

Multi-objective optimization of the separation performance of an overflow re-separation hydrocyclone

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Abstract: An overflow re-separation hydrocyclone was experimentally investigated to reduce coarse-particle contamination in the overflow and improve size classification. Four variables were considered: feed pressure, vortex finder insertion depth, underflow outlet diameter, and connecting tube diameter. The connecting tube diameter mainly controlled the amount of solids entering the overflow separation chamber; increasing this diameter increased the overall classification efficiency. A greater vortex finder insertion depth mainly increased overflow fineness. Enlarging the underflow outlet caused more fine particles to report to the underflow and decreased the overall classification efficiency. Raising the feed pressure produced a finer overflow, but its contribution to the overall efficiency was limited. Based on the single-factor tests, Box-Behnken response-surface models were developed for overflow solids yield and overall classification efficiency, and the two responses were optimized simultaneously. Experimental validation under the selected conditions yielded an overflow solids yield of 24.76% and an overall classification efficiency of 54.21%.

Keywords: overflow re-separation, classification performance, experimental study, multi-objective optimization

1. Introduction

Hydrocyclone classification relies on centrifugal sedimentation and is widely applied in mineral processing, environmental treatment, and chemical processes because the equipment is compact and simple to operate (Kuang et al., 2012; Wang et al., 2007; Zhao et al., 2021). In practice, however, short-circuit flow and turbulent disturbances may carry coarse particles into the overflow, producing coarse-particle misplacement (Su and Zhang, 2022a; Tan et al., 2026). Once these particles leave with the overflow, the product becomes less narrowly classified and the downstream process must handle a broader particle-size range. Reducing this misplaced coarse fraction without sacrificing the compactness of the hydrocyclone is therefore an important issue in improving classification accuracy (Bourgeois and Majumder, 2013; Frausto et al., 2021; Jiang et al., 2019).

Previous work aimed at improving size selectivity has mainly modified the internal geometry or divided the product into additional streams (Vega-Garcia et al., 2018; Xu et al., 2023; Zhang et al., 2021). For example, Hou et al. (2021) introduced a flat-bottom configuration and showed that an appropriate bottom width reduced fine-particle misplacement to the underflow, whereas excessive widening intensified turbulence and increased coarse-particle misplacement to the overflow. Hou et al. (2026) later examined several vortex finder geometries; the inverted-umbrella and truncated-cone designs redirected short-circuit flow toward the outer vortex and reduced the coarse fraction entering the overflow. Su and Zhang (2022b) inserted an axial solid rod to suppress the air core and found that the variable-diameter rod decreased axial velocity and turbulence intensity while increasing tangential velocity and separation sharpness. A W-shaped hydrocyclone was proposed by Jiang et al. (2020) to modify the flow and particle migration near the underflow region, resulting in less fine-particle entrainment in the underflow and improved classification efficiency. Oliveira et al. (2009) replaced the

conventional solid cone with a porous filtering cone, thereby producing a filtrate in addition to the overflow and underflow. E et al. (2024) modified the inlet using tangential and helical configurations and reported stronger tangential motion, a more stable air core, and reduced short-circuit flow and particle misplacement after optimization. Multi-product hydrocyclones provide another way to separate material that would otherwise remain mixed in a conventional overflow. Obeng and Morrell (2003) added a second vortex finder in the JK three-product cyclone to generate inner overflow, outer overflow, and underflow streams, allowing fine high-density minerals to be enriched in the second overflow. Mainza et al. (2004) showed that the same concept could isolate an intermediate product containing coarse, low-density siliceous material for separate screening or regrinding. Recent investigations have further extended multi-stage and multi-product hydrocyclone classification. Liu et al. (2024) demonstrated that the separation-chamber diameter ratio affected particle circulation and misplacement in a multistage hydrocyclone, with both excessively large and excessively small ratios deteriorating the separation performance. Liu et al. (2025) proposed a self-aspirating three-product hydrocyclone in which an appropriately sized air inlet stabilized the upstream inner vortex and improved first-stage classification. More directly related to the present work, Chen et al. (2024) used the residual rotational energy of the inner vortex in a three-product hydrocyclone with overflow re-separation, allowing some medium and coarse particles carried upward by the inner vortex to be recovered through the sideflow. On this basis, the present hydrocyclone creates a secondary classification region in the overflow passage so that misplaced coarse particles can be discharged laterally before the final overflow is collected. Building on the overflow re-separation configuration investigated by Chen et al. (2024), this study advances the previous work by quantitatively evaluating the coupled effects of key operating and geometric parameters on product yield and size selectivity. Particular attention is given to parameter interactions and the trade-off between overflow solids yield and overall classification efficiency, providing a basis for coordinated parameter selection in overflow re-separation hydrocyclones.

The performance of a hydrocyclone is also sensitive to the combined action of operating conditions and geometric parameters (Fan et al., 2015; Li et al., 2026). Han et al. (2019) changed the apex internals and cone length and retained favorable separation while lowering the Euler number and energy consumption. Wu et al. (2025) showed that vortex finder wall thickness affected the internal flow field, particle stratification, and particle misplacement. In a mini-hydrocyclone, Li et al. (2025) identified a suitable ratio of vortex finder length to inlet height that reduced short-circuit flow and improved separation efficiency. Tian et al. (2019) reported that either an excessively small or an excessively large underflow pipe diameter was unfavorable for separation efficiency and split ratio in a hydrocyclone with a reflux device. Hao et al. (2025) achieved adjustable underflow opening by varying air-chamber pressure, while Pukkella et al. (2025) showed that wall roughness changed velocity, water partition, and solids recovery to underflow. Zheng et al. (2024) further examined regulated underflow rate, auxiliary flow, and split ratio as outlet-control variables. Because such parameters can act jointly, evaluating them one at a time does not fully describe their coupled influence (Feng et al., 2016; Xu et al., 2019). Accordingly, statistical optimization has been increasingly used. Özgen et al. (2009) modeled hydrocyclone performance with a Box–Behnken design involving feed solids content, inlet pressure, vortex finder diameter, and underflow outlet diameter. Song et al. (2020) combined single-factor tests with response surface methodology to optimize inner conical-surface height, suction angle, and vortex finder insertion depth while considering separation efficiency and pressure loss. Barrak et al. (2021) used the same general methodology for an activated-carbon-water system, and E et al. (2025) coupled response surface methodology with multi-objective optimization to quantify trade-offs among separation sharpness, water distribution, cut size, and pressure drop. For the overflow re-separation hydrocyclone considered here, the combined effects of connecting tube diameter, vortex finder insertion depth, underflow outlet diameter, and feed pressure have not yet been established for this specific structural configuration.

Accordingly, this investigation was carried out in three steps. The overflow separation chamber diameter was first varied to examine its effect on particle size, solids yield, and classification efficiency and to determine the chamber size used thereafter. The connecting tube diameter, vortex finder insertion depth, underflow outlet diameter, and feed pressure were then studied separately. Finally, the

single-factor results were used to define a Box-Behnken design, from which quadratic models for overflow solids yield and overall classification efficiency were obtained and used for multi-objective optimization. The work provides experimental evidence for selecting the structure and operating parameters of the overflow re-separation hydrocyclone.

