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Study on the sand separation performance of hydrocyclone with combined cone

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Abstract: Owing to insufficient shear force in conventional hydrocyclones, organic matter adhering to the surface of fine inorganic sand cannot be removed, resulting in excessively high organic matter contents in underflow, which hinder secondary utilisation of inorganic sand. This study proposes a combined cone-type hydrocyclone for reducing the organic matter content in underflow by increasing the shear force and expanding the separation space. Experimental testing was conducted to investigate the influences of structural parameters, such as the apex diameter and spherical-cone radius, on the separation performance of the hydrocyclone. Results indicate that reducing the apex diameter decreases the inorganic content in underflow from 81.36% to 73.51%; meanwhile, increasing the sphere-cone radius and step shrinkage distance first increases and then decreases the inorganic content in underflow. A response surface method was used to construct a prediction model for the separation performance. The optimal parameter combination is obtained, with the inorganic yield in underflow reaching as high as 68.97%.

Keywords: hydrocyclone, combined conical type, experimental study, sand washing, separation performance

1. Introduction

Owing to their simple structure, high separation efficiency, and low energy consumption, hydrocyclones are widely used in the field of wastewater treatment (Liu et al., 2017; Thomas et al., 2021; Konstantinos et al., 2025; Liu et al., 2026; Xu et al., 2026). In sludge treatment, hydrocyclones can separate inorganic and organic matter from activated sludge, enabling recovery and reuse of inorganic sand and gravel (Mansour-Geoffrion et al., 2010; Chen et al., 2016; Liu et al., 2021; Liu et al., 2026). However, owing to the tendency of sludge to adhere and insufficient shear force of conventional hydrocyclones, some organic matter tends to adhere to inorganic sand and gravel and discharge with the underflow, resulting in elevated organic content in the underflow and affecting the subsequent recovery and reuse (Xu and Wang, 2019; Zhang et al., 2025; Wang et al., 2026). Moreover, fine inorganic sand in sludge tends to be carried along with the sludge into overflow because of its small particle size, thereby reducing the recovery rate (Anvar et al., 2023; Jiang et al., 2025; Kostrzewa et al., 2025). In their experiments, some researchers tested a multistage cyclone process using slurry feeds with varying solid contents and increasing proportions of ultrafine particles (-44 μm). They found that, with a two-stage Linatex separator process, the underflow recovery rate was high (Mikhail et al., 1997). Some scholars have studied the effect of 30-mm hydrocyclones on the separation efficiency of sludge removal and found that they proposed an optimal configuration demonstrating a total separation efficiency greater than 89% and a flow ratio less than 35% (Mognon et al.,2015). Some scholars have conducted research on the sand removal performance of hydrocyclones manufactured by Haiwang Group in wastewater treatment plants and found that the FX100 achieves a removal rate of 61.89% for fine sand (Huang et al., 2021).

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In response to these issues, numerous scholars have researched structural optimization and improvement (Yablonskii, 2020; Jiang et al., 2022; Seunggi et al., 2025). Some scholars studied double-cone hydrocyclones with different cone-segment combinations and reported that multi-cone-segment configurations offered greater advantages than those with a single cone angle (Jiang et al.,2023). Some scholars optimized a 25-mm hydrocyclone through numerical simulations and experiments, finding that increasing hc/b from 1 to 3 resulted in a continuous decrease in the tangential velocity of the internal flow field and the axial velocity of the outer vortex (Wang et al.,2023). Some scholars proposed a double-cone microhydrocyclone and analyzed microplastic migration and removal, revealing that the double-cone microhydrocyclone promoted axial enrichment of microplastics, with microplastics with densities lower than that of water almost completely separated (Dai et al.,2025). Some scholars proposed a series of hydrocyclones with composite structures and found that the cutting capacity and classification accuracy of these composite structures were significantly improved compared with traditional hydrocyclones (Liang et al.,2025). Some scholars investigated different conical profiles and reported that compared with a traditional base cone, an involute cone reduced the content of particles smaller than 25 μm in underflow by 6.02 percentage points and increased the classification efficiency by 5.93 percentage points (Li et al.,2025). Some scholars studied hydrocyclones with different conical structures and found that as the conical shape gradually changed from concave to convex, a convex-cone hydrocyclone exhibited optimal performance with the smallest possible deviation (Ghodrat, 2014; Ye et al., 2026). Some scholars investigated the effect of the cone section volume on hydrocyclones and demonstrated that increasing the volume of the conical section reduced the static pressure and tangential velocity, leading to a significant decrease in the centrifugal intensity (Li et al.,2026). In summary, the cone section structure significantly affects the shear force and separation performance (Ghodrat, 2016; Li et al., 2018).

Therefore, this study proposes an improvement based on the conical segment structure, introducing a combined spherical-cone and stepped-cone structure, as shown in Fig. 1. When a material enters a hydrocyclone, it moves downwards along the outer wall under the influence of the outer vortex. It is blocked at the inner conical step, causing organic sludge particles with a lower density and larger particle size to pass through the zero-velocity envelope and enter the inner vortex prematurely. Moreover, changes in the conical section geometry may increase the velocity gradient within the conical section, thereby increasing the shear stress. This causes more organic matter adhering to sand and gravel to be separated, releasing some of the fine sand entrained by organic matter and thereby increasing the fine sand content in the bottom flow. Furthermore, under the action of the bottom cone, a material entering the lower cone section is thoroughly dispersed, allowing for secondary separation of organic and inorganic materials. As more organic matter enters the inner vortex, the organic matter concentration near the underflow outlet decreases, and more inorganic matter discharges through the underflow outlet. Ultimately, the goal is to reduce the organic content in the underflow by increasing the shear force via the use of stepped cones and expanding the separation space by the use of spherical cones.

