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A PHYSICAL MODEL OF SEPARATION PROCESS BY MEANS OF JIGS

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Abstract: In the case of ideal separation of minerals, partition into products is conducted according to a specific partition feature which is, for instance, the density of raw material. Usually, enrichment in a jig is described by means of the particle density as a partition feature. However, the degree of particle loosening in the jig's bed is influenced by, among others, the particle free settling velocity. After some time of the pulsating movement duration, particle segregation along the vertical axis according to the settling velocity will occur. It can be said that the particle free settling velocity constitutes a feature characteristic of the feed heterogeneous in terms of physical and geometric properties in the jigging process. In the article on the basis of heuristic considerations, a physical model of the partition function (recovery of the i^{-th} fraction), in which interactions between particles in the working bed of the jig are taken into account, is derived. A cause of the formation of the mechanism of particle dispersion around equilibrium layers is given and the accuracy of particle partition for a narrow size fraction in two variations, i.e. in conditions when a partition feature is, accordingly, the particle density and settling velocity, is calculated. These calculations allowed for the analysis of causes of process and inherent dispersion formation which takes place during the jigging process.

Keywords: *jigging process, physical model, Maxwell distribution, probable error, imperfection*

Introduction

The condition necessary for the correct separation of particles in a separating device, a jig, is sufficient loosening of the feed particles in a working bed, leading to a proper stratification of the feed particles into subsets with similar properties (size, density, settling velocity). The diversified particle settling velocity in the working bed of the jig determines the correct course of the partition process. In order to enable the stratification of material into fractions differing in the settling velocity to occur in the jig, particles must be able to move freely between each other. Therefore, conditions of a pulsating movement must be chosen in such a way as to ensure appropriate loosening of the material layer on jig's sieve. The degree of loosening of particles in

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the jig's bed is influenced by, among others, the particle free settling velocity. A distance traversed by a particle during one cycle depends on the settling velocity. Therefore, after long duration of the pulsating movement, if partition was accurate, segregation of particles will occur along the vertical axis according to the free settling velocity (Willis and Napier-Munn, 2006). Thus, it can be said that the particle free settling velocity at the accurate partition constitutes a characteristic feature of the feed heterogeneous in terms of physical and geometric properties in the partition process in the jig.

The particle partition and dispersion mechanism

A characteristic feature of partition in a jig is the free settling velocity. The particle stratification into elemental fractions (layers) occurs according to the settling velocity. The pulsating movement of liquid is a mechanism generating such partition. The movement of particles in the jig's bed is a random process. The value of a random variable denoting a particle location along the vertical axis in a non-stationary state is a function of time. Particles move between each other under deterministic forces, resulting from the pulsating movement, suffering collisions. These interactions at the moment of collisions are a source of particle dispersion around grouping centres, being a function of time, because the deterministic movement intensity is a function of time. If the pulsating movement velocity was homogeneous in the whole volume of the jig's working space and there were no interactions between particles, then in a stationary state, that is after the conducted partition, the state of ideal stratification according to the settling velocity would be achieved. However, this is not the case. Due to the heterogeneity of the liquid velocity field in the vertical direction of the jig's bed, the permeation of particles to layers improper to them occurs. It can be thus said that two independent random processes occur. One is connected to the heterogeneity of the liquid velocity field, independent of time and the other connected to particles' collisions during the deterministic particles movement with different properties to their equilibrium layers, dependent on time. The latter movement is described by the Fokker-Planck equation (Tichonov, 1968; 1973):

$$\frac{\partial f(z,t)}{\partial t} = -p\frac{\Delta z}{\Delta t}\frac{\partial f(z)}{\partial z} + \frac{1}{2}p\frac{(\Delta z)^2}{\Delta t}\frac{\partial^2 f(z,t)}{\partial z^2} \tag{1}$$

where: $f(z,t)\Delta z$ – the probability of a particle with given density to occur in a layer with thickness Δz at time t or a fraction of particles which at time t are present in layer Δz with coordinate z, p – the probability of a particle group transition from a layer to a layer.