2. Research methodology

2.1. Design principle

Fig. 1 summarizes the particle-separation path in the overflow re-separation hydrocyclone. The lower part acts as the main separation chamber, whereas the upper part provides an additional separation stage for the overflow stream. Slurry enters the lower chamber tangentially under feed pressure and develops a swirling flow. Most coarse particles move downward with the outer vortex and leave through the underflow outlet. The inner vortex carries fine particles, together with a fraction of medium and coarse particles entrained by short-circuit flow and turbulent disturbances, upward through the connecting tube. Although the flow has entered the upper chamber, it still retains rotational motion. Relatively coarse particles therefore migrate toward the wall under centrifugal action and are removed through the tangential sideflow. Finer particles remain with the central upward flow and are discharged through the overflow outlet. In this way, the upper chamber removes part of the material that would otherwise become coarse-particle contamination in the overflow.

A 75-mm overflow re-separation hydrocyclone was fabricated for the classification tests. Its main geometric dimensions are shown in Fig. 1 and listed in Table 1.

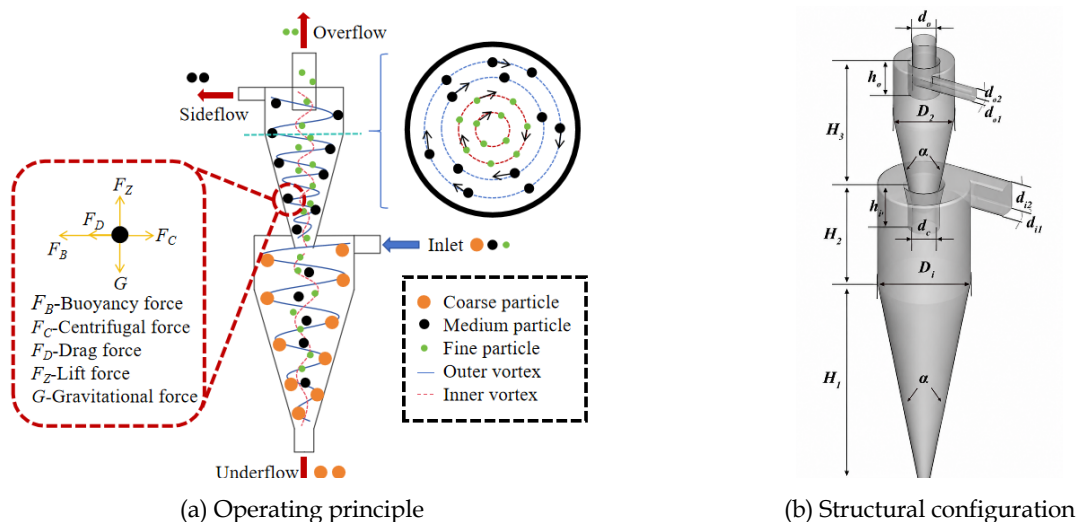


Fig. 1. Schematic of the operating principle and structural configuration of the overflow re-separation hydrocyclone

2.2. Experimental system and materials

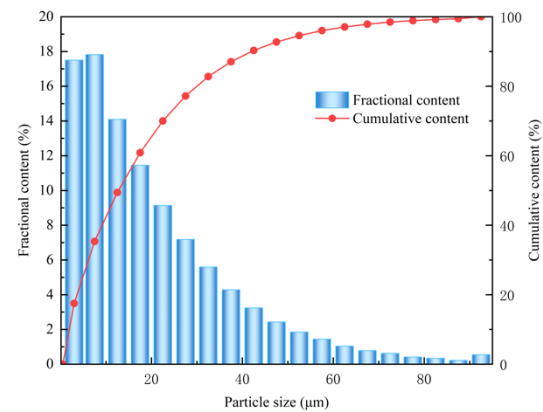
Experiments were performed in a closed-loop system. Quartz sand and clean water were mixed to prepare a feed containing 10 wt% solids, and continuous agitation was used to prevent settling in the mixing tank. The slurry was supplied to the hydrocyclone by a pump. All three products—underflow, sideflow, and overflow—were returned to the tank so that the feed concentration and particle-size composition could remain stable during a test. Feed pressure was controlled by changing pump speed with a variable-frequency drive. For each experimental condition, the system was restarted and the operating parameters were readjusted. After stable operation was reached, the feed and three product streams were sampled simultaneously in at least three successive sets. The samples were kept separate and analyzed individually. Volumetric flow rate, solids mass concentration, and particle-size distribution were determined for each set. The quartz sand density was 2650 kg/m³ and its median particle size was approximately 15.2 μm . Particle-size analysis was conducted using a Mastersizer 3000E laser diffraction particle-size analyzer. The test system and feed size distribution are presented in Fig. 2.

Table 1. Structural parameters of the overflow re-separation hydrocyclone

Structure parameters	Size
Hydrocyclone diameter D_i / mm	75
Overflow separation chamber diameter D_2 / mm	50
Conical-section height of the main separation chamber H_1 / mm	185
Cylindrical-section height of the main separation chamber H_2 / mm	90
Overall height of the overflow separation chamber H_3 / mm	125
Feed inlet dimensions $d_{i1} \times d_{i2}$ / mm	25 × 15
Overflow outlet diameter d_o / mm	20
Underflow outlet diameter d_u / mm	10
Sideflow outlet dimensions $d_{o1} \times d_{o2}$ / mm	7 × 9
Connecting tube diameter d_c / mm	20
Connecting tube insertion depth h_c / mm	40
Vortex finder insertion depth h_o / mm	30
Cone angle α / °	20



(a) Photograph of the experimental system



(b) Particle-size distribution

Fig. 2. Experimental system and feed particle-size distribution

2.3. Experimental design

The comparative tests were first conducted with the feed pressure at 0.15 MPa, the underflow and overflow outlet diameters at 10 and 20 mm, respectively, the connecting tube diameter at 20 mm, and the feed solids concentration at 10 wt%. Under these conditions, the conventional hydrocyclone was compared with overflow re-separation hydrocyclones having chamber diameters of 75, 60, 50, and 45 mm. After the chamber diameter was selected, the four single factors were tested one at a time. The reference condition for these tests was 0.15 MPa feed pressure, 20 mm connecting tube diameter, 30 mm vortex finder insertion depth, 10 mm underflow outlet diameter, and a 7 mm × 9 mm sideflow outlet. All variables other than the factor being examined were kept unchanged. In the Box-Behnken tests, the sideflow outlet dimensions and feed solids concentration remained fixed at 7 mm × 9 mm and 10 wt%, respectively. Table 2 gives the factor levels.

Table 2. Experimental Factors and Parameters

Experimental factors	Parameters
Connecting tube diameter (mm)	16, 18, 20, 22, 24
Vortex finder insertion depth (mm)	20, 25, 30, 35, 40
Underflow outlet diameter (mm)	8, 10, 12, 14, 16
Feed pressure (MPa)	0.11, 0.13, 0.15, 0.17, 0.19

2.4. Evaluation metrics

Classification performance was evaluated using solids yield, size-class recovery, and overall classification efficiency. Solids yield describes how the solids are distributed among the three products, whereas the latter two indices characterize the particle-size separation.