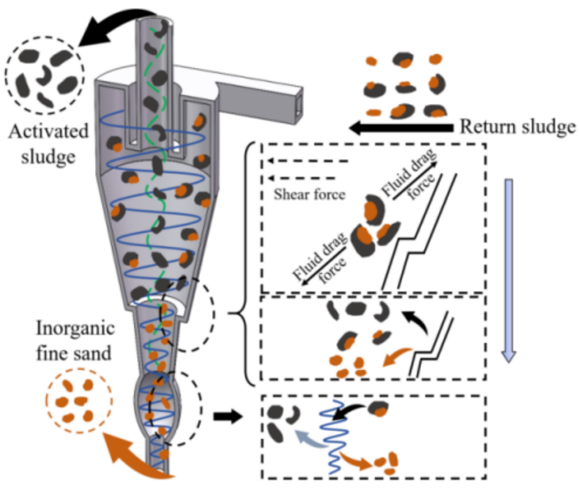


Fig. 1. Principle of combined cone-type hydrocyclone

Although hydrocyclones have a simple structure, numerous factors influence their separation performance (Ni, 2019; Zhao et al., 2026; Júnior et al., 2026). Some scholars studied the effect of the underflow pipe diameter on the separation performance of a novel sludge removal hydrocyclone (Ni et al., 2016). Some scholars studied the effect of the feed particle size distribution on the hydrocyclone separation performance for different apex diameters, finding that an outlet diameter that was either too small or too large caused a significant number of particles to be misclassified into overflow or underflow fractions (Zhang et al.,2019). Some scholars investigated the effect of the apex diameter on the separation performance of parabolic hydrocyclones and reported that decreasing the apex diameter significantly reduced the content of 5 μm fine particles in underflow (Liu et al.,2023). Some scholars studied the effects of the inlet velocity and apex diameter on the separation performance of double-cone hydrocyclones, demonstrating a strong interaction between the apex diameter and inlet velocity, which affected the separated particle size (Xiong et al.,2023). Some scholars investigated the effect of the apex diameter on the separation performance of hydrocyclones with involute feed bodies and reported that increasing the apex diameter reduced the sludge removal efficiency (Lin et al.,2025). Some scholars investigated the effects of the geometric parameters of a conical filter hydrocyclone on its separation performance, achieving an optimal balance between minimising water loss and preventing a decrease in the separation efficiency (Morimoto et al.,2026). They found that as the underflow pipe diameter increased, energy consumption increased slightly, whereas the concentration of overflow decreased slightly.

Therefore, this study employs experimental testing to investigate the effects of structural parameters (such as the apex diameter, step shrinkage distance, and sphere-cone radius) on separation performance metrics, including the inorganic content and yield of underflow.

2. Materials and methods

2.1. Experimental equipment and apparatus

To investigate the effects of factors such as the apex diameter and spherical-cone radius on the separation performance of a combined cone hydrocyclone, a Φ50 mm combined cone hydrocyclone separation test system was designed in this study. Fig. 2 shows the specific structural parameters, and Table 1 lists the hydrocyclone parameters. The experimental system primarily consisted of a mixing tank, an underflow tank, an overflow tank, a vertical pump, piping, ball valves, pressure gauges, turbine flowmeters, a variable frequency drive, and the hydrocyclone, as shown in Fig. 3. The experiment was conducted in an open-loop operation to facilitate sampling and observation. After the system was stabilised, samples were collected from the feed line, underflow outlet, and overflow pipe for analyses. The slurry collected from the interface was weighed, filtered under vacuum, dried, and weighed again to determine the concentrations of each product; yields were calculated, and each product was incinerated in a muffle furnace to determine its organic matter content and enrichment ratio.

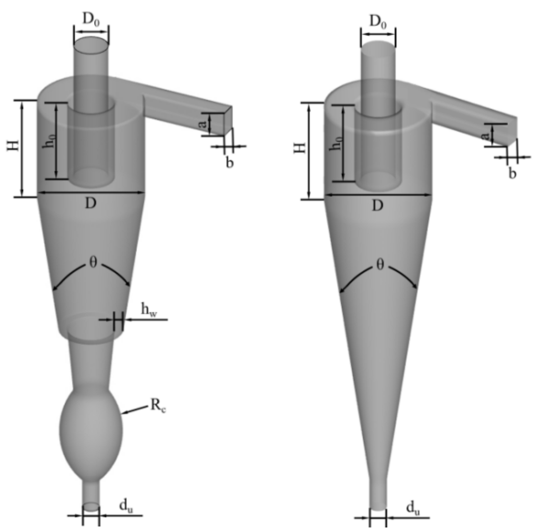
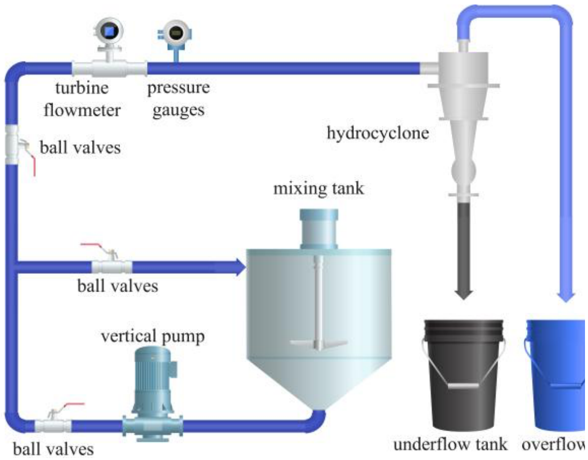