This movement is characterised by a movement of particle grouping centres with specified settling velocities to their equilibrium layers and dispersion of those particles to adjacent layers around grouping centres, dependent on time. Particle dispersion mechanisms in both cases are similar. They are particle interactions due to collisions. However, sources of these collisions are different. In the first case, the source of collisions is the heterogeneity of the liquid velocity field, while in the second case, the source of collisions are particle deterministic movements to their equilibrium layers with different physical and geometric properties, which the value and sense of the velocity vertical component depend on. After stratification of particles according to the settling velocity, this movement stops and this component of particle dispersion is equal to zero. However, the first component of dispersion, which is always present when there is a pulsating movement of liquid between particles, remains. This is proven by experimental facts referred to in chapter 3 according to which the standard deviation of particles with specified properties around their equilibrium centres decreases with time; however, not to zero, but after long time of separation it achieves a constant value.

The steady (stationary) state of a particle system in a jig's chamber is understood as the state in which grouping centres do not change their position over time. Then potential energy of the system would be minimal if there were no random transitions of particles connected to the heterogeneity of the liquid velocity field. Each of the random processes enumerated above generates the particles' velocity distribution independent of each other. It can be thus assumed that a random variable denoting the particle velocity in the jig's chamber is the sum of two independent random variables:

$$W = W_1 + W_2 \tag{2}$$

where: W_1 – a random variable of the particle velocity conditioned by the heterogeneity of the liquid velocity field, independent of time, W_2 – a random component of the particle velocity conditioned by a movement of particle grouping centres, dependent on time.

Random variables W_1 and W_2 have normal distributions (as a vertical component of the particle velocity distributions, which will be discussed in chapter 5). Therefore, the sum (composition) of two independent random variables with normal distributions will have normal distribution whose variance is equal to (Gerstenkorn and Srodka, 1972):

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \tag{3}$$

where: σ_1 – the standard deviation of a random variable W_1 distribution, independent on time, σ_2 – the standard deviation of a random variable W_2 distribution dependent on time.

The standard deviation in a general case depends on particles' geometric properties, the average particle density, the partition duration time and hydrodynamic conditions of the partition (Rafales – Lamarka, 1962).

Dispersing interactions and particle velocity distribution variance in stationary state

The force under the effect of which particles obtain additional kinetic energy is the force of interaction between particles. It is equal to the change in momentum of a particle in a unit of time:

$$F_T = \rho V \frac{v_p}{t} \tag{4}$$

where: ρ – particle density, V – particle volume, t – the average time between collisions, v_p – root mean square velocity of a particle conditioned by the heterogeneity of the liquid velocity field.

The average time between collisions is equal to:

$$t = \frac{\lambda}{v_p} \tag{5}$$

where: λ – the average distance covered by a particle between collisions (the average free distance) is equal to (Tichonov, 1968; 1973):

$$\lambda = \frac{1}{n S} \tag{6}$$

where: n – a number of particles in a unit of volume, S – a projective plane of a particle onto the plane perpendicular to the direction of movement.

Since the number of particles in a unit of volume is expressed by a formula:

$$n = \frac{C}{V} = \frac{6C}{k_1 \pi d_p^3}\ , \tag{7}$$

therefore, the average free distance is equal to:

$$\lambda = \frac{2}{3}\frac{k_1 d_p}{k_2 C} \tag{8}$$

where: C – the particle volume concentration in the jig's bed, k_1 – the volume shape coefficient of particle, k_2 – the dynamic shape coefficient of particle.

Therefore, the average time between collisions after taking expression (8) into account is as follows:

$$t = \frac{2 k_1 d_p}{3 k_2 C v_p}\ . \tag{9}$$

After the substitution of expression (9) into formula (4) we obtain:

$$F_T = \frac{\pi}{4}\rho k_2 v_p^2 d_p^2 C = 0.78\rho k_2 v_p^2 d_p^2 C \,. \qquad (10)$$

This force depends on the particle density. If it is assumed that the root mean square velocity of the particle movement is proportional to the additional, random value of liquid velocity v_f conditioned by the heterogeneity of the velocity field, local turbulences, that is $v_p = bv_f$ (b – proportionality coefficient) and taken into account that $C = 1-\theta$ (θ – loosening coefficient), then:

$$F_T = 0.78 k_2 \rho b v_f^2 d_p^2 (1-\theta)\,. \qquad (11)$$

It can be seen from formula (11) that the greater degree of loosening, the smaller the force of interactions between particles.