For each outlet stream, the measured volumetric flow rate and solids mass concentration were used with the slurry-density relationship to calculate the corresponding solids mass flow rate. Because underflow, overflow, and sideflow account for the three products, their solids mass flow rates were summed to obtain the feed solids mass flow rate. The solids yield of an individual product was then calculated from its share of this total solids flow.

$$m_i = m_u + m_o + m_s \quad (1)$$

$$m_u = \rho_u V_u C_u m_o = \rho_o V_o C_o m_s = \rho_s V_s C_s \quad (2)$$

$$E_u = \frac{m_u}{m_i} \quad (3)$$

$$E_o = \frac{m_o}{m_i} \quad (4)$$

$$E_s = \frac{m_s}{m_i} \quad (5)$$

In Eqs. (1)–(5), subscripts i , u , o , and s represent the feed, underflow, overflow, and sideflow, respectively. The symbols m , V , ρ , and C denote solids mass flow rate, slurry volumetric flow rate, slurry density, and solids mass fraction, respectively. The yields E are expressed as fractions in Eqs. (3)–(5) and reported as percentages in the results.

For each particle-size class, recovery to the underflow was calculated from the amount of that class reporting to the underflow relative to its amount in the feed. The efficiency curve was obtained by plotting these recovery values against the representative particle sizes, as expressed in Eq. (6):

$$G(d_s) = E_u \frac{f_u(d_s)}{f_i(d_s)} \times 100\% \quad (6)$$

where E_u is the underflow solids yield expressed as a fraction, as defined in Eq. (3); d_s is the particle diameter (μm); and $f_i(d_s)$ and $f_u(d_s)$ are the mass fractions of the corresponding size class in the feed and underflow, respectively.

Overall classification efficiency was calculated using the Hancock criterion (Liu et al., 2026):

$$E_h = \frac{(\alpha - \gamma)(\beta - \alpha)}{\alpha(\beta - \gamma)(100 - \alpha)} \times 10000\% \quad (7)$$

In Eq. (7), α and γ are the $\leq 15 \mu\text{m}$ contents of the feed and underflow, respectively (%), and β is the corresponding content of the combined overflow and sideflow, calculated from Eq. (8).

For this three-product hydrocyclone, the equivalent non-underflow product was defined by combining overflow and sideflow on a solids-yield basis:

$$\beta = \frac{\beta_o E_o + \beta_s E_s}{E_o + E_s} \quad (8)$$

In Eq. (8), β_o and β_s are the $\leq 15 \mu\text{m}$ contents of the overflow and sideflow, respectively (%), and E_o and E_s are their solids yields, expressed on the same basis. Thus, E_h evaluates the underflow versus the combined upper products; it does not resolve the secondary split between overflow and sideflow. That split was assessed separately using product yields, particle-size distributions, and size-class recoveries.

3. Results and discussion

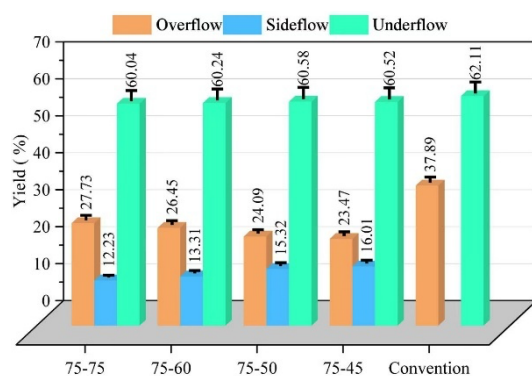
3.1. Comparative experiments

The chamber-diameter tests had two purposes: (i) to determine whether the upper chamber could provide secondary classification and (ii) to select an appropriate chamber diameter for the subsequent experiments. The conventional hydrocyclone and the four re-separation configurations (75, 60, 50, and 45 mm chamber diameters) were compared at the same operating condition: 0.15 MPa feed pressure, 10

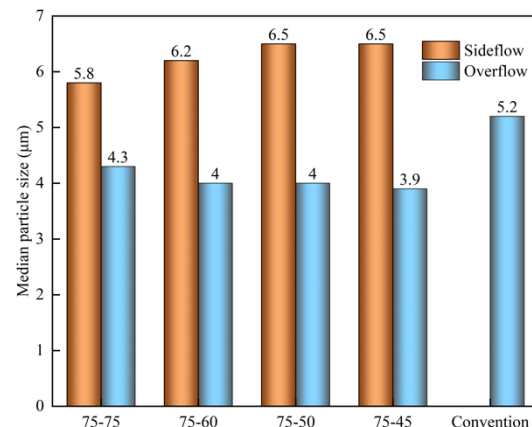
mm underflow outlet diameter, 20 mm overflow outlet diameter, 20 mm connecting tube diameter, and 10 wt% feed solids concentration. The corresponding results are given in Figs. 3 and 4.

A change in the product split can be seen directly in Fig. 3 after the overflow separation chamber was introduced. Part of the solids that entered the conventional overflow was removed through the sideflow, and the overflow solids yield consequently decreased. For every chamber diameter, the median particle size of the sideflow was higher than that of the overflow, which is consistent with preferential removal of the medium and coarse particles carried upward by the inner vortex. When the chamber diameter was reduced from 75 to 45 mm, sideflow solids yield increased from 12.23% to 16.01%, while overflow solids yield decreased from 27.73% to 23.47%. The sideflow median particle size changed from 5.8 to 6.5 μm and the overflow median size from 4.3 to 3.9 μm . By comparison, the conventional overflow had a median particle size of 5.2 μm . The re-separation chamber therefore produced a finer overflow, and reducing its diameter strengthened the transfer of relatively coarse material to the sideflow.

The underflow curves in Fig. 4 almost overlap for the different chamber diameters, showing that changing the upper chamber had little effect on the primary classification in the lower chamber. The difference appears mainly in the two upper products. As chamber diameter decreased, recovery of the medium and coarse size fractions to the sideflow increased and the sideflow and overflow curves became more separated. At chamber diameters of 50 and 45 mm, the sideflow median particle size was the same (6.5 μm), and the two overflow median sizes differed by only 0.1 μm . Further reduction below 50 mm therefore provided little additional separation benefit. A smaller chamber could also reduce throughput, increase the resistance encountered by the upward inner vortex, and make fabrication more difficult. For these reasons, 50 mm was used in the subsequent tests.

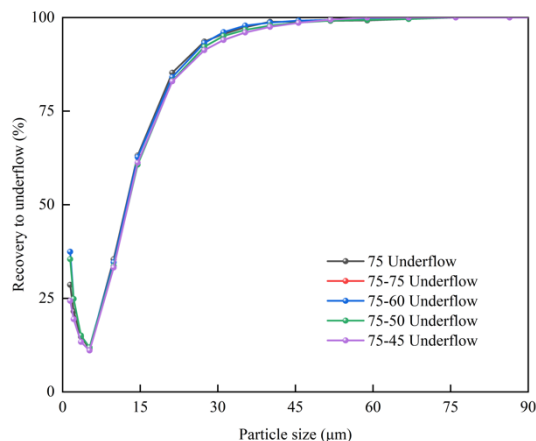


(a) Solids yield

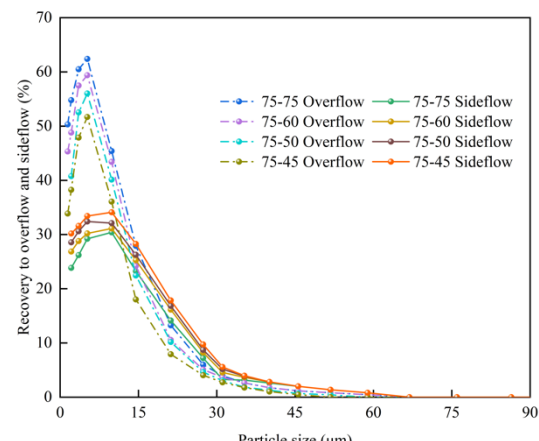


(b) Median particle size

Fig. 3. Solids yields and median particle sizes at different overflow separation chamber diameters



