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APPLICATION OF THE OBSERVATIONAL TUNNELS METHOD TO SELECT A SET OF FEATURES SUFFICIENT TO IDENTIFY A TYPE OF COAL

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Abstract: Coal is a material which has many features deciding about its quality. Among them, the decisive ones are mainly ash contents, sulfur contents and combustion heat. The paper presents the investigation of coal characteristics of three selected coal types in the context of their energetic value. For this purpose samples were collected from three different Polish mines: coal types 31, 34.2 and 35 (Polish classification of coals). Each of these materials was separated into particle size fractions (9 fractions) and then into 8 density fractions by separation in heavy liquids. For each size-density fractions obtained in this way, chemical analyses were performed which allowed for determination of such features as combustion heat, sulfur contents, ash contents, volatile parts contents and analytical moisture. Altogether, seven dimensions of grained material characteristics were obtained. The data prepared in this way was subsequently analyzed for correlation with the purpose of determining significant relations between investigated features. It was stated that the most correlated coal features are density, combustion heat, ash contents and volatile parts contents.

For multidimensional analysis and identification of coal type, the modern image visualization technique, the Observational Tunnels Method, was applied. After performing seven-dimensional analysis aimed at the proper recognition of coal type, it was decided to determine the minimum amount of random variables, which describe a particular material in order to identify its type. It was stated that the crucial coal identification parameter is “analytical moisture”. Due to existing correlation between individual features, three of them were selected for testing: analytical moisture, sulfur contents and volatile parts contents. On the basis of the obtained images, it was stated that it was possible to obtain a view with the data concerning each type of coal being located in other part of the space. Subsequently, it was checked if a similar result is possible when the parameter “volatile parts contents” is replaced with highly correlated parameters “combustion heat” and “ash contents”. In both cases the exchange of these variables did not produce good enough results. This can be explained by a different scale of empirical data making it impossible to obtain a clear multidimensional image for which all three types of coal would be located in

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other parts of space. However, it was proved that the modern graphical and computer methods can be successfully applied to identify the types of particulate materials.

Keywords: *multidimensional statistical analysis, observational tunnels method, coal, image visualization, energetic materials*

Introduction

Coal as energetic material is characterized by multiple parameters determining its quality. Depending on the studied type of coal (energetic, coking or semi-coking), these parameters differ significantly and allow for identifying the type of coal on the basis of its characteristics. In the paper a wide analysis of coals from three selected mines located in the Upper Silesia is carried out. They included coals types 31, 34.2 and 35 (according to Polish classification of coals). To evaluate their quality correctly, it is necessary to conduct the multidimensional statistical analysis. There are many techniques of such analysis, including:

- multidimensional distribution functions of random vector X (Lyman, 1993; Niedoba, 2009; 2011; Olejnik et al., 2010; Niedoba and Surowiak, 2012), where X is the vector describing coal properties,
- multidimensional regressive equations with the analysis of matrix coefficients of linear correlation and partial correlation between individual coal features (Niedoba, 2013; Tumidajski and Saramak, 2009),
- factor analysis (Stanisz, 2007; Tumidajski and Saramak, 2009),
- other methods including visualization by observational tunnels method (Jamroz, 2001), parallel coordinates and visualization of relations between multidimensional blocks (Jamroz, 2009).

The multidimensional distributions of vector X treated as random vector and their practical applications are widely described in the literature and will not be the subject of this paper. The other methods listed above are connected to some extent with the contents of this paper.

To carry out analyses for more than 3 dimensions, it is suitable to apply modern visualization methods which allow simultaneous analysis of many features of grained materials. One of such methods is observational tunnels method which was applied in the paper. These methods are becoming more and more useful in modern applications, which is reflected in many papers (Aldrich, 1998; Assa et al., 1997; 1999; Chatterjee et al., 1993; Chou et al., 1999; Cook et al., 1995; Heike, 2000; Hurley and Buja, 1990; Kim et al., 2000; Kraaijveld et al., 1995; Li et al., 2000). The observational tunnels method is connected with existing relations between individual coordinates of vector X (where X represents individual coal features). However, while the classical analysis of correlation is based on calculating the individual partial correlation coefficients without considering other features, the observational tunnels method considers projections of all coordinates taken together, giving a graphical image as a result. The competent change of the viewpoint gives the possibility of obtaining a view based on which it is possible to identify the type of coal and information concerning differences between investigated materials. It also allows to select features which decide about the type of coal the investigated material is to be qualified for.