The following product:

$$F_T \cdot \lambda = W_L \qquad (12)$$

presents the work of the dispersion force over a distance equal to the particle average free distance. Therefore, the variance σ_1^2 of the particles' velocity distribution in the stationary state is equal to (Smirnova, 1980):

$$\sigma_1^2 = \frac{F_T \lambda}{m} = \frac{0.78 k_2 b \rho v_f^2 d_p^2 (1-\theta)\lambda}{m} \qquad (13)$$

where: $m = k_1 \dfrac{\pi d_p^3}{6}\rho$ – the mass of a particle.

After the substitution of m into formula (13) we obtain:

$$\sigma_1^2 = \frac{1.5 k_2 b v_f^2 (1-\theta)\lambda}{k_1 d_p}\,. \qquad (14)$$

Variance conditioned by a movement of particle grouping centres

Experience shows that the standard deviation characterising the distribution of particles with given properties around grouping centres decreases over the partition duration time (Fig. 1). The following expressions (Tichonov, 1968; 1973):

$$p\frac{\Delta z}{\Delta t} = v \qquad (15)$$

$$\frac{1}{2}p\frac{(\Delta z)^2}{\Delta t}=\frac{1}{2}v\Delta z=\sigma^2 \tag{16}$$

present the velocity of a grouping centre movement and the standard deviation, respectively. Both the velocity of movement and standard deviation are proportional to the probability of transition. This probability decreases with approaching the equilibrium layer and is expressed by the formula:

$$p=1-\frac{i}{n}=1-\frac{z_i}{z_{\max}} \tag{17}$$

where: i – layer's number, n – amount of layers, z_i – the distance from the equilibrium layer to the position of i^{th} layer, z_{max} – the distance from the beginning of the system to the equilibrium layer (Fig. 2).

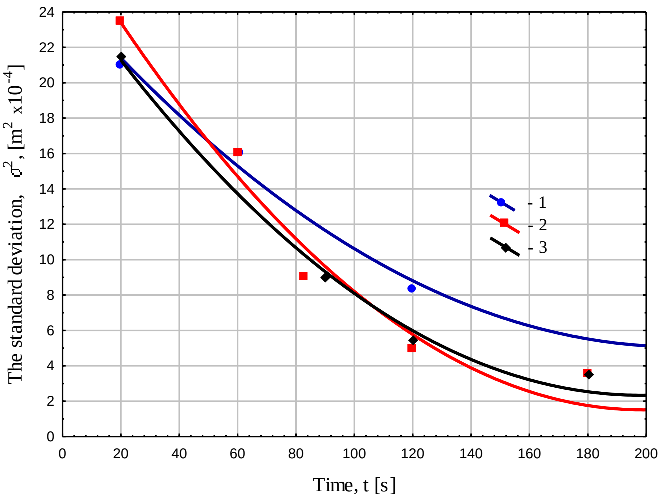

Fig. 1. Dependence of standard deviation on time:
1–3 densities accordingly 1.4–1.5; 1.6–1.7; 1.9–2.0 Mg/m^3

Taking formula (17) into account, expressions (15) and (16) take the form of:

$$v=v_{\max}\left(1-\frac{z}{z_{\max}}\right)=\frac{dz}{dt} \tag{18}$$

$$\sigma^2=\frac{1}{2}v_{\max}\left(1-\frac{z}{z_{\max}}\right)z_{\max} \tag{19}$$

where: v_{max} – the maximum value of the velocity of equilibrium centre movement.

The dependence of variance on time is developed on the basis of stochastic processes is as follows (modified formula 19) (Nawrocki, 1972):

$$\sigma_2^2 = \frac{1}{2} z_{\max}^2 k^2 e^{-kt} = \frac{1}{2} z_{\max}^2 k^2 e^{-\eta} \tag{20}$$

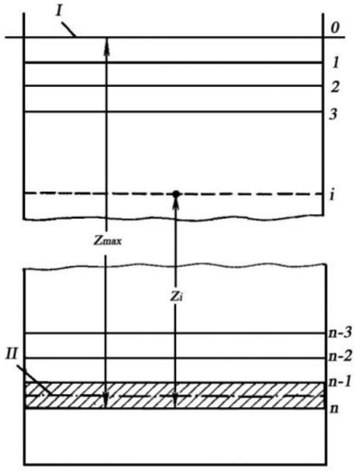

Fig. 2. System of elemental layers in a jig. I – a reference level, II – the equilibrium layer (Nawrocki, 1972)

The constant *k* occurring in formula (20) presents the correct velocity of a grouping centre (partition rate constant) of particles with given physical and geometric properties to its equilibrium layer. Its value should be proportional to the velocity of particle movement in conditions of constrained movement v_{smax}, because the higher velocity of those particles' movement, the faster the grouping centre moves to its equilibrium layer and it should be inversely proportional to the distance of the equilibrium layer location from the feed level. At the established partition duration time determined by the process efficiency, more distant equilibrium layers will be more blurred and the boundary between centres of particle equilibrium with different properties less sharp.

