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Introducing a novel, and fast matrix-independent method to determine least sampling points for mass balancing of flotation circuits

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Abstract: Mass balancing of mineral processing plants refers to material distributions to different streams of a circuit. Raw data required to produce mass balancing equations mainly includes elemental assays, moistures, mass rates, water, and material flowrates. Since running a complete sampling program to obtain all required information from all streams of a circuit is practically and economically impossible, a minimum number of streams should be determined to conduct a comprehensive mass balancing. In a conventional approach, this value is calculated using the number of feed streams and separating nodes obtained from a matrix presenting the relationship between these streams and nodes. Arrangement of such a connection matrix for complicated circuits is difficult; therefore, the present study aimed to introduce simple equations to calculate the minimum number of sampling points using the knowledge of the total number of streams and nodes included in the circuit. The suggested method eliminated the need for generating a connection matrix and relevant tables. Comparing the results of utilizing the conventional method and the presented new equation for different flotation circuits confirmed the superiority of the proposed method from both simplicity and rapidness points of view.

Keywords: mass balancing, mineral processing plant, minimum sampling points, connectivity matrix

1. Introduction

Mass balancing in mineral processing is essential for systematically tracking material flow through each stage (crushing, separation, etc.), ensuring input equals output plus losses. It optimizes efficiency by identifying inefficiencies, minimizing waste, and improving recovery of valuable minerals. Accurate mass balances enable data-driven decisions for process adjustments, troubleshooting underperforming units, and enhancing profitability. They also ensure compliance with environmental regulations by monitoring waste streams and emissions. Economically, mass balancing supports precise resource accounting, financial reporting, and cost management. By providing a clear picture of material distribution, it underpins operational transparency, sustainability, and informed strategic planning in mineral extraction and processing (Sadeghi et al., 2018; Ling et al., 2023; Gama et al., 2025). It involves calculating all measures that can be used for metallurgical assessment of different streams within the plant. Accurate mass balancing is essential for optimizing plant performance, ensuring efficient resource utilization, and enhancing overall operational efficiency (Jarkani et al., 2014; Wills and Finch, 2015). Mass balancing process relies on raw data obtained from a comprehensive sampling program. Generally speaking, conducting a complete sampling program to gather all necessary information from every stream within the circuit is practically challenging and economically unfeasible (Hodouin, 2010; Gholami et al., 2022). Therefore, it is imperative to determine the minimum number of streams that need to be sampled to achieve a comprehensive and accurate mass balance. Determining the minimum number of streams to sample in mineral processing is critical to balance cost, efficiency, and accuracy.

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Sampling too many streams wastes resources and time, while insufficient sampling risks missing key data, leading to unreliable mass balance calculations and process insights. By identifying essential streams (e.g., feed, concentrate, tailings), operators ensure representative data for evaluating recovery, grade, and losses without redundancy. This minimizes analytical costs and operational downtime while maintaining precision in troubleshooting, optimizing separation efficiency, and complying with environmental or financial reporting standards. Strategically limiting sampling points also simplifies data management, enabling focused adjustments to enhance profitability and sustainability in mineral operations (Berneder et al., 2017; Bakalarz and Duchnowska, 2024, Gharaei et al., 2024).

Determining the minimum streams to sample in mineral processing involves methods of mass balance equations, which use input-output relationships to identify critical streams (e.g., feed, concentrate, tailings) needed to solve for unknowns, to calculate the number of independent variables, ensuring sampled streams match the equations required for accuracy, and to optimize sample size based on desired precision (Gholami and Khoshdast, 2025). Traditionally, the minimum number of sampling points is calculated using the number of feed streams and simple separating nodes derived from a connection matrix. This matrix represents the relationship between streams and nodes within the circuit (Laplante and Cleary, 1986). While, constructing such a connection matrix for complex circuits can be a daunting and intricate task often requiring significant time and effort. Therefore, in all mineral processing plants, sampling of different streams is routinely carried out as part of the plant or process performance evaluation or audit. Such steady-state sampling-based assessments often result in the identification and subsequent solution of operating problems. Further, steady-state sampling is an essential step in building both steady-state and dynamic models of unit operations in mineral processing (Salama, 2003; Silva et al., 2014). However, due to experimental errors and the over-determined nature of mass balances, where more data is available than the minimum necessary to determine a unique solution, it is essential to adjust the experimentally determined data to satisfy the mass balance constraints (Smith and Frew, 1983). Therefore, to generate data for plant flowsheet mass balancing, various streams should be sampled according to a well-designed sampling scheme that was improved by Salama (1999). In relatively simple flotation circuits, it is not difficult to determine the number of sampling streams to produce data for a unique set of equations for the system. However, to calculate a steady-state mass balance for an entire complicated circuit, a more analytical method of generating n linear equations for n unknowns is required. A plant flowsheet can be reduced to a series of nodes where each simple node is either a separator, S (one input and two outputs), or a junction, J (two inputs and one output) (Wills and Finch, 2015). Smith and Frew (1983) showed that providing the mass flow of a reference stream (usually the feed) is known, the minimum number of streams (N) that must be sampled to ensure the production of a complete circuit mass balance is:

$$N = 2(F + S) - 1 \tag{1}$$

where F and S are the number of feed streams and simple separators, respectively.

For example, the flotation bank shown in Fig. 1 can be reduced to node-stream form and cascaded into simple nodes. The minimum number of streams that must be sampled is thus:

$$N = 2(1 + 3) - 1 = 7$$

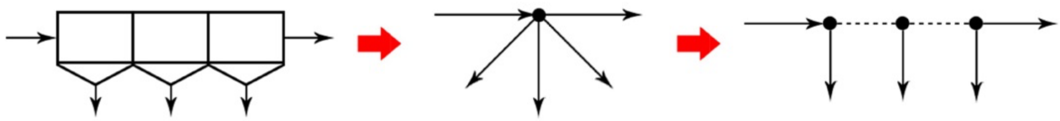

Fig. 1. A flotation bank reduced to node-stream form (Wills and Finch, 2015)

It can be seen from Fig. 1 that a node with two product streams can be divided into three simple separator nodes. In general, if a separator produces n products, then this can be cascaded to n–1 simple nodes. This is useful, as reducing a very complicated plant to simple nodes. A procedure has been developed by Frew (1983) that allows easy automation and provides a check on the count-up of nodes from the flow diagram. The method involves the use of a connection matrix. The number of feed streams and simple separating nodes can be easily determined from the connection matrix; however, constructing the matrix for complicated plants is usually very difficult and needs computer-aided programs.

To address this challenge, the present study introduces a simplified approach to determining the minimum number of sampling points required for mass balancing. The proposed method leverages straightforward equations that only require knowledge of the total number of streams and nodes in the circuit, eliminating the need to generate a connection matrix and associated tables. This novel approach aims to simplify the mass balancing process while maintaining accuracy and reliability. The suggested method also eliminates the need for generating a connection matrix and relevant tables. By focusing on the total number of streams and nodes, the new approach streamlines the process, making it more accessible and less time-consuming for practitioners in the field. This innovation is particularly valuable for complex circuits where traditional methods prove cumbersome and inefficient. Finally, to validate the efficacy of the proposed method, the study compares the results of utilizing the conventional method and the new equation across different flotation circuits. The comparison confirms the superiority of the new method in terms of simplicity and rapidness, offering a more efficient and practical solution for mass balancing in mineral processing plants.

2. A review of the matrix method

The matrix method in mineral processing is a crucial analytical tool used to model and solve complex systems within mineral processing circuits. By representing the relationships between streams and nodes using a connection matrix, it helps organize and solve mass balance equations, ensuring accurate material accounting and process control. This method is particularly beneficial for managing large-scale operations with numerous streams and nodes. Literature highlights its efficiency in automating sampling and mass balancing processes, reducing manual errors, and integrating with real-time monitoring systems (Khoshdast et al., 2017; Smith et al., 2020). Despite its advantages, constructing the matrix for complex circuits can be challenging, prompting the development of automated procedures (Frew, 1986). Overall, the matrix method enhances accuracy and efficiency in mass balance calculations, supporting the optimization of mineral processing operations (Wills and Finch, 2015). The connection matrix, C is a zero-one structure where each element in the matrix is,

$$C_{ij} = \begin{cases} +1 \text{ for stream } j \text{ flowing into the } i^{\text{th}} \text{ node} \\ -1 \text{ for stream } j \text{ flowing out the } i^{\text{th}} \text{ node} \\ \ \ 0 \text{ for stream } j \text{ not appearing at } i^{\text{th}} \text{ node} \end{cases}$$