Fig. 2. Schematic diagram of the structural parameters

Table 1. Structural parameters of hydrocyclone

Structural Parameters	Conventional Hydrocyclone	Combined Cone-Type Hydrocyclone
Diameter of cylinder section D (mm)	50	50
Height of cylindrical section H (mm)	50	50
*Equivalent diameter of feed inlet a*b (mm)*	12*8	12*8
Diameter of vortex finder D_0 *(mm)*	16	16
Insertion depth of vortex finder h_0 *(mm)*	40	40
Apex diameter d_u *(mm)*	8	8
Cone angle θ *(°)*	16	16
Step shrinkage distance h_w *(mm)*	/	4
Sphere-cone radius R_c *(mm)*	/	30



(a) flowchart

(b) photograph of the physical prototype

Fig. 3. Diagram of hydrocyclone test system

2.2. Materials

The material used in this experiment was sourced from a municipal wastewater treatment plant. Testing revealed that inorganic matter accounted for 48.97% of the total composition. Fig. 4 shows the particle size distribution: fine sand (particles smaller than 33 μm) constituted 87.79%, inorganic matter in the range of 33 μm–50 μm accounted for 11.13%, and inorganic matter > 50 μm accounted for 0.84%. Because fine particles smaller than 33 μm tend to entrain organic matter and are easily carried into overflow by the internal vortex due to their small size, resulting in waste of recoverable materials, this study aims to improve the yield of inorganic matter in underflow and reduce organic content. Hereinafter, particles smaller than 33 μm are referred to as fine particles.

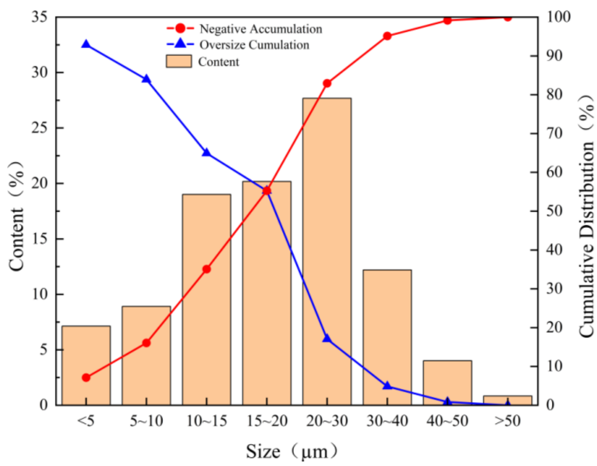


Fig. 4. Granulometric analysis result chart

2.3. Scheme design

To investigate in detail the effects of factors such as the apex diameter and sphere-cone radius on the separation performance—including the inorganic content and yield in underflow—single-factor experiments were first designed. These experiments used the apex diameter, step shrinkage distance, and sphere-cone radius as experimental variables to examine the variation patterns in the separation performance of the combined cone hydrocyclone. Table 2 shows the levels of the experimental factors. Experiments were then conducted using the Box–Behnken response surface method to establish a predictive model for the inorganic matter yield in the leachate (η), with the aim of investigating interactions among various factors and determining the optimal combination of parameters. The specific levels of factors used in the single-factor experiments are shown in Table 3.

Table 2. Single-factor test factor level table of combined cone-type hydrocyclone

Experimental Factors	**Experimental Level**
Feed pressure (MPa)	*0.06*
Feed solids concentration (%)	*5*
Apex diameter (mm)	*6, 7, 8, 9, 10*
Step shrinkage distance (mm)	*2, 3, 4, 5, 6*
Sphere-cone radius (mm)	*26, 28, 30, 32, 34*

Table 3. Factors and levels Box-Behnken Design

Affecting Factors	**Coded Value**	**Level**		
		-1	**0**	**1**
Apex diameter (mm)	*A*	6	8	10
Sphere-cone radius (mm)	*B*	26	30	34
Step shrinkage distance (mm)	*C*	2	4	6

3. Results and discussion

3.1. Comparative experiments

Underflow inorganic content and underflow inorganic yield are key indicators for evaluating the sand-discharge performance of hydrocyclones. The inorganic content in the underflow refers to the mass percentage of inorganic matter in the underflow product. This indicator directly reflects the quality of the underflow product; a higher content indicates that organic matter has been more thoroughly separated, and the inorganic matter in the underflow is of higher purity. The inorganic matter recovery rate refers to the mass of inorganic matter recovered in the underflow as a percentage of the total mass of inorganic matter in the feed material. This indicator reflects the recovery efficiency of the separation process; a higher recovery rate indicates that a greater proportion of the inorganic matter from the feed material is directed into the underflow, resulting in less loss to the overflow. Therefore, this paper focuses on the sludge and sand separation performance of the combined conical cyclone and examines the trends in performance indicators such as underflow inorganic content and underflow fine particle content.