The matrices of linear coefficients and partial correlations are usually connected with existing linear models of relations between researched random variables of vector X. The coefficients of linear correlation are determined for pairs of random variables totally irrespective of other variables. The partial correlation coefficients are determined on the basis of the matrix of coefficients of linear correlation taking the role of other variables in certain equation of linear regression into consideration. In the case of analysis of three random variables from which one is treated as a dependent variable and two others as independent ones, it leads to determination of correlation coefficients for projections of points parallel to regressive plane. It allows to determine the hierarchy (power of influence) of relations between variables in researched system. On the basis of matrix of linear coefficients of correlation the factor analysis can be performed which allows for grouping the existing variables into the so-called factors representing joint influences of variables according to the results of investigated processes. Consequently, some sort of classification must be provided.

This paper also presents the methods of visualization of multidimensional data which make it possible to draw comparisons between the investigated data sets and suggest possibilities of their classification. They are a sort of continuation and development of the methods discussed above.

Observational tunnels method

The theoretical underpinnings of the Observational Tunnels Method were described in a paper by Jamroz (2001). Intuitively, it may be said that the method of observational tunnels makes use of a parallel projection with a local orthogonal projection of an extent limited by the maximal radius of the tunnel. This solution makes it possible to observe selected parts of space containing important information, which is not possible in the case of orthogonal projection. The method of projection used in this paper is roughly presented in Fig. 1. The observational plane P will be used as a screen through which any object placed in space X will be viewed. This observational plane $P \subset X$ is defined as: $P=\delta(w,\{p_1,p_2\})$, where:

$$\delta\left(w,\{p_1,p_2\}\right) \overset{def}{=} \{x \in X : \exists \beta_1, \beta_2 \in F, \text{ such that } x = w + \beta_1 p_1 + \beta_2 p_2\}, \qquad (1)$$

X is any n-dimensional ($n \geq 3$) vector space, over an F field of real numbers, with a scalar product.

Vector w will indicate the position of the screen midpoint, whereas p_1,p_2 will indicate its coordinates. Let us assume for the moment that the space X is 3-dimensional (an example assuming a space with more dimensions would be more difficult to conceive) and that observational plane P is 1-dimensional (i.e. it is possible to observe the pertinent reality not through a segment of 2-dimensional plane but through a segment of a line). Additionally, let us take vector r, being the proper direction of projection onto the observational plane P (The proper direction of projection r onto the observational plane $P=\delta(w,\{p_1,p_2\})$ is defined as any vector $r \in X$ if vectors $\{p_1, p_2, r\}$ are an orthogonal system). Let's determine $k_{a,r}$ (i.e. a line parallel to r and passing through a) for observed point a. As shown in Fig. 1, the line $k_{a,r}$ need not have common points with P. However, $k_{a,r}$ always has one common point with hypersurface S containing P and being orthogonal to r. (the hypersurface $S_{(s,d)}$, anchored in $s \in X$ and directed towards $d \in X$ is defined as:

$$S_{(s,d)} \stackrel{def}{=} \{x \in X : (x - s, d) = 0\}). \tag{2}$$

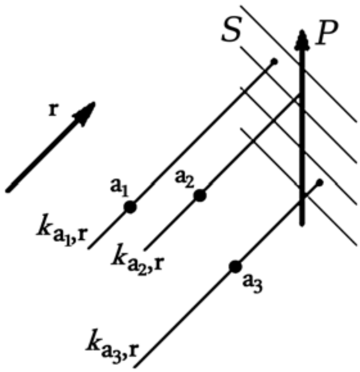

Fig. 1. Presentation of the rules of projection on plane P in the observational tunnels method
A line parallel to r and passing through a does not have to have common points with P. However, it always has exactly one common point with hypersurface S containing P and being orthogonal to r. In the above mentioned case only point a_2 will be visible using observational plane P.

In practice, some points could be viewed only at some orientations of observational plane P . This implies that in the majority of cases, when viewing a set of points using observational plane P, nothing will be seen. In order to avoid such situation, let us assume that the points visible on observational plane P do not only include points situated on lines parallel to r and passing through P, but also the points which are situated on lines parallel to r and passing through S (i.e. the hypersurface containing P and orthogonal to r) within a smaller distance from observational plane P than a certain fixed value. This distance for observed point a will be represented by vector b_a called the tunnel radius:

$$b_a = \psi r + a - w - \beta_1 p_1 - \beta_2 p_2, \tag{3}$$

where:

$$\psi = \frac{(w-a\,,r)}{(r\,,r)},\ \beta_1 = \frac{(\psi r + a - w,\, p_1)}{(p_1\,,p_1)},\ \beta_2 = \frac{(\psi r + a - w,\, p_2)}{(p_2\,,p_2)}\,. \tag{4}$$

In Eq. 4 $r \in X$ denotes a proper direction of projection onto observational plane P.