Taking the above notes into account, it was assumed that the partition rate constant is equal to (Surowiak, 2007):

$$k = \frac{v_{s\max}}{z_{\max}} \tag{21}$$

where: $v_{s\max}$ – the maximum velocity of particle movement under constrained conditions.

The velocity of particle movement in constrained conditions is connected to the free settling velocity *v*. For Reynolds numbers *Re > 400* and the degree of loosening θ from range (0.5-1.0), it is the following dependence (Minc, 1953):

$$v_{s\max} = \left[-0.362(1-\theta) + \sqrt{\left[0.362(1-\theta)\right]^2 + \theta^3}\right] v \,. \tag{22}$$

After taking the above connection and dependence presenting the settling velocity of irregular particle $v = 5.33\sqrt{x}\sqrt{d_p}\sqrt{\left(\frac{k_1}{k_2}\right)}$ into account (Brozek and Surowiak, 2010), the partition rate constant is expressed by the formula:

$$k = \frac{5.33\sqrt{xd_p \frac{k_1}{k_2}}}{z_{\max}} \left[-0.362(1-\theta) + \sqrt{\left[0.362(1-\theta)\right]^2 + \theta^3}\right] \tag{23}$$

where: $x = \frac{\rho - \rho_o}{\rho_o}$ - reduced particle density, ρ – particle density, ρ_o – medium density.

By replacing η in expression (20) from the following dependence:

$$q = \frac{Q}{bl} = \frac{3600\bar{\rho}h}{t} = \frac{3600\bar{\rho}hk}{\eta} \tag{24}$$

we obtain:

$$\sigma_2^2 = \frac{1}{2} z_{\max}^2 k^2 \exp\left[-\frac{3600\bar{\rho}hk}{q}\right] \tag{25}$$

where: *q* – capacity per unit of jig bed surface, *Q* – jig capacity, $\bar{\rho}$ – mean particles density, *h* – height of jig's bed, *b* – jig bed's width, *l* – jig bed's length.

The dispersion function

In the non-stationary state of the particle system in the jig's chamber, the movement of random nature, resulting from collisions between particles, overlaps the particle movement conditioned by deterministic forces. Local turbulences created in spaces between particles transfer energy of whirls to particles adjacent to them (Vinogradov, 1965, after Samylin 1976). It is a source of the additional random movement between particles. A result of this random, additional kinetic energy is the particle movement velocity distribution around the most probable value.

It was assumed that the distribution of the random particle velocity component along the vertical axis (axis z), that is the dispersion function, has the Maxwell distribution of the velocity component (Smirnova, 1980):

$$t(v) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(v - v_r)^2}{2\sigma^2} \right] \tag{26}$$

where: v_r – the most probable value of the particle velocity, while $\sigma^2 = \sigma_1^2 + \sigma_2^2$.

It is thus a normal distribution. Taking dependences (14) and (24) into account, the standard deviation is equal to:

$$\sigma^2 = \frac{1.5k^2 b v_f^2 (1-\theta)\lambda}{k_1 d_p} + \frac{1}{2} z_{\max}^2 k^2 \exp\left[-\frac{3600 \rho_{śr} h k}{q} \right]. \tag{27}$$

After a long time of partition (infinitely long in theory), taking formula (20) into account:

$$\lim_{t \to \infty} \sigma^2 = \sigma_1^2 = \frac{1.5 k_2 b v_f^2 (1-\theta)\lambda}{k_1 d_p}. \tag{28}$$

It is the standard deviation in the stationary state. When velocity v_f goes to zero, that is when the value of the dispersing force goes to zero ($v_f \to 0$), then the standard deviation σ_1 goes to zero. The dispersion function for such a situation, that is the following limit:

$$\lim_{\sigma_1 \to 0} \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left[-\frac{(v - v_r)^2}{2\sigma_1^2} \right] = \delta(v - v_r) \tag{29}$$

is equal to function δ-Dirac (Byron and Fuller, 1975), that is:

$$\lim_{v_f \to 0} t(v) = \delta(v - v_r). \tag{30}$$

It means that all particles with settling velocity v_r are in their equilibrium layer.