Using the inorganic content in underflow, inorganic yield, and enrichment ratio as evaluation criteria, a comparative analysis was conducted between the combined cone hydrocyclone and conventional columnar hydrocyclone. Table 4 shows the results. As shown in the table, compared with the conventional hydrocyclone, the combined cone hydrocyclone increased the inorganic content in underflow by 5.46% and inorganic yield by 6.4%. The fine-particle inorganic matter content in underflow reached 80.02%, while the organic matter enrichment ratio in overflow of the combined cone hydrocyclone increased to 3.19. The results indicate that both the stepped-cone and spherical-cone structures effectively increase inorganic matter yield and reduce organic content in the underflow.

Table 4. Comparative test results table

Evaluation Criteria	Conventional Hydrocyclone	Combined Cone-Type Hydrocyclone
Underflow inorganic content (%)	72.96	78.42
Underflow fine particle content (%)	71.87	80.02
Underflow inorganic yield (%)	22.89	29.29
Inorganic enrichment ratio	2.00	2.12
Organic enrichment ratio	1.60	3.19

3.2. Effect of apex diameter on separation performance

The underflow outlet diameter affects flow distribution within the hydrocyclone and significantly influences separation performance. For a step shrinkage distance of 4 mm and a sphere-cone radius of 30 mm, the study investigated the effects of different apex diameters (6, 7, 8, 9, and 10 mm) on the separation performance of the combined cone hydrocyclone.

Fig. 5 shows the effect of apex diameter on the inorganic matter and fine-particle contents in the underflow. When the apex diameter was increased from 6 to 10 mm, more particles and fluid entered the underflow, weakening the 'classification' effect. Consequently, more fine particles entered the underflow, increasing fine-particle content from 68.7% to 82.1% (an increase of 19.5%). Furthermore, because the surfaces of fine particles carried a relatively large amount of organic matter, the inorganic matter content in the underflow decreased from 81.36% to 73.51%, a decrease of 9.65%. Fig. 6 illustrates the effect of the apex diameter on the enrichment ratio. As the apex diameter gradually increased, the downward flow rate increased, causing more sand and gravel that would otherwise flow downward externally to appear inside the zero-velocity envelope. This reduced the organic matter enrichment ratio from 3.09 to 2.45. Meanwhile, the inorganic matter enrichment ratio first increased and then decreased, reaching its maximum value when the apex diameter was 8 mm.

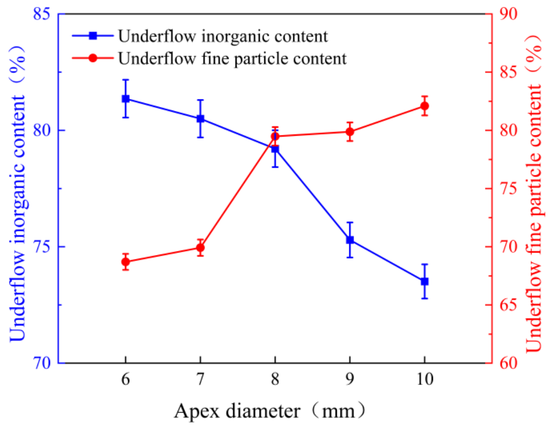


Fig. 5. The influence of apex diameter on inorganic matter content in underflow

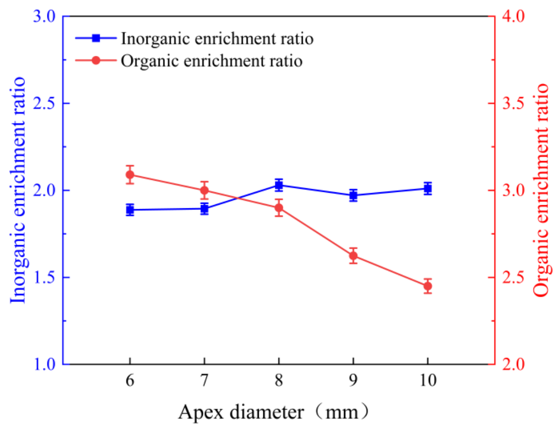


Fig. 6. The influence of apex diameter on enrichment ratio

Fig. 7 shows the effect of the apex diameter on the underflow inorganic yield. As shown in Fig. 7, an increase in the apex diameter might intensify turbulence in the internal flow field of the hydrocyclone, leading to a reduced separation efficiency. This caused some organic matter to be carried into the underflow, resulting in a decrease in the inorganic matter recovery rate from 57.49% to 32.44%. Furthermore, sludge returned to the biological treatment tank still contained a high level of inorganic matter, preventing achievement of the desired recovery efficiency.

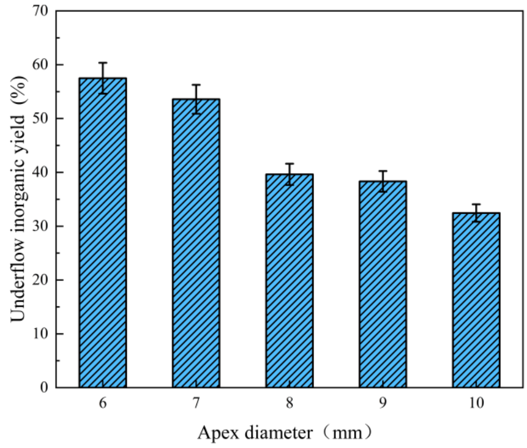


Fig. 7. The influence of apex diameter on underflow inorganic matter yield

3.3. Effect of sphere-cone radius on separation performance

The sphere-cone radius plays a crucial role in re-separating material in the conical section and affects the degree of sludge loosening in that section. With the apex diameter set at 8 mm and step shrinkage distance at 4 mm, the study investigated the effects of different sphere-cone radii (26, 28, 30, 32, and 34 mm) on sludge separation in the combined conical hydrocyclone.