In the case presented in Fig. 2, at the point e of observational plane P, all points situated in the tunnel whose intersection is a segment of and which is spreading along r will be visible. However, generally, at the point e on the observational plane P, all points situated in the tunnel whose intersection is n-3 dimensional sphere and spreading along the direction of projection r will be visible.

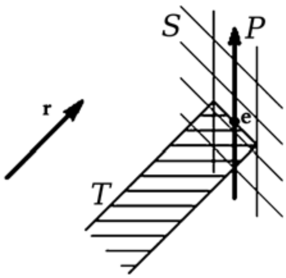

Fig. 2. Way of choosing observational tunnel T
Tunnel T for point e is shown. (The area hatched with horizontal lines). All points that belong to tunnel T will be visible at point e of observational plane P

The algorithm below should be followed in order to draw the projection of observed point a consistently with the direction of projection r onto observational plane $P = \delta(w,\{p_1,p_2\})$:

1. the distance of projection of observed point a is to be calculated using the formula:

$$\psi = (w - a,r)/(r,\, r) \tag{5}$$

2. the position of the projection (i.e. the pair β_1, $\beta_2 \in F$) of observed point a is to be calculated using the formula:

$$\beta_1=(\psi r + a - w,p_1)/(p_1, p_1),\ \beta_2=(\psi r + a - w,p_2)/(p_2, p_2) \tag{6}$$

3. the tunnel radius b_a of point a is to be calculated using the definition (3)
4. at this point it should be verified whether the scalar product (b_a,b_a) is lower than the maximum tunnel radius determined at a given time and whether the distance of the projection of observed point a is shorter than the maximum range of view

determined at a given time. If this is the case, then one should draw a point on observational plane $P=\delta(w,\{p_1,p_2\})$ in the position of coordinates (β_1,β_2), otherwise the point should not be drawn.

The scalar product is to be calculated using the formula:

$$(x, y) = \sum_{i=1}^{n} x_i y_i \,, \tag{7}$$

where: $x = (x_1, x_2, ..., x_n)$, $y = (y_{1,}, y_2, ...y_n)$, n - number of dimensions, $n \geq 3$.

Experimental

Three types of coal, types 31 (energetic coal), 34.2 (semi-coking coal) and 35 (coking coal) in the Polish classification were used in the investigation. They originated from three various Polish coal mines and all of them were initially screened on a set of sieves of the following sizes: –1.00, –3.15, –6.30, –8.00, –10.00, –12,50, –14.00, –16.00 and –20.00 mm. Then, the size fractions were additionally separated into density fractions by separation in dense media using zinc chloride aqueous solution of various densities (1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 g/cm^3). The fractions were used as a basis for further consideration and additional coal features were determined by means of chemical analysis. For each density-size fraction such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents and analytical moisture were determined, making up, together with mass of these fractions, seven various features for each coal. The examples of such data were presented in tables 1–3 showing the data for size fractions 1.00–3.15 mm for each type of coal.

Table 1. Data for size fraction 1.00–3.15 mm – coal, type 31

Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
<1.3	4187.8	7367	1.25	0.63	36.02	4.15
1.3–1.4	2864.0	7021	3.35	0.66	32.14	4.33
1.4–1.5	310.0	5939	18.78	1.33	27.54	2.55
1.5–1.6	102.3	5547	23.83	1.66	26.87	2.80
1.6–1.7	111.9	4911	30.54	1.91	25.98	2.65
1.7–1.8	91.3	4177	39.94	1.93	25.17	2.35
1.8–1.9	80.9	3462	47.43	1.74	24.00	2.29
>1.9	1051.8	762	82.20	1.72	13.05	1.14

Table 2. Data for size fraction 1.00–3.15 mm – coal, type 34.2

Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
<1.3	1803.0	8345	1.10	0.31	31.87	0.85
1.3–1.4	794.0	8032	3.70	0.38	26.52	0.88
1.4–1.5	83.3	6972	14.70	0.60	25.66	0.64
1.5–1.6	40.6	5971	24.20	0.84	24.30	0.80
1.6–1.7	25.0	5093	31.60	0.42	25.48	0.42
1.7–1.8	20.8	4571	37.00	0.86	22.08	0.56
1.8–1.9	6.7	4228	40.20	0.96	24.77	0.63
>1.9	213.7	887	79.30	0.89	13.75	0.55

Table 3. Data for size fraction 1.00–3.15 mm – coal, type 35

Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
<1.3	3476.2	8297	2.22	0.38	21.94	1.07
1.3–1.4	791.1	7781	7.84	0.46	19.56	0.85
1.4–1.5	264.7	6836	17.61	0.51	18.65	0.97
1.5–1.6	119.2	5830	27.70	0.62	18.22	0.93
1.6–1.7	117.0	5029	35.57	0.66	17.40	1.05
1.7–1.8	92.1	4222	43.45	0.76	16.99	1.08
1.8–1.9	72.9	3516	50.64	0.74	16.12	1.16
>1.9	1422.2	630	81.31	0.35	11.53	1.05

Searching for significant coal features

With a view to checking the relations between individual coal features, the partial correlation matrix was calculated for each type of coal. For each matrix the data concerning each particle density-size fraction (9 size fractions × 8 density fractions = 72 data), for coal type 34.2 several measurements were performed, which resulted in the number of the data = 61 in this case) for each type of investigated coals. The correlation matrices were presented in Tables 4–6.