(a) Underflow



(b) Overflow and sideflow

Fig. 4. Efficiency curves at different overflow separation chamber diameters

3.2. Effect of connecting tube diameter on classification performance

Figs. 5 and 6 and Table 3 show that the connecting tube diameter strongly changed the solids split between the lower and upper sections. As the diameter increased from 16 to 24 mm, underflow solids yield fell from 71.17% to 58.20%. Over the same range, overflow solids yield increased from 17.94% to 25.52% and sideflow solids yield from 10.89% to 16.28%. The connecting tube is the passage followed by the inner vortex into the overflow separation chamber; increasing its cross-sectional area therefore allowed more solids to enter the upper chamber and reduced the fraction discharged directly through the underflow.

The redistribution of solids was accompanied by corresponding changes in the particle-size composition of the products. The $\leq 15 \mu\text{m}$ content of the underflow decreased from 35.20% to 25.87%, while the overall classification efficiency rose from 33.63% to 56.81%. Overflow fineness reached its highest value at 22 mm, where the $\leq 15 \mu\text{m}$ content was 86.54%; at 24 mm it was 86.13%. The overall classification efficiency also changed only slightly between 22 and 24 mm, increasing by 0.89 percentage points. This weak marginal improvement indicates that the effect of further enlargement was already limited. Therefore, 16, 20, and 24 mm were selected as the levels for the response surface design.

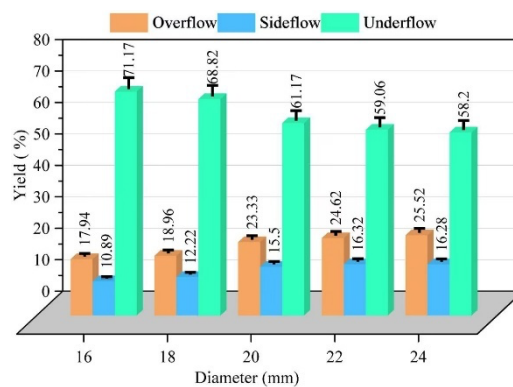


Fig. 5. Solids yields at different connecting tube diameters

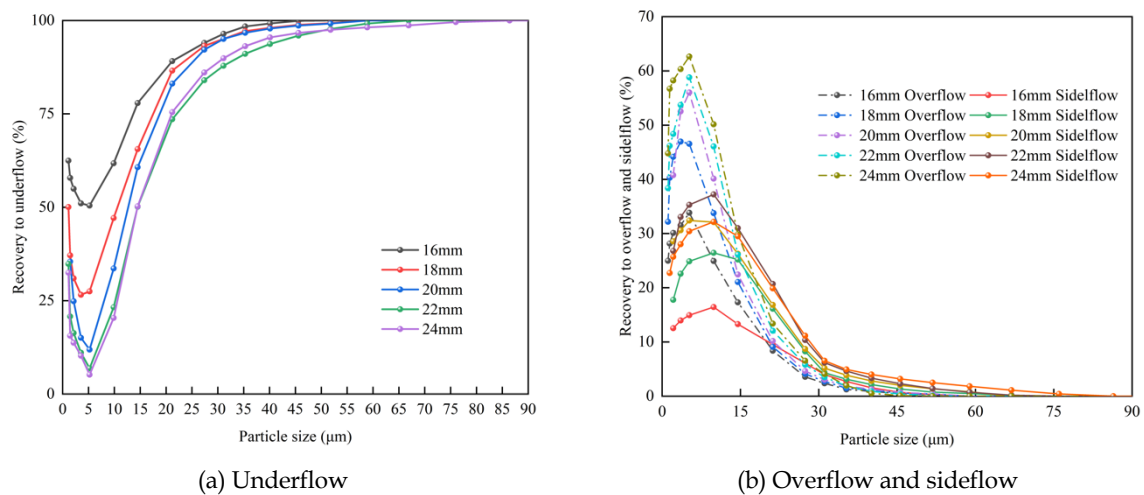


Fig. 6. Efficiency curves at different connecting tube diameters

3.3. Effect of vortex finder insertion depth on classification performance

The vortex finder insertion depth determines the axial position at which the final overflow is collected from the upper chamber and therefore affects particle selection in that region. Insertion depths of 20, 25, 30, 35, and 40 mm were examined with the other variables fixed (Figs. 7 and 8; Table 4). Unlike the connecting tube diameter, changing the insertion depth caused only small variations in the solids yields of the three products. Its main influence was consequently on product particle-size composition rather than on the overall solids split.

From 20 to 40 mm insertion depth, the $\leq 15 \mu\text{m}$ fraction in the overflow increased from 78.77% to 87.23%. The same fraction in the underflow remained within a narrow range of 27.31%–28.41%. Overall classification efficiency was 47.28% at 20 mm and 53.61% at 40 mm, with the highest measured value, 53.84%, occurring at 30 mm. These data show that increasing insertion depth primarily refined the overflow. Because the overall efficiencies from 30 to 40 mm differed little, 20, 30, and 40 mm were chosen as the response-surface levels.

Table 3. Effect of connecting tube diameter on hydrocyclone classification performance

Connecting tube diameter (mm)	Content of particles $\leq 15 \mu\text{m}$ (%)			Overall classification efficiency (%)
	Underflow	Overflow	Sideflow	
16	35.2	72.77	65.38	33.63
18	31.4	78.49	73.15	43.22
20	27.3	86.46	79.78	53.84
22	26.14	86.54	81.19	55.92
24	25.87	86.13	81.56	56.81