Fig. 8 shows the effect of sphere-cone radius on inorganic and fine-particle contents in the underflow. As the sphere-cone radius increased, the separation space inside the ball-cone expanded. The hydrocyclone achieved more thorough sedimentation and separation of inorganic fine sand and organic sludge particles, resulting in increases in both the fine particle content and inorganic content in the underflow. The fine particle content in the underflow increased from 76.46% to 81.77% (6.94% increase), while the inorganic content increased from 79.58% to 81.57% (2.5% increase). Fig. 9 shows the effect of the spherical-cone radius on the enrichment ratio. As the sphere-cone radius increased, the fluid in the inner vortex may have gained a greater upward velocity, enhancing the lifting effect on organic sludge; consequently, the organic matter enrichment ratio increased from 2.86 to 3.61. Moreover, because of the high density of inorganic fine sand, it could overcome the downward resistance and settle into the underflow, causing the inorganic matter enrichment ratio to increase from 1.81 to 2.45.

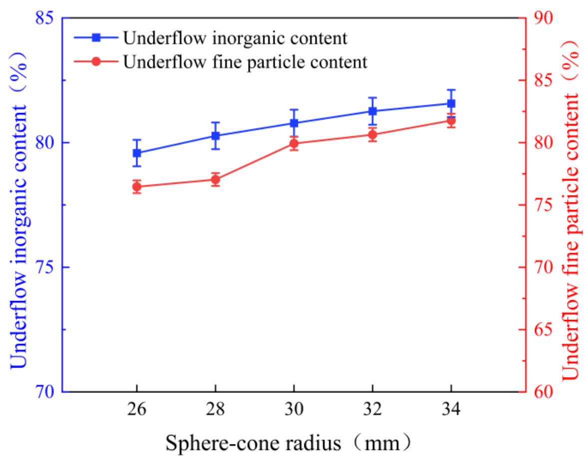


Fig. 8. The influence of sphere-cone radius on the inorganic content of underflow

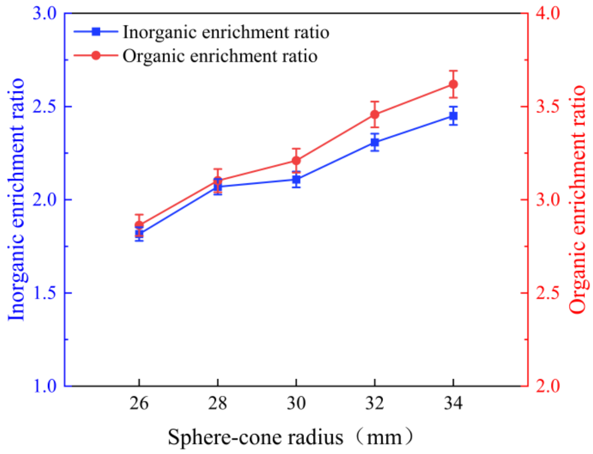


Fig. 9. The influence of sphere-cone radius on enrichment ratio

Fig. 10 shows the effect of the sphere-cone radius on the yield of inorganic matter in underflow. As the sphere-cone radius increased, the space available for material movement expanded, resulting in smaller movement speeds and easier settling and accumulation. This further enhanced the separation efficiency of the hydrocyclone, allowing more inorganic fine sand to overcome the resistance and enter the outer vortex. Consequently, the yield of inorganic material in underflow increased from 34.77% to 56.45%, effectively reducing the organic content in underflow.

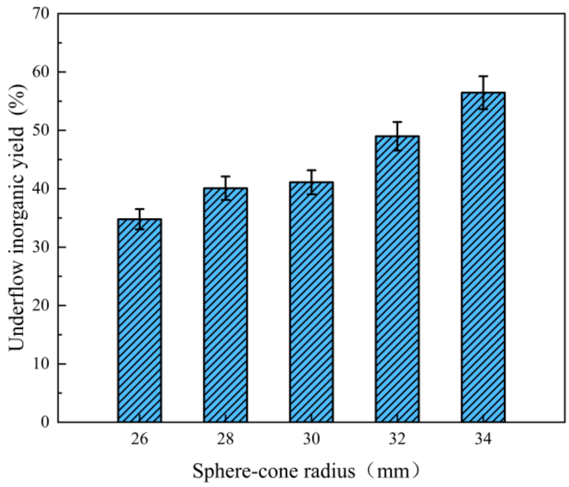


Fig. 10. The influence of sphere-cone radius on yield of inorganic matter in underflow

3.4. Effect of step shrinkage distance on separation performance

Different step shrinkage distances in a stepped cone result in varying separation efficiencies for sludge and fine sand particles; therefore, selecting an appropriate step shrinkage distance is particularly important for removing organic matter from inorganic sand and gravel. With the apex diameter set at 8 mm and sphere-cone radius set at 30 mm, this study investigated the effects of different step shrinkage distances (2, 3, 4, 5, and 6 mm) on the separation performance of the combined cone hydrocyclone.

