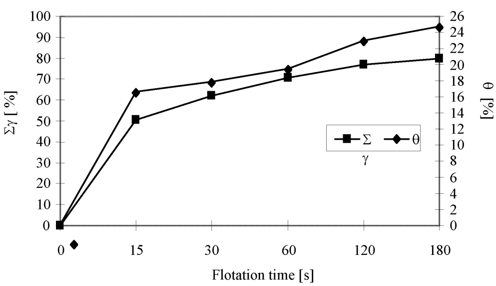

Fig. 5. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.1–0.2 mm

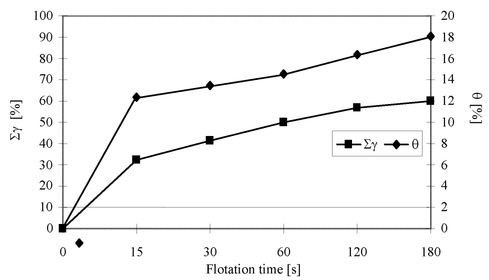

Fig. 6. Flotation yield and ash content
in the concentrate in the function of time
for particle size 0.1–0.63 mm

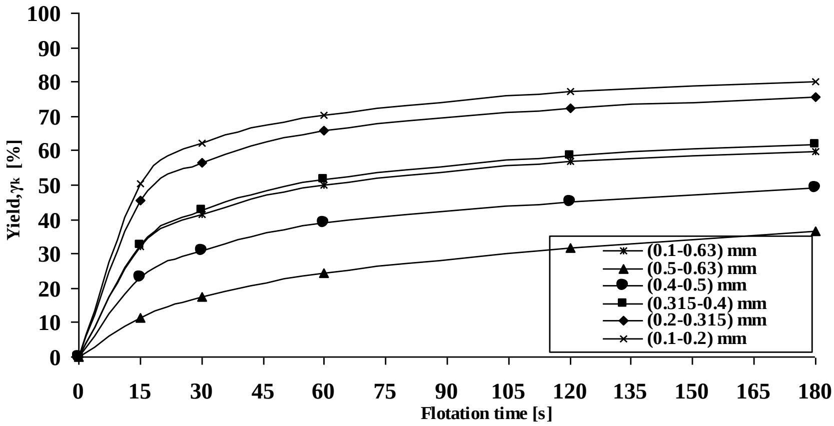

Fig. 7. Flotation kinetic curves of narrow size fractions

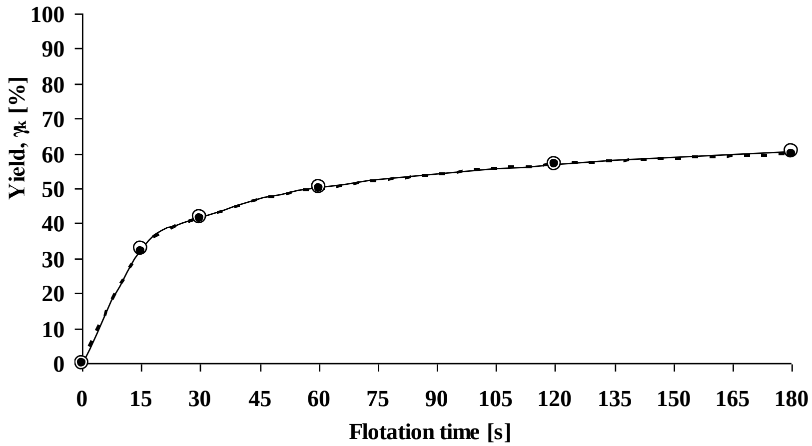

Fig. 8. Comparison of flotation kinetics of wide size fraction and sums of flotation of all narrow size fractions

Float and sink analysis of flotation products

The results of the float and sink analysis of concentrates and tailings of respective size fractions, in the liquid of density 1.6 Mg/m^3, are presented in Table 2.

The results given in Table 2 will be used to calculate, the probability of particle detachment from an air bubble in the next section.

As it results from the experimental data contained in Figures 9-13, in all cases the ash content in the flotation concentrate is higher than the ash content in the heavy liquid concentrate at the same yield. Differences in ash contents are generally larger for fine grain classes to the disadvantage of flotation. As it will be discussed in the next part of the paper, it is the result of the phenomenon of floating of hydrophilic particles with the high mineral substance content to the froth product.