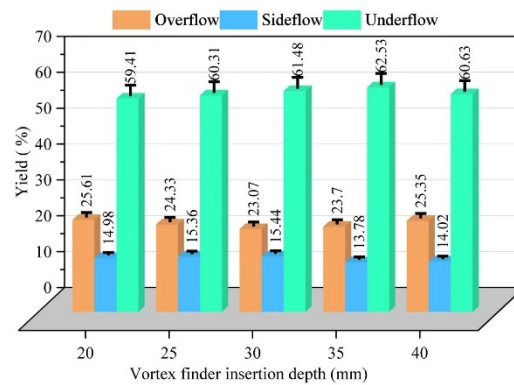


Fig. 7. Solids yields at different vortex finder insertion depths

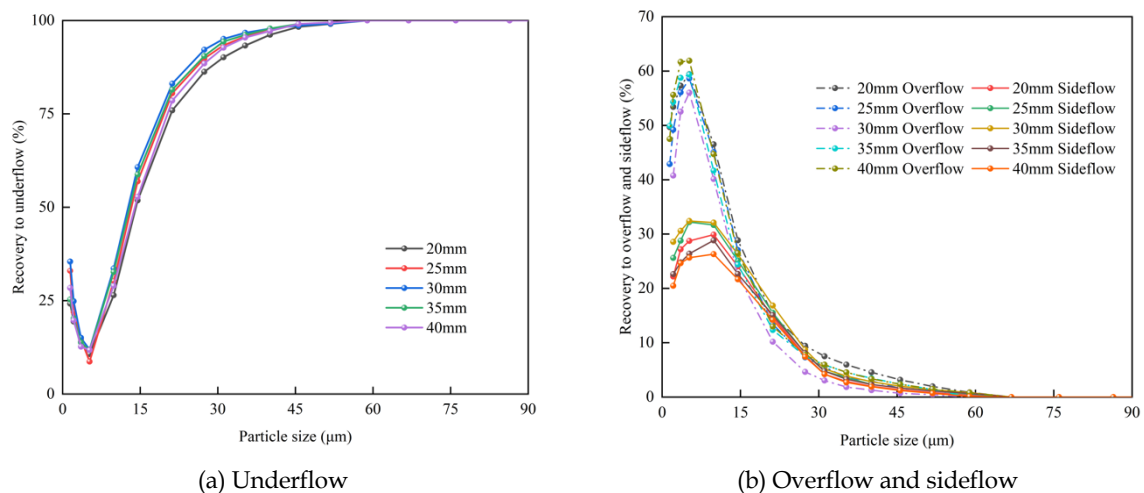


Fig. 8. Efficiency curves at different vortex finder insertion depths

3.4. Effect of underflow outlet diameter on classification performance

Changing the underflow outlet diameter altered the discharge capacity of the main separation chamber and produced a pronounced redistribution of solids (Figs. 9 and 10; Table 5). Increasing the diameter from 8 to 16 mm raised the underflow solids yield from 52.33% to 71.21%, while the overflow and

sideflow yields declined from 28.21% and 19.46% to 15.06% and 13.73%, respectively. With a larger outlet, the resistance to downward discharge was reduced, so more particles followed the outer vortex to the underflow and less material was carried into the overflow separation chamber.

This change in solids partition was accompanied by greater fine-particle loss to the underflow. The $\leq 15 \mu\text{m}$ content of the underflow increased from 24.14% at an 8 mm outlet to 35.47% at 16 mm, whereas the corresponding overflow content decreased from 88.47% to 78.71%. Overall classification efficiency consequently fell from 59.54% to 35.86%. An excessively large underflow outlet therefore favors discharge capacity at the expense of size selectivity and also leaves less solids available for re-separation in the upper chamber. Its selection must take both overflow solids yield and overall classification efficiency into account.

Table 4. Effect of vortex finder insertion depth on hydrocyclone classification performance

Vortex finder insertion depth (mm)	Content of particles $\leq 15 \mu\text{m}$ (%)			Overall classification efficiency (%)
	Underflow	Overflow	Sideflow	
20	28.41	78.77	72.38	47.28
25	27.90	83.49	76.15	50.97
30	27.31	86.46	79.78	53.84
35	28.14	86.54	81.19	53.03
40	27.87	87.23	80.56	53.61

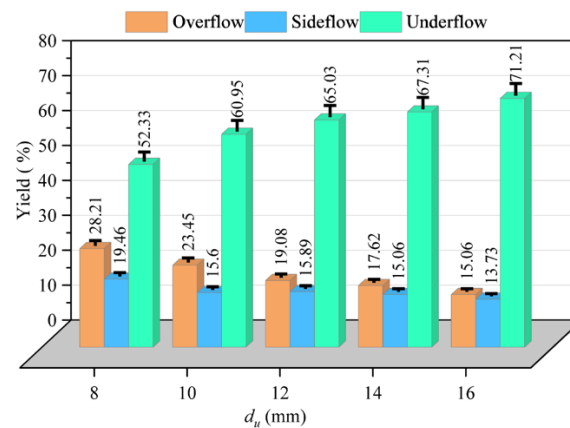


Fig. 9. Solids yields at different underflow outlet diameters

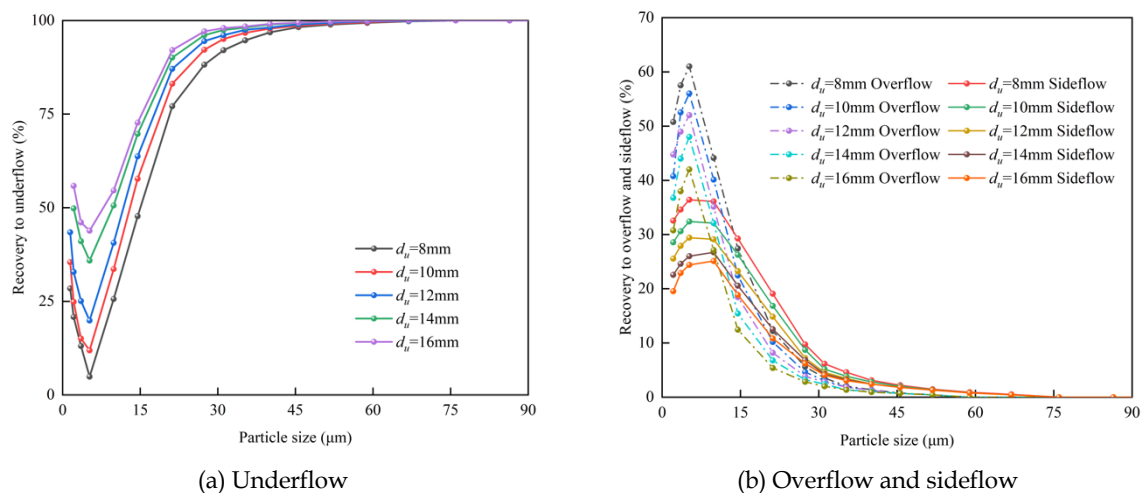


Fig. 10. Efficiency curves at different underflow outlet diameters

Table 5. Effect of underflow outlet diameter on hydrocyclone classification performance

Underflow outlet diameter (mm)	Content of particles $\leq 15 \mu\text{m}$ (%)			Overall classification efficiency (%)
	Underflow	Overflow	Sideflow	
8	24.14	88.47	81.47	59.54
10	27.31	86.46	79.78	53.84
12	29.7	83.89	76.14	48.19
14	32.14	80.73	71.61	42.22
16	35.47	78.71	69.87	35.86

3.5. Effect of feed pressure on classification performance

Feed pressure changed the inlet momentum and, in turn, the intensity of particle migration in the swirling field. Tests were conducted at 0.11, 0.13, 0.15, 0.17, and 0.19 MPa with the geometry unchanged (Figs. 11 and 12; Table 6). Raising the pressure increased underflow solids yield from 56.91% to 62.47%, while the overflow and sideflow yields decreased slightly. The higher inlet momentum strengthened centrifugal action and increased the tendency of particles to move toward the wall and report to the underflow.

The effect on particle-size composition was not entirely favorable. When feed pressure increased from 0.11 to 0.19 MPa, the $\leq 15 \mu\text{m}$ content of the overflow rose from 82.12% to 89.74%, indicating a finer overflow; however, the same size fraction in the underflow also increased from 26.40% to 28.20%. Thus, the improvement in overflow fineness was accompanied by a small increase in fine-particle entrainment to the underflow. Overall classification efficiency increased only from 52.55% to 54.09%. The limited change in this index shows that pressure alone could not produce a large improvement and should be considered together with the geometric parameters.