Fig. 11 shows the effect of step shrinkage distance on inorganic and fine-particle contents in the underflow. As the step shrinkage distance increased from 2 to 4 mm, the retention effect of the stepped structure on organic sludge particles increased. Both the inorganic matter and fine particle contents in the underflow increased, reaching their maximum values at a step shrinkage distance of 4 mm. The maximum inorganic matter content in underflow was 80.57%, and the maximum fine particle content was 82.19%. As the step shrinkage distance increased from 4 to 6 mm, the stepped structure retained both organic sludge particles and sand and gravel particles, reducing both inorganic matter and fine particle contents in the underflow. Fig. 12 shows the effect of the step shrinkage distance on the enrichment ratios. As shown in Fig. 12, as the step shrinkage distance increased, both the inorganic and organic enrichment ratios first increased and then decreased. Consistent with the trends shown in Fig. 11, both the inorganic matter enrichment ratio and organic matter enrichment ratio reached their maximum values simultaneously at a step shrinkage distance of 4 mm, with the inorganic matter enrichment ratio peaking at 2.12 and the organic matter enrichment ratio peaking at 3.19.

Fig. 13 shows the effect of the step shrinkage distance on the inorganic matter yield in underflow. As shown in Fig. 13, the inorganic matter yield in underflow reached its maximum of 41.92% when the step shrinkage distance was 4 mm. This indicates that when the step shrinkage distance is too small,

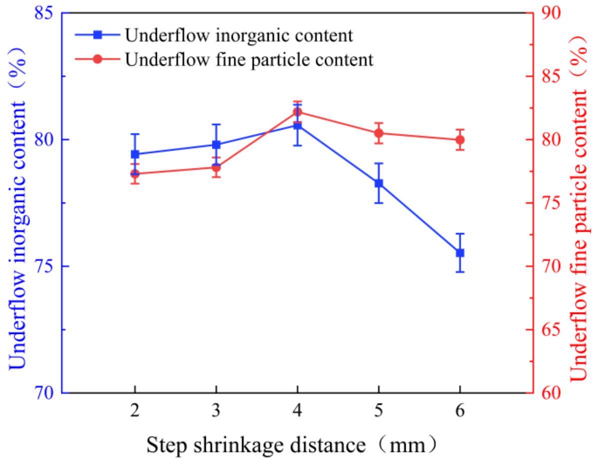


Fig. 11. The influence of step shrinkage distance on inorganic content in underflow

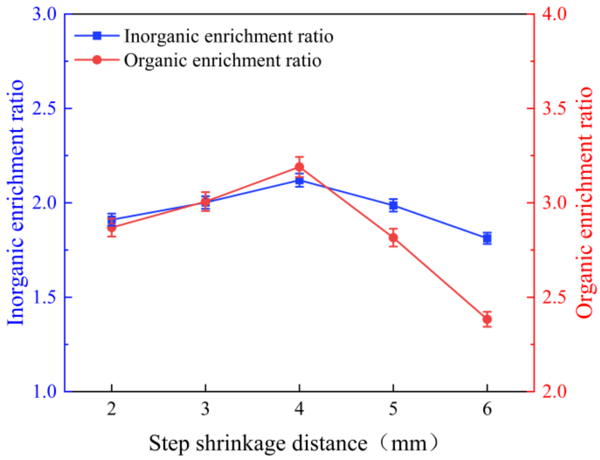


Fig. 12. The influence of step shrinkage distance on product enrichment ratio

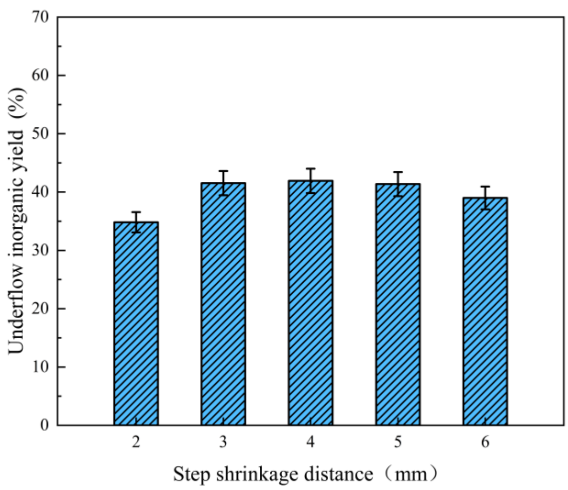


Fig. 13. Influence of step shrinkage distance on yield of inorganic matter in underflow

organic sludge particles are not sufficiently blocked; conversely, when the step shrinkage distance is too large, the separation space within the hydrocyclone becomes too small, preventing adequate separation of organic sludge from inorganic sand and gravel and thereby reducing the separation efficiency. The variation in the inorganic sand yield also demonstrates that the combined cone hydrocyclone achieves the best separation and de-sanding performance at a step shrinkage distance of 4 mm.

3.5. Mathematical modelling

Based on the Box–Behnken response surface method and using the inorganic matter yield in underflow (η) as the evaluation criterion, this study further investigated the interactive effects of the apex diameter, sphere-cone radius, and step shrinkage distance on the separation performance of the hydrocyclone. Table 5 presents the results of the multifactorial response surface experiments. As the parameters varied, the underflow inorganic yield ranged from a maximum of 68% to a minimum of 18.36%. Table 6 shows the analysis of variance results for the model. A model P-value less than 0.05 indicates a significant effect, whereas a value greater than 0.05 indicates a nonsignificant effect. As shown in Table 6, all response model indicators were less than 0.05, suggesting that multiple linear regression analysis based on the experimental data could be accepted as the results of response surface analysis.