Table 2. Results of float and sink analysis in the liquid of density of 1.6 Mg/m^3

Size fraction [mm]	Density CCl_4 [Mg/m^3]	Products name	Density fraction [Mg/m^3]	Mass [g]
0.5–0.63	1.6	concentrate	< 1.6	50.95
			>1.6	3.60
		tailings	< 1.6	38.72
			>1.6	60.93
0.4–0.5	1.6	concentrate	< 1.6	66.05
			>1.6	7.30
		tailings	< 1.6	8.96
			>1.6	70.65
0.315–0.4	1.6	concentrate	< 1.6	80.65
			>1.6	13.41
		tailings	< 1.6	2.09
			>1.6	57.11
0.2–0.315	1.6	concentrate	< 1.6	87.49
			>1.6	27.12
		tailings	< 1.6	0.20
			>1.6	37.27
0.1–0.2	1.6	concentrate	< 1.6	99.77
			>1.6	18.36
		tailings	< 1.6	0.68
			>1.6	29.63

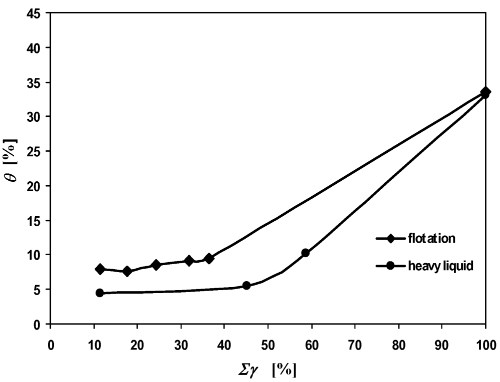

Fig. 9. Ash content in concentrate as a function of yield for particle size 0.5–0.63 mm

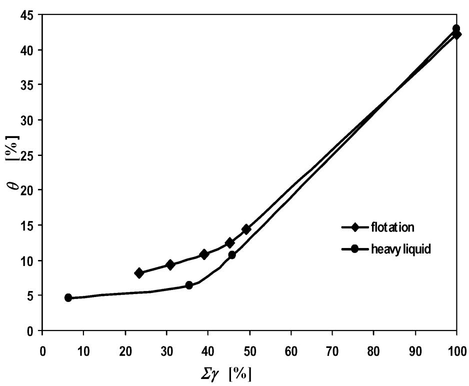

Fig. 10. Ash content in concentrate as a function of yield for particle size 0.4–0.5 mm

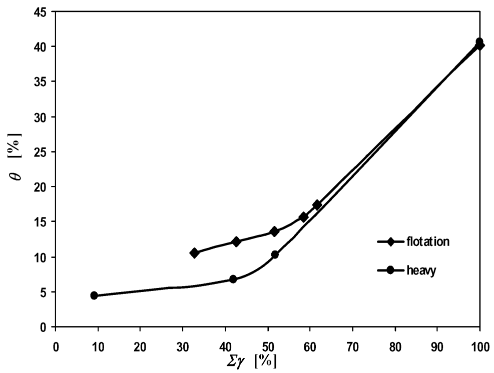

Fig. 9. Ash content in concentrate as a function of yield for particle size 0.315–0.4 mm

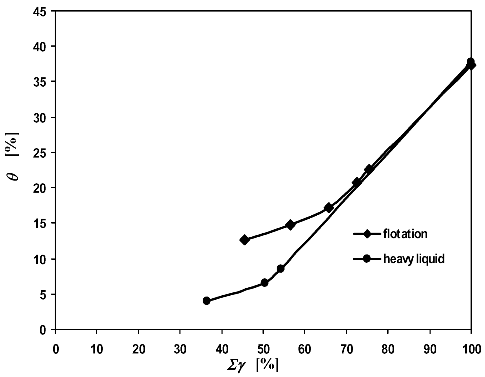

Fig. 10. Ash content in concentrate as a function of yield for particle size 0.2–0.315 mm