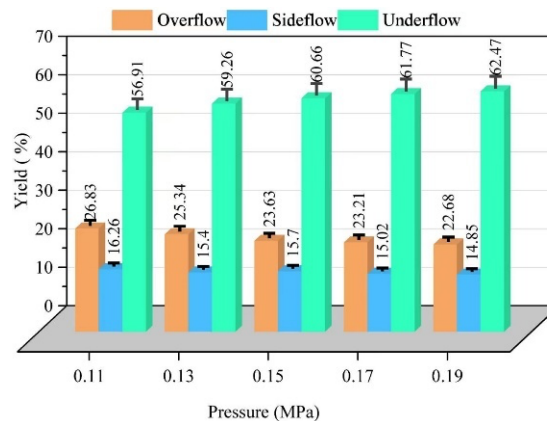


Fig. 11. Solids yields at different feed pressures

Table 6. Effect of feed pressure on hydrocyclone classification performance

Feed pressure (MPa)	Content of particles $\leq 15 \mu\text{m}$ (%)			Overall classification efficiency (%)
	Underflow	Overflow	Sideflow	
0.11	26.4	82.12	76.54	52.55
0.13	26.9	84.79	78.99	53.67
0.15	27.3	86.46	79.78	53.84
0.17	27.9	88.73	80.67	53.96
0.19	28.2	89.74	82.01	54.09

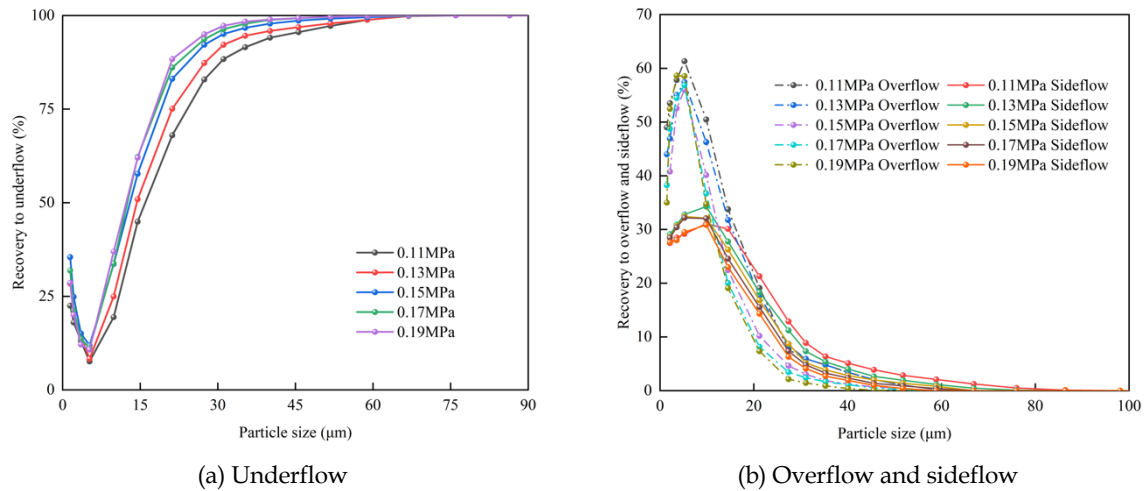


Fig. 12. Efficiency curves at different feed pressures

3.6. Response surface analysis

On the basis of the single-factor results, a four-factor, three-level Box-Behnken design was used to evaluate the combined parameter effects. Feed pressure, vortex finder insertion depth, underflow outlet diameter, and connecting tube diameter were coded as A, B, C, and D, respectively. The two responses were overflow solids yield (E_o) and overall classification efficiency (E_h). The coded levels and the 27 experimental combinations are listed in Tables 7 and 8.

Table 7. Factors and levels of the Box-Behnken design

Factor	Code	Level		
		-1	0	1
Feed pressure (MPa)	A	0.11	0.15	0.19
Vortex finder insertion depth (mm)	B	20	30	40
Underflow outlet diameter (mm)	C	8	12	16
Connecting tube diameter (mm)	D	16	20	24

Table 8. Experimental matrix and response values for the Box-Behnken design

Run	Feed pressure (MPa)	Vortex finder insertion depth (mm)	Underflow outlet diameter (mm)	Connecting tube diameter (mm)	E_o (%)	E_h (%)
1	0.19	30	12	24	21.14	50.17
2	0.11	30	16	20	15.67	34.76
3	0.19	30	16	20	11.74	44.15
4	0.15	30	8	24	29.11	52.05
5	0.19	30	12	16	12.98	42.02
6	0.15	40	12	24	24.11	51.15
7	0.15	30	16	24	17.11	41.13
8	0.11	30	12	24	25.14	48.01
9	0.11	30	8	20	28.64	44.54

Run	Feed pressure (MPa)	Vortex finder insertion depth (mm)	Underflow outlet diameter (mm)	Connecting tube diameter (mm)	E_o (%)	E_h (%)
10	0.15	20	12	16	14.87	33.15
11	0.11	40	12	20	24.16	48.13
12	0.15	30	12	20	21.15	51.54
13	0.11	30	12	16	15.98	29.64
14	0.15	20	16	20	14.48	41.15
15	0.19	40	12	20	18.96	51.13
16	0.15	40	12	16	14.77	39.14
17	0.15	20	12	24	24.43	50.13
18	0.15	20	8	20	28.14	47.13
19	0.15	30	12	20	18.89	48.11
20	0.19	20	12	20	18.44	48.09
21	0.11	20	12	20	24.26	47.18
22	0.19	30	8	20	23.98	46.13
23	0.15	30	8	16	22.15	38.12
24	0.15	40	8	20	27.97	52.12
25	0.15	30	12	20	17.64	47.04
26	0.15	40	16	20	14.67	45.96
27	0.15	30	16	16	10.01	31.12

Overflow solids yield:

$$E_o = 19.23 - 2.22A + 0.0017B - 6.36C + 4.19D + 0.1550AB + 0.1825AC - 0.25AD + 0.09BC - 0.0550BD + 0.035CD + 0.4017A^2 + 1.42B^2 + 0.7242C^2 - 0.7596D^2 \quad (9)$$

Overall classification efficiency:

$$E_h = 48.9 + 2.45A + 1.73B - 3.49C + 6.62D + 0.5225AB + 1.95AC - 2.56AD - 0.045BC - 1.24BD - 0.98CD - 1.72A^2 + 0.8467B^2 - 3.67C^2 - 5.23D^2 \quad (10)$$

3.6.1. Regression models and significance analysis

Tables 9 and 10 summarize the analysis of variance. For both response models, the model P-values were below 0.0001, whereas the lack-of-fit P-values were 0.9384 for E_o and 0.681 for E_h . The significant regressions and non-significant lack-of-fit terms indicate that, within the investigated factor ranges, the fitted equations represented the experimental responses adequately for prediction and optimization.

For overflow solids yield, the significant linear terms were feed pressure, underflow outlet diameter, and connecting tube diameter. Vortex finder insertion depth did not show a significant linear contribution but had a significant quadratic effect. The main-factor influence followed $C > D > A > B$, and none of the two-factor interactions was significant. For overall classification efficiency, all four linear terms were significant and their influence followed $D > C > A > B$. Among the interaction terms, only AD, representing feed pressure $\times$ connecting tube diameter, reached statistical significance.