Table 4. Correlation matrix for coal type 31

	Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a	Particle size d
Density [Mg/m^3]	1.00	–0.22	–0.92	0.92	0.53	–0.81	–0.86	–0.08
Mass [g]	–0.22	1.00	0.21	–0.19	–0.29	0.21	0.21	–0.29
Combustion heat [cal]	–0.92	0.21	1.00	–0.97	–0.36	0.89	0.89	–0.08
Ash contents [%]	0.92	–0.19	–0.97	1.00	0.36	–0.93	–0.92	0.06
Sulfur contents [%]	0.53	–0.29	–0.36	0.36	1.00	–0.24	–0.37	–0.31
Volatile parts contents V^a	–0.81	0.21	0.89	–0.93	–0.24	1.00	0.86	–0.03
Analytical moisture W_a	–0.86	0.21	0.89	–0.92	–0.37	0.86	1.00	–0.10
Particle size d	–0.08	–0.29	–0.08	0.06	–0.31	–0.03	–0.10	1.00

Table 5. Correlation matrix for coal type 34.2

	Density [Mg/m^3]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a	Particle size d
Density [Mg/m^3]	1.00	–0.47	–0.96	0.93	0.20	–0.70	–0.44	–0.08
Mass [g]	–0.47	1.00	0.37	–0.35	–0.30	0.29	0.06	–0.18
Combustion heat [cal]	–0.96	0.37	1.00	–0.99	–0.12	0.82	0.42	0.01
Ash contents [%]	0.93	–0.35	–0.99	1.00	0.12	–0.85	–0.41	–0.02
Sulfur contents [%]	0.20	–0.30	–0.12	0.12	1.00	–0.05	–0.12	–0.29
Volatile parts contents V^a	–0.70	0.29	0.82	–0.85	–0.05	1.00	0.29	–0.07
Analytical moisture W_a	–0.44	0.06	0.42	–0.41	–0.12	0.29	1.00	0.44
Particle size d	–0.08	–0.18	0.01	–0.02	–0.29	–0.07	0.44	1.00

Table 6. Correlation matrix for coal type 35

	Density [Mg/m³]	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a	Particle size d
Density [Mg/m³]	1.00	–0.14	–0.98	0.97	0.30	–0.88	0.32	–0.08
Mass [g]	–0.14	1.00	0.05	–0.04	–0.40	0.02	–0.18	–0.28
Combustion heat [cal]	–0.98	0.05	1.00	–1.00	–0.15	0.93	–0.29	0.04
Ash contents [%]	0.97	–0.04	–1.00	1.00	0.13	–0.94	0.28	–0.04
Sulfur contents [%]	0.30	–0.40	–0.15	0.13	1.00	0.01	0.18	–0.10
Volatile parts contents V^a	–0.88	0.02	0.93	–0.94	0.01	1.00	–0.21	0.05
Analytical moisture W_a	0.32	–0.18	–0.29	0.28	0.18	–0.21	1.00	0.55
Particle size d	–0.08	–0.28	0.04	–0.04	–0.10	0.05	0.55	1.00

It is worth looking at statistical description of the considered random variables. Their characteristics were presented in Tables 7–9.

Table 7. Statistical description for coal 31

Parameter	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
mean value	502.84	4827.68	30.59	1.14	26.13	2.88
standard deviation	859.77	1928.20	22.80	0.49	6.58	0.94
skewness	2.71	–0.71	0.97	0.54	–1.03	–0.21
curtosis	7.47	–0.14	0.48	–0.77	0.72	0.02
max	4187.80	7518.00	86.59	2.28	37.04	5.41
min	7.10	433.00	1.25	0.39	9.30	0.91
interval	4180.70	7085.00	85.34	1.89	27.74	4.50

Table 8. Statistical description for coal 34.2

Parameter	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
mean value	170.46	5515.98	27.86	0.57	24.66	0.94
standard deviation	345.01	2390.50	23.89	0.35	5.66	0.28
skewness	3.60	–0.75	1.03	1.29	–1.48	0.35
curtosis	14.49	–0.22	0.35	2.27	1.58	0.75
max	1817.90	8367.00	81.97	1.81	31.87	1.87
min	2.20	591.00	0.79	0.05	9.77	0.37
interval	1815.70	7776.00	81.18	1.76	22.10	1.5