Parameter A represents the apex diameter, parameter B represents the sphere-cone radius, and parameter C represents the step shrinkage distance. Box-Behnken data reveal patterns of interactive effects among the three core structural parameters of the combined cone hydrocyclone on inorganic underflow yield. Among these, the sphere-cone radius is the most critical factor; increasing this radius significantly improves separation efficiency, with the compensatory effect being most pronounced under conditions of a large underflow outlet, as its gently sloping conical structure may provide a more thorough separation path. The effect of the step shrinkage distance exhibits a threshold effect: a moderate increase may enhance internal shear forces and promote organic matter detachment, but the marginal benefit diminishes when the underflow outlet is too large, or the sphere-cone radius has already reached its optimal value.

A multiple linear regression analysis of the experimental data using the underflow inorganic yield (η) as the response variable yielded the specific prediction equation shown in Eq. (1). By comparing the coefficients preceding each influencing factor in the prediction equation, the degree of influence of each factor can be determined. For underflow inorganic yield, the order of influence was: sphere-cone radius (B) > apex diameter (A) > step shrinkage distance (C). Underflow inorganic yield:

$$\eta = 44.59 - 9.9\mathrm{A} + 15.75\mathrm{B} + 7.04\mathrm{C} + 5.28\mathrm{AB} - 0.51\mathrm{AC} + 3.86\mathrm{BC} + 4.11\mathrm{A}^2 - 0.2392\mathrm{B}^2 - 8.71\mathrm{C}^2 \quad (1)$$

Table 5. Test plan and response values of Box-Behnken Design

Level	Apex Diameter (mm)	Sphere-Cone Radius (mm)	Step Shrinkage Distance (mm)	Underflow Inorganic Yield (%)
1	6	26	4	44.96
2	10	26	4	18.36
3	6	34	4	68
4	10	34	4	62.52
5	6	30	2	41.04
6	10	30	2	18.48
7	6	30	6	62.52
8	10	30	6	37.92
9	8	26	2	20.94
10	8	34	2	42.62
11	8	26	6	20.94
12	8	34	6	58.05
13	8	30	4	45.63
14	8	30	4	41.57
15	8	30	4	43.76
16	8	30	4	42.32
17	8	30	4	49.65

Table 6. Analysis of variance table of quadratic model (underflow inorganic yield)

Source	Sum of Squares	Df	Mean Square	F-Value	p-Value	Significance
Model	3714.86	9	412.76	18.04	0.0005	significant
A	784.87	1	784.87	34.31	0.0006	
B	1984.19	1	1984.19	86.74	< 0.0001	
C	396.92	1	396.92	17.35	0.0042	
AB	111.51	1	111.51	4.87	0.063	
AC	1.04	1	1.04	0.0455	0.8372	
BC	59.52	1	59.52	2.6	0.1508	
A^2	71.24	1	71.24	3.11	0.121	
B^2	0.241	1	0.241	0.0105	0.9211	
C^2	319.37	1	319.37	13.96	0.0073	
Residual	160.13	7	22.88			
Lack of Fit	118.48	3	39.49	3.79	0.1152	not significant
Pure Error	41.65	4	10.41			
Cor Total	3874.99	16				

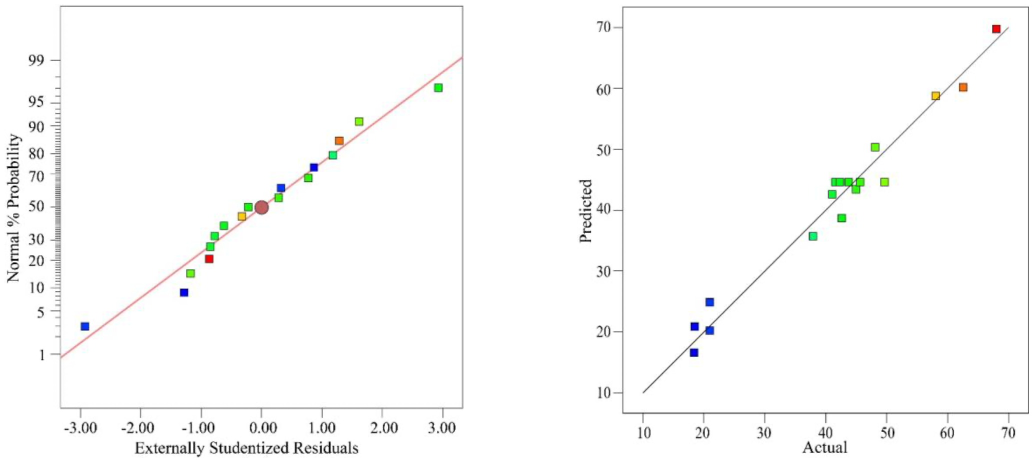
Fig. 14 shows the residual distribution plot for the inorganic matter yield in the leachate. The data points were concentrated along a single straight line, indicating good test results. Furthermore, a two-dimensional contour plot was used to analyse the influences of the experimental factors on the response surface for the inorganic matter yield; the results are shown in Fig. 15.

Fig. 15(a) shows the response surfaces of the apex diameter and step shrinkage distance for the underflow inorganic yield. As shown in Eq. (1), the apex diameter was negatively correlated with the underflow inorganic matter yield, whereas the step shrinkage distance was positively correlated. However, the effect of the step shrinkage distance on the underflow inorganic matter yield was much smaller than that of the apex diameter.