3.6.2 Response surface analysis

Fig. 13 shows the overflow solids yield surfaces obtained by varying two factors while holding the other two at their center levels. Increasing feed pressure or underflow outlet diameter reduced E_o , whereas increasing connecting tube diameter raised it; the largest variation was associated with the underflow

Table 9. Analysis of variance for the E_o regression model

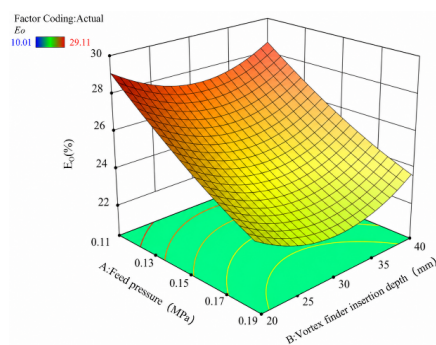
Source	Sum of squares	Degrees of freedom	Mean square	F-value	P-value	Significance
Model	777.51	14	55.54	45	< 0.0001	Significant
A	59.01	1	59.01	47.81	< 0.0001	
B	0	1	0	0	0.9959	
C	485.27	1	485.27	393.17	< 0.0001	
D	210.67	1	210.67	170.69	< 0.0001	
AB	0.0961	1	0.0961	0.0779	0.7850	
AC	0.1332	1	0.1332	0.1079	0.7482	
AD	0.2500	1	0.2500	0.2026	0.6607	
BC	0.0324	1	0.0324	0.0263	0.8740	
BD	0.0121	1	0.0121	0.0098	0.9228	
CD	0.0049	1	0.0049	0.0040	0.9508	
A ²	0.8605	1	0.8605	0.6972	0.4201	
B ²	10.8000	1	10.8000	8.7500	0.0120	
C ²	2.8000	1	2.8000	2.2700	0.1581	
D ²	3.0800	1	3.0800	2.4900	0.1403	
Residual	14.8100	12	1.2300			Not significant
Lack of fit	8.4800	10	0.8481	0.2680	0.9384	
Pure error	6.3300	2	3.1700			
Total	792.32	26				

Table 10. Analysis of variance for the E_h regression model

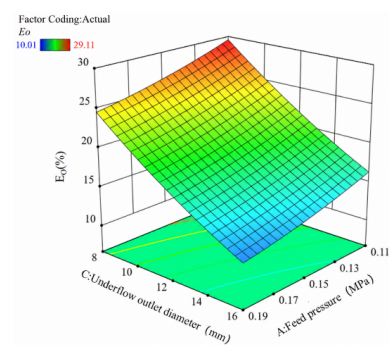
Source	Sum of squares	Degrees of freedom	Mean square	F-value	P-value	Significance
Model	1061.04	14	75.79	16.81	< 0.0001	Significant
A	72.18	1	72.18	16.01	0.0018	
B	36.05	1	36.05	7.99	0.0152	
C	145.74	1	145.74	32.32	0.0001	
D	526.03	1	526.03	116.65	< 0.0001	
AB	1.09	1	1.09	0.2422	0.6315	
AC	15.21	1	15.21	3.37	0.0912	
AD	26.11	1	26.11	5.79	0.0331	
BC	0.0081	1	0.0081	0.0018	0.9669	
BD	6.18	1	6.18	1.37	0.2646	
CD	3.84	1	3.84	0.8519	0.3742	
A ²	15.72	1	15.72	3.49	0.0865	
B ²	3.82	1	3.82	0.8478	0.3753	
C ²	71.67	1	71.67	15.89	0.0018	
D ²	146	1	146	32.38	0.0001	
Residual	54.11	12	4.51			Not significant
Lack of fit	43.06	10	4.31	0.7791	0.681	
Pure error	11.05	2	5.53			
Total	1115.15	26				

outlet diameter. This response agrees with the single-factor behavior. A larger underflow opening facilitates downward discharge, while a larger connecting tube increases the amount of inner-vortex solids entering the overflow separation chamber. The effect of vortex finder insertion depth was mainly nonlinear. In addition, the contour lines showed no strong distortion attributable to factor interaction, which is consistent with the non-significant interaction terms in Table 9. Hence, E_o was controlled mainly by the individual linear and quadratic effects.

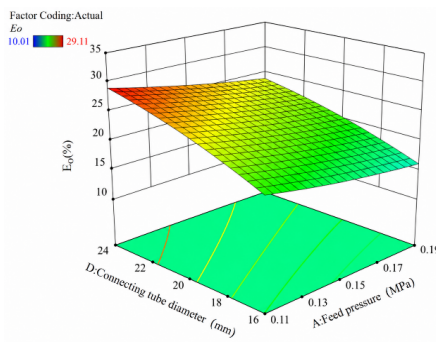
A different pattern is evident for overall classification efficiency in Fig. 14. Increasing the connecting tube diameter generally increased E_h , whereas enlarging the underflow outlet reduced it. The connecting tube diameter exerted the strongest influence, followed by the underflow outlet diameter. A larger connecting tube provides more opportunity for medium and coarse particles carried by the inner vortex to enter the re-separation chamber and leave through the sideflow. A larger underflow outlet, on the other hand, increases the probability that fine particles are discharged in the underflow. The response surfaces also agree with the ANOVA result that only the feed pressure–connecting tube diameter interaction was significant. The remaining pairwise interactions were weak compared with the main and quadratic effects.



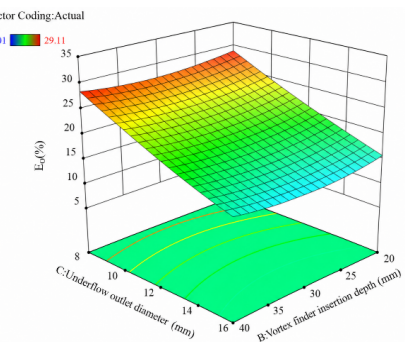
(a) Feed pressure and vortex finder insertion depth



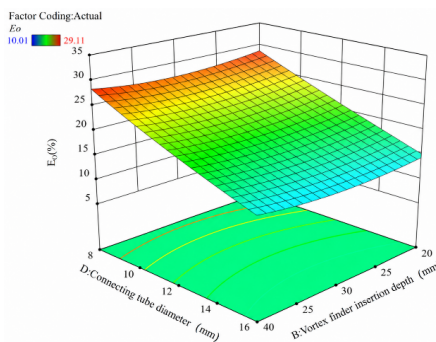
(b) Feed pressure and underflow outlet diameter



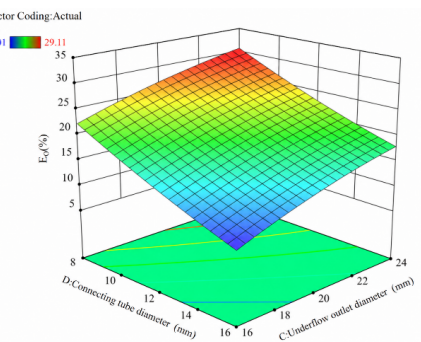
(c) Feed pressure and connecting tube diameter