Table 9. Statistical description for coal 35

Parameter	Mass [g]	Combustion heat [cal]	Ash contents [%]	Sulfur contents [%]	Volatile parts contents V^a	Analytical moisture W_a
mean value	367.64	5418.26	31.60	0.56	17.65	1.27
standard deviation	631.53	2286.93	23.40	0.247	2.82	0.18
skewness	3.14	–0.67	0.73	1.09	–1.19	–0.005
curtosis	10.91	–0.27	–0.19	1.06	1.004	–0.58
max	3476.20	8383.00	82.02	1.26	22.18	1.65
min	21.00	600.00	1.84	0.18	10.60	0.85
interval	3455.20	7783.00	80.18	1.08	11.58	0.80

From the correlation analysis results it is clearly visible that the most correlated coal features are density, combustion heat, ash contents and volatile parts contents. It is presumed then that for coal type identification it is not necessary to choose all of these features, but only one of them. To perform the multidimensional analysis of the data presented above, which describes coal parameters, the observational tunnels method was applied. Each of seven dimensions was treated as one. At the beginning, it was examined whether the information contained in all parameters is sufficient to the correct identification of coal type (Niedoba and Jamroz, 2013). As a result, the 7-dimensional space was created. It turned out that the accepted parameters were sufficient for proper identification if a given sample originated from coal type 31, 34.2 or 35. Figure 3 shows an example of experiment result considering data from all types of coal: 34.2 (61 samples), 35 (72 samples) and 31 (72 samples). From this it occurs that the data representing coal of type 35 is located in other part of the space than data representing coal of type 34.2. However, it is impossible to conclude about possibility of separation of coal of type 31. Furthermore, it was impossible to achieve one view from which the conclusion about proper identification of each of three analyzed types of coal was possible. Only joined conclusions occurring from several views allowed to state this (Niedoba and Jamroz, 2013). It proves that the nature of analyzed data is complicated.

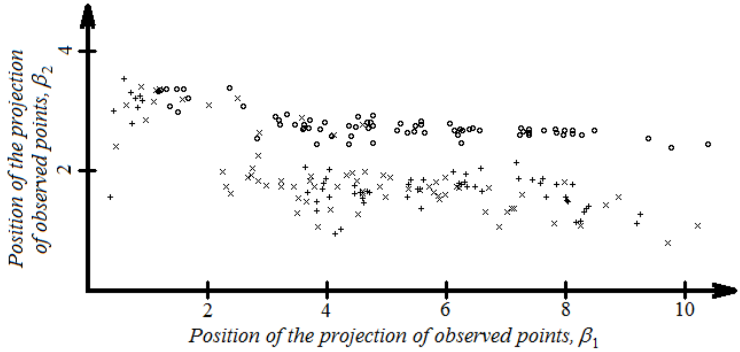

Fig. 3. Obtained view which shows that 7-dimensional data for coal 35 ("circles") gather in other part of the space than coal 34.2 ("pluses"). From this view it is not possible to conclude about possible separation of coal of type 31 (×)

Next, the last parameter, "analytical moisture", was deleted from the set of data. Consequently, the six-dimensional space was created. The Figs 4–5 show the experimental results for 6-dimensional data created in this way. From the view obtained in Fig. 4 it can be seen that the data concerning coal type 31 is located in other part of the space than coal type 35. On this basis it can be stated that the accepted parameters are sufficient to identify properly if certain sample origins from coal type 31 or 35. On the basis of this figure it is impossible to conclude about possible separation of the coal of type 34.2. In Fig. 5 the obtained view made it possible to state that the data concerning coal type 35 is located in other part of the space than coal type 34.2. It is possible then to state that the accepted parameters are sufficient for the proper identification whether certain sample origins from coal type 35 or 34.2. However, it was impossible to get the view from which the conclusion could be made that data concerning coal type 31 is located in other part of the space than coal of type 34.2. So, it is impossible to accept that these parameters are sufficient to the proper identification of coal type.

As it was noticed before, the seven-dimensional data created from the seven coal features described above is sufficient to the proper identification of coal type, but the same data is not sufficient for this purpose after removal of the parameter "analytical moisture". The conclusion is that the parameter "analytical moisture" is essential to the proper identification of coal type. It was the reason for constructing the next set of data on the basis of this parameter.