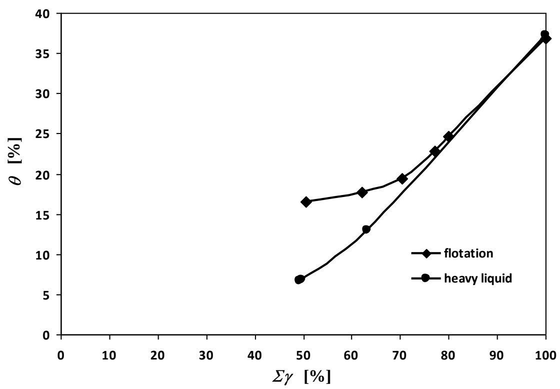

Fig. 13. Ash content in concentrate as a function of yield for particle size 0.1–0.2 mm

Evaluation of flotation

Partition curve

According to the dependence of the flotation rate constant on particle size, the yield of K1 product, as shown in Table 1, after flotation time of 15 s, decreases with the increase in size fraction from the value of 50.47% for the size fraction 0.1–0.2 mm to the value of 11.37% for the size fraction 0.5–0.63 mm.

The concentrate yield after flotation time of 180 s (up to empty froth) decreases with the increase in particle size from the value of 80.03% for the size fraction 0.1–0.2 mm to the value of 36.45 % for the size fraction 0.5–0.63 mm. The decrease in the concentrate yield with the increase in particle size is equivalent to the increase in the tailings yield. The increase in the tailings yield is related to the increase in the probability of particle detachment.

As it result from comes out of the data (Table 2), the flotation tailings of the finest particle size fraction 0.1–0.2 mm, contain merely 0.68 g of fraction with the density below 1.6 Mg/m^3, while the concentrate contains 99,77 g. For this reason, it was assumed that floated particles are these with hydrophobic properties corresponding to the density (related to the mineral matter content) below 1.6 Mg/m^3. It is coal for which the density of particles is connected with the content of mineral matter which can be described by means of ash content. The lower the density of particles, the higher the content of organic coal matter, low ash content and better floatability. The mutual relations of the mentioned features have been described in the following works Brozek (1995a, b, c), Brozek and Mlynarczykowska (2005).

Therefore, the probability of detachment of a particle from a given particle size fraction, according to formula (1), was calculated as a relation between the mass of particles with the density below 1.6 Mg/m^3 in tailings and the mass of particles with this density in the feed. Particle size, being the arithmetic mean of lower and upper limit of the size fraction, was assigned to the probability calculated in this way. The particle size 0.2–0.315 was omitted due to too weak partition for tailings. The detachment probability are as follows:

- for the size fraction 0.1–0.2 mm

$$P_d(0.15\ \mathrm{mm}) = \frac{0.68\ \mathrm{g}}{100.45\ \mathrm{g}} \cdot 100\% = 0.68\% \tag{3a}$$

- for the size fraction 0.315–0.4 mm

$$P_d(0.357\ \mathrm{mm}) = \frac{2.09\ \mathrm{g}}{82.74\ \mathrm{g}} \cdot 100\% = 2.52\% \tag{3b}$$

- for the size fraction 0.4–0.5 mm

$$P_d(0.45\ \mathrm{mm}) = \frac{8.96\ \mathrm{g}}{75.01\ \mathrm{g}} \cdot 100\% = 11.94\% \tag{3c}$$

- for the size fraction 0.5–0.63 mm

$$P_d(0.57\ \mathrm{mm}) = \frac{38.72\ \mathrm{g}}{89.67\ \mathrm{g}} \cdot 100\% = 43.17\%\,. \tag{3d}$$

In Fig. 14, the dependence of detachment probability on particle size and at the same time the partition curve giving the probability for a particle with the density below 1.6 Mg/m^3 of getting to tailings was plotted. The partition size approximated from the partition curve and simultaneously maximum floatable particle size in the physicochemical conditions in the flotation cell, assumed in this paper, is equal to d_{50} = 0.58 mm.