(d) Vortex finder insertion depth and underflow outlet diameter



(e) Vortex finder insertion depth and connecting tube diameter



(f) Underflow outlet diameter and connecting tube diameter

Fig. 13. Response surfaces for the overflow solids yield

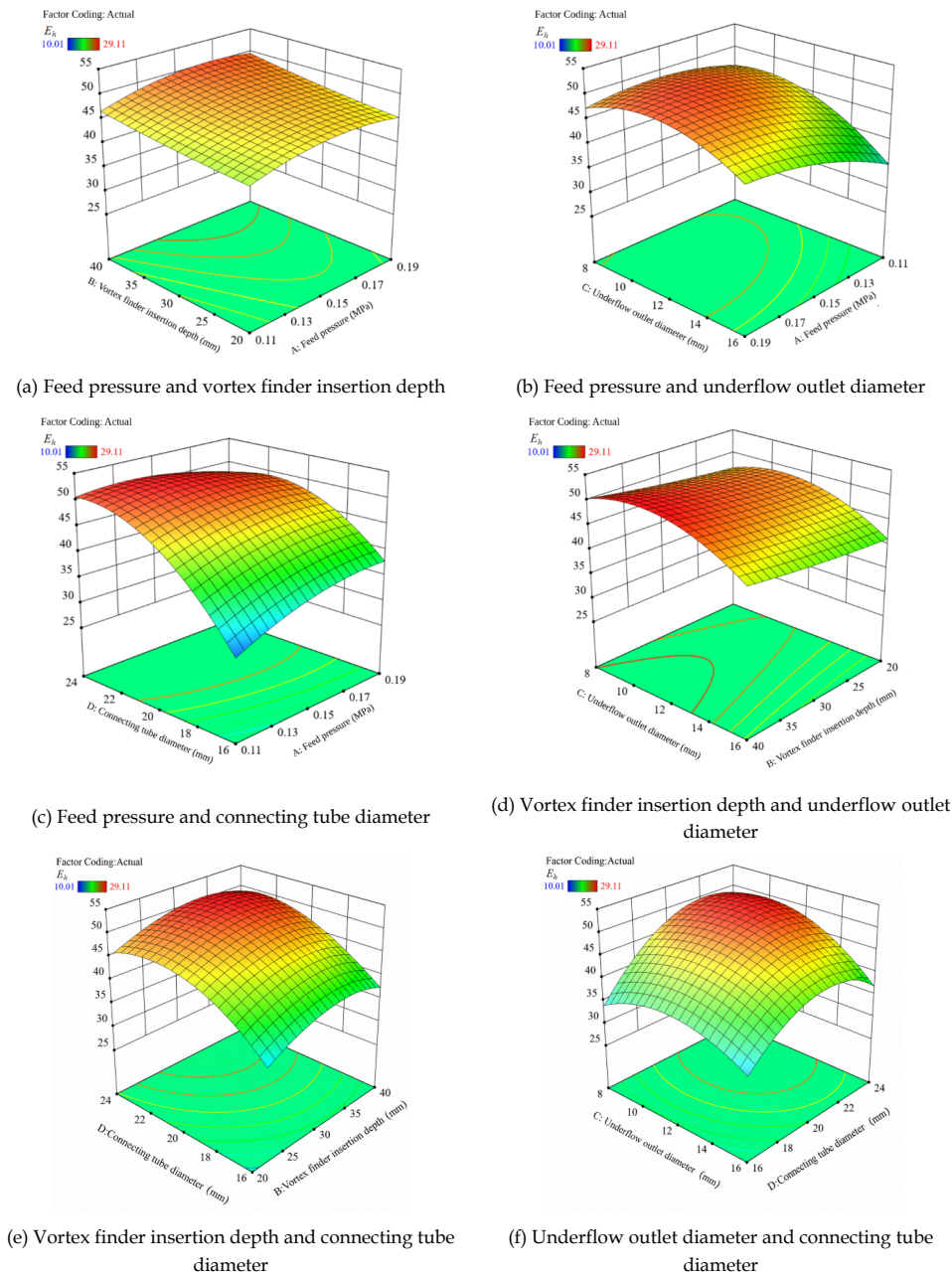


Fig. 14. Response surfaces for the overall classification efficiency

3.6.3 Multi-objective optimization and experimental validation

Multi-objective optimization was carried out in Design-Expert using the two fitted response models. The overflow solids yield was constrained within 22%–32%, while the overall classification efficiency was set to be maximized. The solution with the highest composite desirability of 0.877 corresponded to a feed pressure of 0.159 MPa, a vortex finder insertion depth of 40 mm, an underflow outlet diameter of 10.022 mm, and a connecting tube diameter of 22.002 mm. At this theoretical optimum, the predicted overflow solids yield and overall classification efficiency were 25.32% and 54.03%, respectively.

Considering the precision of experimental adjustment and component fabrication, the parameter combination used for experimental validation was rounded to 0.15 MPa, 40 mm, 10 mm, and 22 mm, respectively. Under these conditions, the experimentally measured overflow solids yield and overall classification efficiency were 24.76% and 54.21%, respectively. Compared with the predictions at the theoretical optimum, the corresponding deviations were 2.21% and 0.33%, indicating that the optimized parameter combination was experimentally reproducible and that the response surface models provided reliable guidance for parameter optimization.

Table 11. Candidate parameter combinations obtained from multi-objective optimization

Number	Feed pressure (MPa)	Vortex finder insertion depth (mm)	Underflow outlet diameter (mm)	Connecting tube diameter (mm)	E_o (%)	E_h (%)	Desirability
1	0.159	40	10.022	22.002	25.318	54.028	0.877
2	0.119	40	9.125	23.858	30.925	52.774	0.798
3	0.129	20	9.023	23.703	30.675	52.528	0.783
4	0.135	20	9.355	23.378	29.362	52.468	0.779
5	0.126	20.015	8.108	22.894	32	51.999	0.75
6	0.128	20.001	8	22.407	31.655	51.551	0.722

4. Conclusions

Comparative tests, single-factor tests, and response-surface optimization were used to examine the separation behavior of the overflow re-separation hydrocyclone. The main conclusions are as follows:

- (1) The upper overflow separation chamber provided a measurable secondary-classification effect. Fine-particle recovery in the overflow was higher in the 0–5 μm range, whereas the sideflow showed higher recovery for the relatively coarse 10–15 μm fraction. This difference indicates that medium and coarse particles carried upward by the inner vortex were partly removed through the sideflow. With a chamber diameter of 50 mm, the median particle sizes of the overflow and sideflow were 4.0 and 6.5 μm , respectively.
- (2) The four single factors played different roles in the classification process. Increasing the connecting tube diameter allowed more material to enter the upper chamber and improved overall classification efficiency. Increasing vortex finder insertion depth mainly refined the overflow. Enlarging the underflow outlet increased fine-particle entrainment in the underflow and reduced overall classification efficiency. Higher feed pressure also increased overflow fineness, but the improvement in overall classification efficiency was limited.
- (3) The influence on overflow solids yield decreased in the order underflow outlet diameter > connecting tube diameter > feed pressure > vortex finder insertion depth. For overall classification efficiency, the corresponding order was connecting tube diameter > underflow outlet diameter > feed pressure > vortex finder insertion depth. Among the two-factor interactions, only feed pressure $\times$ connecting tube diameter was significant.
- (4) Multi-objective optimization was conducted with overflow solids yield and overall classification efficiency as the two responses. Experimental validation of the selected parameter combination resulted in an overflow solids yield of 24.76% and an overall classification efficiency of 54.21%.

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