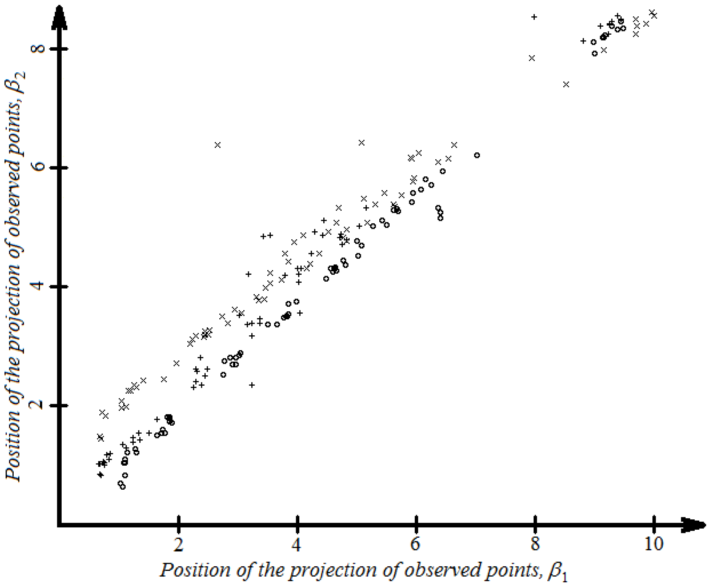

Fig. 4. View of 6-dimensional set of data created after removal of parameter „analytical moisture". It is visible that the data representing coal of type 35 ("circles") is located in other part of the space than data representing coal type 31 ("crosses"). From this view it is not possible to conclude about possible separation of coal of type 34.2 (+)

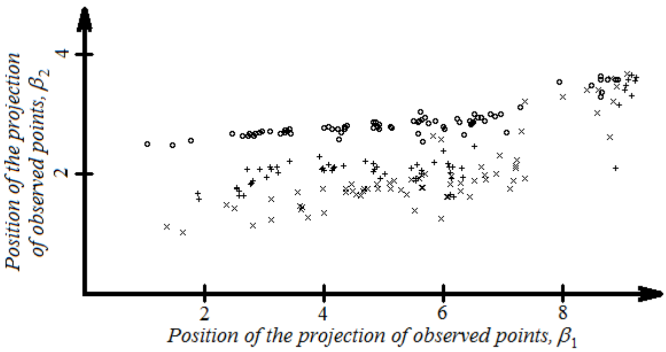

Fig. 5. View of 6-dimensional data created after the removal of parameter „analytical moisture". It is visible that the data representing coal type 35 ("circles") is located in other part of the space than data representing coal type 34.2 ("pluses"). From this view it is not possible to conclude about possible separation of coal of type 31 (×)

Based on correlation matrix results, 4 coal features were removed from the seven-dimensional set of data presented above and only 3 parameters left: analytical moisture, sulfur contents and volatile parts contents. Each of these parameters was treated as one dimension. Therefore, the three-dimensional space was created. Figures

6–7 present the results of these experiments. From the view obtained in Fig. 6 it can be seen that data concerning coal type 31 is located in other part of the space than coal type 35. On this basis it can be stated that the accepted parameters are sufficient for the proper identification if certain sample origins from coal type 31 or 35. On the basis of this figure it is impossible to conclude about possible separation of coal of type 34.2. Based on the view obtained in Fig. 7 it can be stated that data concerning coal type 34.2 is located in other part of the space than coal type 35 and simultaneously in other part of the space than coal type 31. From these two views, it can be concluded that the accepted parameters are sufficient to identify properly if certain sample origins from coal type 31, 34.2 or 35. At the same time it was impossible to get one view from which such conclusion could be made. It can be the proof that the structure of analyzed data is complicated.

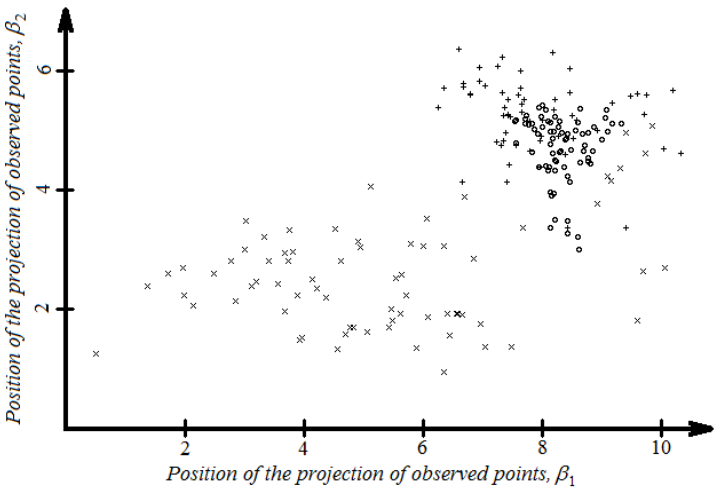

Fig. 6. View of 3-dimensional data: analytical moisture, sulfur contents and volatile parts contents. It is visible that the data representing coal type 35 ("circles") is located in other part of the space than data representing coal type 31 ("crosses"). From this view it is not possible to conclude about possible separation of coal of type 34.2 (+)