Partition function (recovery of the i^{-th} fraction) and probable error

The partition function (recovery of the i^{-th} fraction) specifying the probability of getting a particle with terminal velocity v to tailings (in the case of coal) is presented by the formula:

$$T(v) = \int_{-\infty}^{v} t(v)dv = \int_{-\infty}^{v} \exp\left[-\frac{(v-v_r)^2}{2\sigma^2}\right]dv = \varphi\left(\frac{v-v_r}{\sigma}\right) \tag{31}$$

where: ϕ – the Laplace function, σ – the standard deviation.

The partition function (recovery of the i^{-th} fraction) expressed by formula (31) constitutes the distribution function of a normal distribution. In Fig. 3, there are presented forms of the partition curve for three values of velocity v_f in the stationary state.

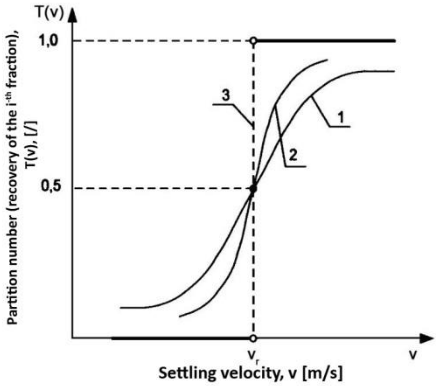

Fig. 3. Partition curve for three velocity values: $v_{1f} > v_{2f}$, $v_{3f} = 0$

The probable error being a measure of the partition accuracy according to the definition is equal to:

$$E_p = \frac{v(T=0.75) - v(T=0.25)}{2} \tag{32}$$

It is the so-called quartile deviation equal to a half of the difference between the third and first quartile and for the normal distribution it is equal to:

$$E_p = 0.67\sigma\,. \tag{33}$$

Taking (27) into account, the probable error is expressed by the formula:

$$E_p = 0.67\sqrt{\frac{1.5k_2 b v_f^2 (1-\theta)\lambda}{k_1 d_p} + \frac{1}{2} z_{\max}^2 k^2 \exp\left(-\frac{3600\bar{\rho} hk}{q}\right)}\,. \tag{34}$$

Based on formulas (28) and (23), the probable error in the general case depends on physical and geometric properties of a particle, the bed height, specific yield, hydrodynamic conditions of partition (through θ). It decreases with the increase in particle size, a degree of bed loosening and the partition rate constant, while it increases with the increase in the bed height. It also depends on the shape of the mixture particles.

For the stationary state:

$$E_p = 0.82 v_f \sqrt{\frac{k_2 b(1-\theta)\lambda}{k_1 d_p}} \, . \tag{35}$$

When velocity v_f increases, then in accordance with formula (21) the standard deviation σ_1 increases. Therefore, with the unlimited increase in velocity v_f is

$$\lim_{v_f \to \infty} T(v) = \frac{1}{2} \, . \tag{36}$$

The partition curve has thus a form of a horizontal line, parallel to v-axis. It is equivalent to no partition. Such a situation can occur in the case of partition of fine particles with too high frequency and amplitude of the pulsating movement.

In the stationary state, particle dispersion is connected to the existence of interactions between particles conditioned by the heterogeneity of the liquid velocity field in the jig's bed which is characterised by the value of v_f. At no interactions, that is for the ideal partition, the partition function (recovery of the i^{-th} fraction) with consideration of dependence (30) is expressed by the following formula (Byron and Fuller, 1975):

$$T(v) = \int_{-\infty}^{v} t(v)dv = \int_{-\infty}^{v} \delta(v - v_r)dv = H(v - v_r) \tag{37}$$

where $H(v - v_r)$ is the Heaviside function. The Heaviside function is specified as follows:

$$H(v - v_r) = \begin{cases} 0 & \text{for} \quad v < v_r \\ 1 & \text{for} \quad v > v_r \end{cases} \tag{38}$$

Considering the fact that according to formula (31) $T(v = v_r) = \frac{1}{2}$ the partition function at the ideal stratification is presented as follows:

$$T(v) = \begin{cases} 0 & \text{for} \quad v < v_r \\ \dfrac{1}{2} & \text{for} \quad v = v_r \\ 1 & \text{for} \quad v > v_r \end{cases} . \tag{39}$$

In Fig. 3, there is a graph of the partition curve described by formula (39) (curve 3). It is thus a curve for the ideal partition. Its equation results directly from the general equation of the dispersion function and partition function through the crossing point corresponding to the condition of the ideal partition. Particles with settling velocity $v = v_r$ with the identical probability equal to 1/2 reach the concentrate and tailing. The probable error is equal to zero.