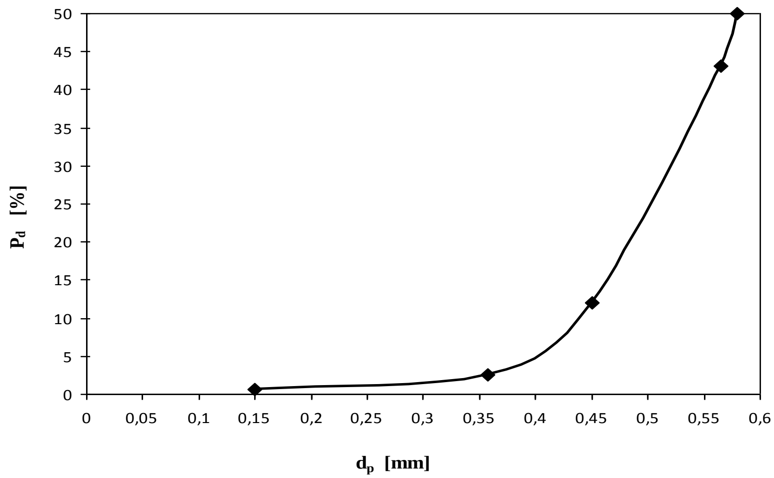

Fig. 14. Dependence of the detachment probability on particle size

Qualitative evaluation of flotation effectiveness

The qualitative data of flotation products are given in Table 1. This is the ash content in respective fractions *A*, concentrate grade θ, the yield of concentrate $\sum\gamma\downarrow$, recovery of combustible and volatile matter in the concentrate ε, tailings grade β , the yield of tailings $\sum\gamma\uparrow$ and recovery of ash in tailings ε'. On the basis of this data, the following observations con be found. The ash content in tailings decreases with the increase in particle size. It is caused by the fact that particles with the low ash content of the density below 1.6 Mg/m^3 get to tailings as a result of detachment, intensity of which increases with the increase in particle size (as shown in Fig. 14). Also, for this reason the yield of tailings increases with the increase in particle size. With the increase of particle size, the recovery of ash in tailings increases as well. This happens at the expense of a significantly greater increase in the yield of tailings (through the phenomenon of detachment) with the increase of particle size (from 19.97% for the size fraction 0.1–0.2 mm to 63,55% for the size fraction 0.5–0.63 mm, that is the increase by 3.18 times) than the decrease in the ash content with the increase in particle size (from 86.07% for the size fraction 0.1–0.2 mm to 47.30 for the size fraction 0.5–0.63 mm, that is the decrease by 1.8 times). This is, with the increase in particle size, tailings with relatively low, in comparison to the feed, the ash content (upgrade ratio β/α = 1.4 for the size fraction 0.5–0.63 mm) but with high yield are obtained. On the other hand, for the finest particle fraction, tailings with the high ash content (upgrade ratio β/α = 2.33) and low yield are obtained. Such a situation causes the increase in the combustible and volatile matter loss in tailings with the increase in particle size from 4.61% for the size fraction 0.1–0.2 mm to 50.38% for the size fraction 0.5–0.63 mm. As shown in Table 1, with the increase in particle size, the yield of the concentrate decreases as well as the ash content decreases in the concentrate. The decrease in the yield is related to the phenomenon of low-ash particles' detachment, which get to tailings and increase the loss of combustible and volatile matter in tailings. On the other hand, the decrease of the concentrate grade with the increase of particle size or, the other way round, the increase of the concentrate grade with the decrease of particle size is the result of entrainment, called mechanical flotation of gangue particles (hydrophilic particles) to the concentrate (Konopacka, 2005). The intensity of this phenomenon increases with the decrease in floated particle size as well as with the increase in flotation rate. Mechanical flotation causes deterioration of the concentrate quality and increase in ash remnants in the concentrate at decreasing the floated particle size from 10.31 % for the size fraction 0.5–0.63 mm to 54.40% for the size fraction 0.1–0.2 mm. To sum up, it can be said that the phenomenon of detachment concerns larger particles and causes the loss in combustible and volatile matter in tailings; on the other hand, the phenomenon of entrainment, characteristic of smaller particles, increases ash remnants in the coal concentrate. Both phenomena decrease the flotation effectiveness.