Next, it was decided to examine the influence of potential replacement of one of the coal features with another one highly correlated with it. The correlation index of "volatile parts contents" parameter was equal to 0.93 with parameter "combustion heat" for coal type 31, 0.82 for coal type 34.2 and 0.89 for coal type 35 (see: Tables 4–6). They are parameters highly related to each other. That is why in a set of three coal features (analytical moisture, sulfur contents and volatile parts contents), which are sufficient for the correct identification of coal type as shown above, the parameter

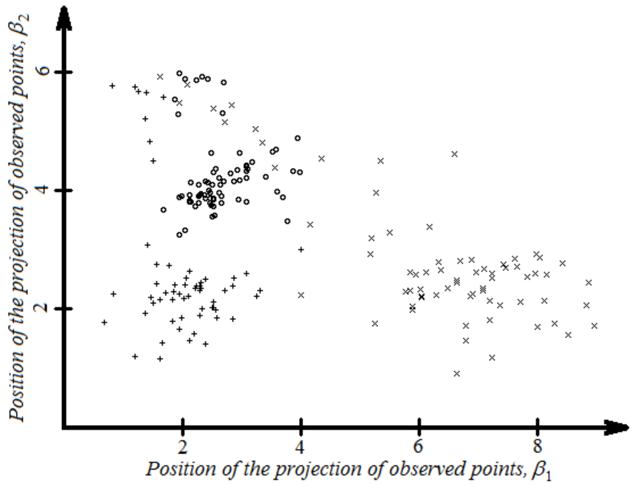

Fig. 7. View of 3-dimensional data created from features: analytical moisture, sulfur contents and volatile parts contents. It is visible that the data representing coal type 34.2 (”pluses”) is located in other part of the space than data representing coal type 31 (“crosses”) as well data representing coal type 34.2 (“pluses”) is located in other part of the space than data representing coal type 35 (“circles”)

“volatile parts contents” was replaced with parameter “combustion heat”. Figures 8-9 present the obtained results. The view obtained in Figure 8 allows to state that data concerning coal type 31 is located in other part of the space than coal type 34.2, and at the same time it is located in other part of the space than coal type 35. On this basis it can be said that the accepted parameters are sufficient to the proper identification if a particular sample originates from coal type 31 or not. All the same, it was impossible to find a view stating that the data concerning coal type 34.2 is located in other part of the space than coal type 35. The best obtained view concerning the possibility of separating the data into coals types 34.2 and 35 was shown in Figure 9. I occurs from it that the accepted parameters are sufficient to proper identification if the sample origins from coal of type 31 or 34.2. It cannot be said then that these three accepted parameters are sufficient to the proper identification of the type of coal. It proved that as a result of replacement of parameter “volatile parts contents” with “combustion heat”, the information allowing for the proper identification of coal type was lost. It occurred despite the high value of correlation index between these two coal features.

Similarly, the parameter „volatile parts contents” was replaced with parameter “ash contents”. The correlation index between these two features was high and was equal to –0.94 for coal type 31, –0.85 for coal type 34.2 and –0.93 for coal type 35 (see: Tables 4–6). Consequently, the set of three parameters was created: analytical moisture, sulfur contents and ash contents. The view presented in Figure 10 allows to state that data concerning the coal type 31 is located in other part of the space than data concerning coal type 34.2 and at the same is located in other part of the space than

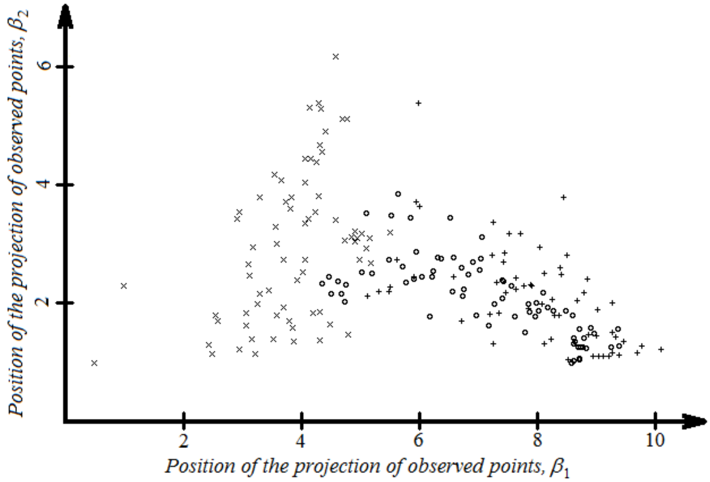

Fig. 8. View of 3-dimensional data created from features: analytical moisture, sulfur contents and combustion heat. It is visible that data representing type coal 31 ("crosses") is located in other part of the space than data representing coal type 34.2 ("pluses") and other part of the space than data representing coal type 35 ("circles")