Empirical verification

Based on data obtained from the industrial experiment which consisted in testing a fines jig upgrading bituminous coal, distributions of geometric (sizes of particles and their shape coefficients) and densimetric properties (density) of the feed particles and the partition products were calculated (Niedoba, 2013; Surowiak, 2007; Surowiak and Brozek, 2014a,b). In order to calculate the partition accuracy, the partition curve coordinates for two variations: when the partition property is the particle density and the particle settling velocity were calculated. For the purposes of this work, one size fraction 8 -10 mm was chosen and partition numbers and the partition accuracy were calculated within this size fraction for both variations.

With functions of the settling velocity distribution density in tailing and the feed in narrow size fraction 8 – 10 mm (Surowiak, 2014), it is possible to calculate partition numbers (recovery of the i^{-th} fraction) for tailing and a given value of the settling velocity using the formula (Surowiak, 2007):

$$T(v) = \gamma_o \frac{h_o(v)}{h(v)} \tag{40}$$

where: $T(v)$ – a partition number (recovery of the i^{-th} fraction) equal to probability of transferring particles of certain settling velocity v to tailing, γ_o – tailing yield, $h_o(v)$ – the probability density function of the settling velocity in tailing of size fraction 8–10 mm, $h(v)$ – the probability density function of the settling velocity in the feed of size fraction 8–10 mm.

For the empirical partition curve obtained in this way, a model dependence, which is expressed by the normal distribution function, is matched. From the match, a value of the probable error and value of the cut point are obtained. In Fig. 4, the partition curve obtained from the match is presented. Its equation is expressed by the following formula in accordance with dependence (31):

$$T(v) = \varphi\left(\frac{v - 0.159}{0.018}\right). \tag{41}$$

The curvilinear correlation index is greater than 0.99. The value of the cut point is equal to v_r = 0.159 m/s, while the probable error on the basis of formula (33) E_p = 0.012 m/s. Imperfection in accordance with the definition expressed by the formula is equal to $I = \frac{E_p}{v_r} = 0.076$.

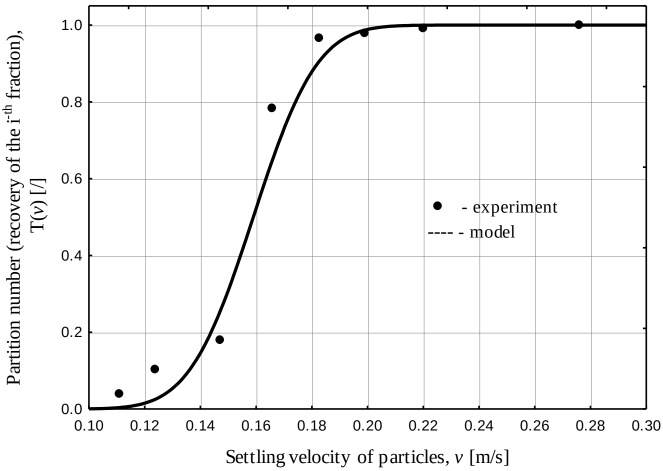

Fig. 4. Partition curve for tailing (8–10 mm size fraction)

In the practice of upgrading and evaluation of partition results in a jig, using partition curves is adopted when the partition feature is the particle density. Density is the partition feature in the case of upgrading of the monodisperse spherical particles sample. Then geometric properties have established identical values for all particles, and the settling velocity distribution depends only on the particle density distribution. Although the particle density distribution in a sample is independent of the geometric properties distribution, the evaluation of the partition accuracy of the device, in which particle stratification according to the settling velocity occurs, on the basis of density is burdened with some error due to the existence of the inherent (pre-existents) probable error, independent of the process course conditions and dependent on the geometric properties distribution.

In Fig. 5, the partition curve based on results of densimetric analyses provided in the work (Brozek and Surowiak, 2006; Surowiak, 2007) for size fraction 8–10 mm is drawn.