For determining the optimum enriched particle size, because of the phenomena of large particles detachment and entrainment of fine particles, on the basis of the results given in Table 1, the degree of heterogeneity was calculated for flotation of particular size fractions with splitting into the concentrate and tailings. The degree of heterogeneity can be the measure of differentiation of a certain feature or the component content (characteristic of a given raw material) in the separation products. With the splitting into the concentrate and tailings, the degree of heterogeneity is expressed by the following formula (Stepinski 1964):

$$\sigma_a = \sqrt{\gamma_k (\theta-\alpha)^2 + (1-\gamma_k)(\beta-\alpha)^2} \tag{4}$$

where: γ_k – the yield of the concentrate, α – feed grade, θ –the concentrate grade, β – the tailings grade.

From the mathematical point of view, this is the standard deviation of a random variable, such as the ash content in the case of coal. The larger value of the heterogeneity index, the more diverse the separation products, i.e. the better separation. With the accurate separation, the ash content in the concentrate and tailings, respectively, are equal to: $\theta = 0$ and $\beta = 1$.

In this case the degree of heterogeneity is as follows:

$$\sigma_a = \sqrt{\beta^2 + \alpha^2 - 2\beta\alpha - \gamma_k\beta^2 + 2\beta\alpha\gamma_k}\ . \tag{5}$$

On the basis of the balance equation

$$\alpha = \gamma_k\theta + (1-\gamma_k)\beta \tag{6}$$

for the ideal separation $\alpha = (1 - \gamma_k) = \gamma_o$ and $\gamma_k = (1 - \alpha)$ the degree of heterogeneity has the maximum value and is equal to:

$$\sigma_{th} = \sqrt{\alpha - \alpha^2}\,. \tag{7}$$

With the lack of separation $\theta = \alpha$ and $\beta = \alpha$ and from the formula (4) it follows that $\sigma_a = 0$.

On the basis of flotation results given in Table 1, the real σ_0 and theoretical σ_{th} degree of heterogeneity and the relation of those values σ_a / σ_{th} as a measure of flotation efficiency. The calculation results are included in Table 1. With the increase in particle size, the degree of heterogeneity increases and reaches the maximum value for particle fractions 0.315–0.4 mm, and then it decreases for larger particles. The characteristics of the dependence of the degree of heterogeneity on particle size is analogous with many flotation processes – the dependence of the flotation rate constant on particle size.

A similar tendency is demonstrated by the dependence of flotation efficiency on particle size. The relation σ_a / σ_{th} has the maximum value for particles from the fraction 0.2–0.315 mm. Thus, it can be said that within this particle size interval, because of the low probability of detachment, there is low scattering of hydrophobic particles to tailings.

Conclusions

1. From the comparison of flotation kinetics of wide particle fractions to kinetics obtained from the sum of kinetic curves for narrow size fraction, with appropriate weights, it appears that particles with different sizes float independently from each other. Knowing the particle size distribution of the feed and flotation kinetics of each size fractions, the flotation course of the feed can be forecast.
2. The ash content in tailings decreases with the increase in particle size. It is caused by the fact that particles with the low ash content with the density below 1.6 Mg/m^3 get to tailings as a result of detachment, intensity of which increases with the increase in particle size. This causes the increase in the loss of combustible and volatile matter in tailings with the increase in particle size.
3. The yield of the concentrate decreases with the increase in particle size. The decrease in the yield is related to the phenomenon of detachment of low-ash particles, which get to tailings increasing by this the loss of combustible and volatile matter in tailings.
4. The ash content increases in the concentrate with the decrease in particle size. It is related to the effect of entrainment of gangue (hydrophilic particles) to the concentrate. The intensity of this phenomenon increases with the decrease in floated particle size as well as with the increase in the flotation rate. This causes the increase in ash remnants in the concentrate.

5. To sum up, it can be said that the phenomenon of detachment concerns larger particles and causes the increase in the loss of combustible and volatile matter in tailings; nevertheless, the phenomenon of mechanical flotation, characteristic of smaller particles, increases ash remnants in the coal concentrate. Both phenomena decrease the flotation effectiveness.
6. The degree of heterogeneity increases with the increase in particle size and reaches the maximum value for particles of the size fraction 0.315 – 0.4 mm, then decreases for larger particles. The characteristics of the dependence of the degree of heterogeneity on particle size is analogous as in many flotation processes – the dependence of the flotation rate constant on particle size.

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