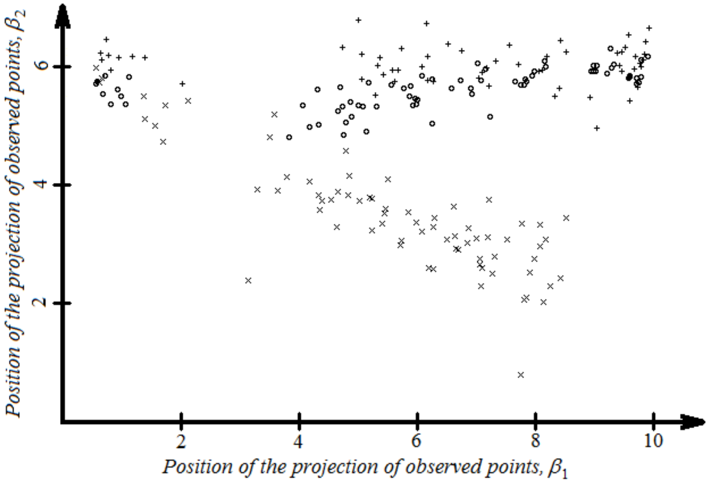

Fig. 9. View of 3-dimensional data created from features: analytical moisture, sulfur contents and combustion heat. The best view does not allow to state if the data representing type coal 34.2 ("pluses") can be separated from the data representing type coal 35 ("circles") – sets of points representing these two types overlap. It can be stated that the accepted parameters are sufficient to proper identification if certain sample origins from coal of type 31 ("crosses") or 34.2 ("pluses")

data concerning coal type 35. On this basis, similarly to the previous case, it can be stated that the accepted set of parameters is sufficient for the correct identification if certain sample originates from coal type 31 or not. All the same, it was impossible to find a view allowing to state that the data concerning coal type 34.2 is located in other part of space than the data concerning coal type 35. The accepted parameters are then not sufficient for the proper identification of the type of coal. This proved that the replacement of the parameter “volatile parts contents” with parameter “ash contents” also resulted in a loss of information allowing for the proper identification of all three coal types despite the high value of correlation index between replaced coal features.

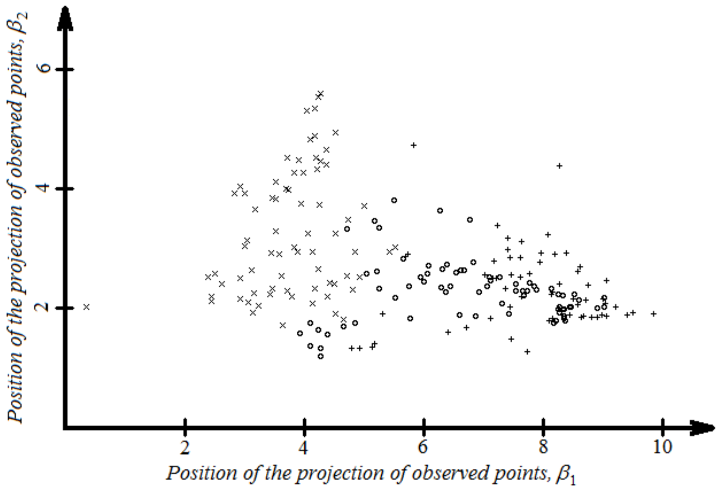

Fig. 10. View of 3-dimensional data created from features: analytical moisture, sulfur contents and ash contents. It is visible that data representing coal type 31 (“crosses”) is located in other part of the space than data concerning coal type 34.2 (“pluses”) and other part of the space than data representing coal type 35 (“circles”)

Conclusions

The visualizations of multidimensional data made it possible to arrive at the following conclusions:

- 6-dimensional data created as a result of removal of the parameter “analytical moisture” is not sufficient for the proper identification of coal type. They only allow to recognize coal types 35 and 31 as well 35 and 34.2 in pairs
- the parameter “analytical moisture” is crucial to the proper identification of coal type. Without this parameter, it would have been impossible to recognize the type of coal successfully
- three-dimensional data created from parameters: analytical moisture, sulfur contents and volatile parts contents are sufficient for the proper identification of coal type
- replacement of one of the parameters in coal features set with other highly correlated parameter does not guarantee to save the information necessary for the identification of coal type

- the system features, including analytical moisture, sulfur contents and combustion heat as well analytical moisture, sulfur contents and ash contents are not sufficient for the proper identification of coal type.

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