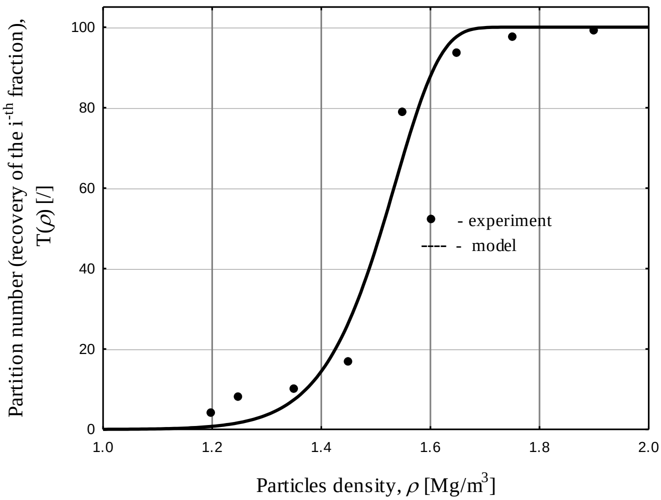

Fig. 5. Partition curve for 8–10 mm particle size fraction

As other authors' (Paul et al., 1998; Gottfried, 1978) studies have shown, partition curves for jigs with density as the partition feature are asymmetric curves and are well approximated by the Weibull distribution. Therefore, the Weibull distribution was matched to the empirical partition curve. Its equation and values of the partition density, probable error and imperfection are as follows:

$$T(\rho) = 100\left\{1 - \exp\left[-\left(\frac{\rho}{1.54}\right)^{19.5}\right]\right\} \tag{42}$$

ρ_r = 1.505 Mg/m^3, E_p = 0.0605 Mg/m^3, I = 0.12.

In Fig. 5, the continuous curve presents the model dependence. The curvilinear correlation index is greater than 0.95. The value of imperfection in this case in greater than in a situation when partition results are analysed on the basis of the particle settling velocity. This difference is a result of the fact that the dispersion in the case of the results analysis according to density is the sum of the inherent and process dispersion. However, when partition results are analysed according to the settling velocity, there is only the process dispersion. It can be thus said that the imperfection difference is a cause coming from the inherent dispersion.

Discussion

The state in which material heterogeneous in terms of physical and geometric properties in the jig's bed is ideally stratified due to the settling velocity is the state with the lowest potential energy. It results from Mayer's energy theory. If material partitioned itself into densimetric fractions and apart from that each fraction into particle size fractions according to the particle pyramid from the biggest to the smallest, then the layer porosity would be the highest and thus the layer height would be greater than before (Kuprin et al., 1983). Therefore, after such partition, potential energy would be greater than before partition. It is contrary to the principle of least action that processes proceed in the direction of minimum potential energy of the system. A decrease in porosity will occur when empty spaces between bigger particles are fulfilled with smaller particles. It will take place in a situation when in the ideally stratified material, as above, smaller particles with greater density move to a higher sublayer of greater particles with lower density or the other way round. There will appear the ideal stratification according to the settling velocity, whose result is the phenomenon of dispersion of particles with given density to other layers. It is the so-called inherent dispersion. It can be thus said that from the physical point of view the inherent dispersion results from the principle of minimising potential energy of the system of particles heterogeneous in terms of physical and geometric properties. It is present always when a mixture of mineral particles heterogeneous in terms of geometric properties is partitioned in a jig or other partitioning devices where the partition argument is the settling velocity. If the feed particles were spherical particles with the same diameters, then at the ideal separation there would be no dispersion of particles to layers inappropriate to them and the value of the inherent dispersion would be equal to zero.

The inherent dispersion constitutes the first mechanism of dispersion of particles with given density to other layers. Another mechanism of dispersion are mutual collisions of particles between each other due to the heterogeneity of the liquid velocity field. It is the so-called process dispersion which is always present regardless of what physical properties is the partition feature (density or settling velocity). If material heterogeneous in terms of physical and geometric properties is partitioned, then the settling velocity is the partition feature. Analysing the partition results on the basis of this feature, the lower value of imperfection is obtained because there is no inherent dispersion in this case. However, in analysing the results of such material partition with the use of the particle density as the partition feature, the greater value of imperfection is obtained because it is the result of the inherent and process dispersion's influence on the partition accuracy.

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