Fig. 15(b) shows the response surfaces of the apex diameter and sphere-cone radius for the underflow inorganic yield. As shown in Eq. (1), the sphere-cone radius was positively correlated with the underflow inorganic yield, whereas the apex diameter was negatively correlated. Furthermore, the sphere-cone radius influenced the underflow inorganic yield more than the apex diameter. As shown in Fig. 15(b), reducing the apex diameter and increasing the sphere-cone radius can effectively improve the underflow inorganic matter yield. However, one must consider the impact of changes in the apex diameter on the fine sand content in the underflow. When the apex diameter is too small, materials accumulate in the conical section, and fine inorganic sand particles are prone to mix with activated sludge and enter the inner vortex. Therefore, when the sphere-cone radius is set to its maximum value, appropriately reducing the apex diameter affords the highest underflow inorganic matter yield.

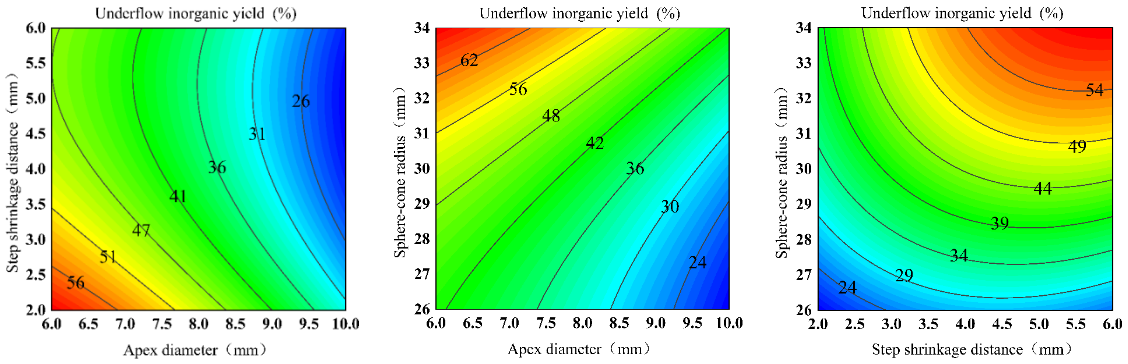
Fig. 15(c) shows the response surfaces for sphere-cone radius and step shrinkage distance for underflow inorganic yield. As shown in Eq. (1), the sphere-cone radius, step shrinkage distance, and bottom-stream inorganic matter yield were all positively correlated. As shown in Fig. 15(c), when the step shrinkage distance was small, the bottom-stream inorganic matter yield of the hydrocyclone increased with an increase in the step shrinkage distance. As the step shrinkage distance increased, both organic sludge particles and inorganic fine sand particles were simultaneously retained, allowing more fine-grained sand particles to enter the inner vortex. By contrast, an excessively large step shrinkage distance reduced the separation space within the hydrocyclone, preventing adequate separation of materials. This decreased the amount of inorganic fine sand entering the outer vortex, thereby lowering the inorganic solid yield in the underflow. Increasing the sphere-cone radius effectively increased the separation space in the conical section. Therefore, at maximum sphere-cone radius, a larger step shrinkage distance improves bottom-stream inorganic matter yield.

After comprehensively considering the effects of various factors on the performance of the hydrocyclone and performing computational optimisation, the optimal combination was determined to be an apex diameter of 6 mm, a step shrinkage distance of 5.7 mm, and a sphere-cone radius of 33 mm. Under these operating conditions, the underflow inorganic yield was 68.97%.



(a) normal probability plot (b) predicted vs actual plot

Fig. 14. Residual distribution of underflow inorganic matter yield



(a) apex diameter-step shrinkage distance (b) apex diameter-sphere-cone radius (c) step shrinkage distance-sphere-cone radius

Fig. 15. Response surface of pairwise test factors to underflow inorganic matter yield

4. Conclusions

This study employed experimental research methods to investigate the ability of combined cone hydrocyclones to reduce organic matter in underflow during sludge-sand separation. The effects of the apex diameter, sphere-cone radius, and step shrinkage distance on the inorganic content, enrichment ratio, and inorganic yield in underflow during the sludge-sand separation process were examined. Furthermore, a response surface method was used to develop a prediction model for separation performance and identify the optimal parameter combination. This study draws the following conclusions:

(1) Compared with conventional hydrocyclones, the combined cone hydrocyclone demonstrates superior separation performance for inorganic sand and gravel in feed sludge, particularly for fine inorganic sand. It effectively reduces the organic content in the underflow and increases the underflow inorganic yield.

(2) As the sphere-cone radius increases, the underflow inorganic yield increases, and the fine particle content and enrichment ratio in underflow also show an upward trend. Although increasing the apex diameter increases the fine particle content and inorganic enrichment ratio in underflow, the inorganic yield in underflow actually decreases; simultaneously, an excessively large apex diameter leads to a lower inorganic enrichment ratio. As the step shrinkage distance increases, the inorganic yield in underflow first increases and then decreases.

(3) The order of influence of these factors on the inorganic matter yield in underflow is as follows: sphere-cone radius > apex diameter > step shrinkage distance. Considering the combined effect of these factors on hydrocyclone performance, the optimal combination obtained through

computational optimization is an apex diameter of 6 mm, a step shrinkage distance of 5.7 mm, and a sphere-cone radius of 33 mm. Under these operating conditions, the inorganic matter yield in the underflow reaches 68.97%.

The findings of this study aim to provide a scientific basis for the widespread application of combined cone hydrocyclones in sludge dewatering and to offer theoretical guidance for the optimized design and engineering practice of hydrocyclone separation equipment.

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