Consider the flowsheet shown in Fig. 2,

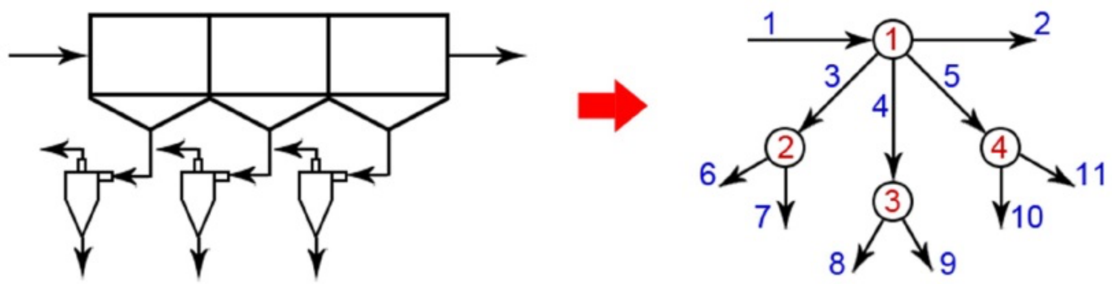

Fig. 2. A simple flotation circuit reduced to node-stream form (Wills and Finch, 2015)

From the node-stream flowsheet (Fig. 2), there are 11 flow streams and four nodes. The connection matrix thus has 11 columns and 4 rows as shown below,

$$C_{ij} = \begin{bmatrix} +1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

The contents of each column represent the individual streams, and when summed must equal +1, –1, or 0, any other result indicating an error in the input of data, i.e.,

$$\text{Column sum} = \begin{cases} +1 \text{ stream is a feed (F)} \\ -1 \text{ stream is a product (P)} \\ \ \ 0 \text{ stream is internal stream (I)} \end{cases}$$

The elements of each row represent the individual nodes, and if the number of "+1" entries (n_p) and the number of "–1" entries (n_n) are counted, then n_p and n_n can be used to assess the number of simple nodes,

- Number of simple junctions (J) = n_p–1
- Number of simple separators (S) = n_n–1

Now, the connection tables can be drawn as given in Tables 1 and 2.

Table 1. Type of streams in the circuit shown in Fig. 2

Column (stream)	1	2	3	4	5	6	7	8	9	10	11
Sum	+1	–1	0	0	0	–1	–1	–1	–1	–1	–1
Type	F	P	I	I	I	P	P	P	P	P	P

Table 2. The connection table indicates the type of nodes in the circuit shown in Fig. 2

Row (node)	n_p	n_n	J	S
1	1	4	0	3
2	1	2	0	1
3	1	2	0	1
4	1	2	0	1
Sum			0	6

There are thus 6 simple separators, no junctions, and one feed stream; the minimum number of streams that must be sampled is,

$$N = 2(1+6)-1 = 13$$

3. Development of a new method

Considering a circuit with n nodes and m streams. The number of streams, either flowing into or out the node i is m_i including F_i feed streams and $n_{n,i}$ product streams. Thus, the circuit includes F feed streams and n product streams,

$$m = F + n_n \tag{2}$$

and,

$$F = \sum_1^n F_i \tag{3}$$

$$n_n = \sum_1^n n_{n,i} \tag{4}$$

Each node with $n_{n,i}$ product streams can be divided into $n_{n,i}$–1 individual simple separating node. Therefore, the total number (S) of simple separators is,

$$S = \sum_1^n \left(n_{n,i} - 1\right) = \sum_1^n n_{n,i} - n = n_n - n \tag{5}$$

Substituting Equation 2 into Equation 5 gives,

$$S = m - F - n \tag{6}$$

and substituting Equation 6 into Frew and Smith's Equation (Equation 1) results in,

$$N = 2(F+S)-1 = 2(F+m-F-n)-1$$

Therefore,

$$N = 2(m - n) - 1 \tag{7}$$

where N is the minimum number of streams required to be sampled, m is the total number of streams and n is the total number of nodes.

To evaluate the accuracy of Equation 7, two simple and complicated circuits were considered and the minimum number of sampling points was calculated using both Frew and Smith`s equation (Equation 1) and the proposed one (Equation 7). To illustrate the applicability of the presented method, two case studies were examined first for a simple flotation circuit and then for a more complicated circuit. The minimum number of sampling points for the flotation circuit shown in Fig. 2 was calculated using the matrix method and found to be 13. Referring to the node-stream form of the circuit, there are 11 streams (m) and 4 nodes (n); therefore, using the new equation,

$$N = 2(m - n) - 1 = 2(11 - 4) - 1 = 13$$

Consider the flotation circuit shown in Fig. 3 as a relatively complicated circuit.

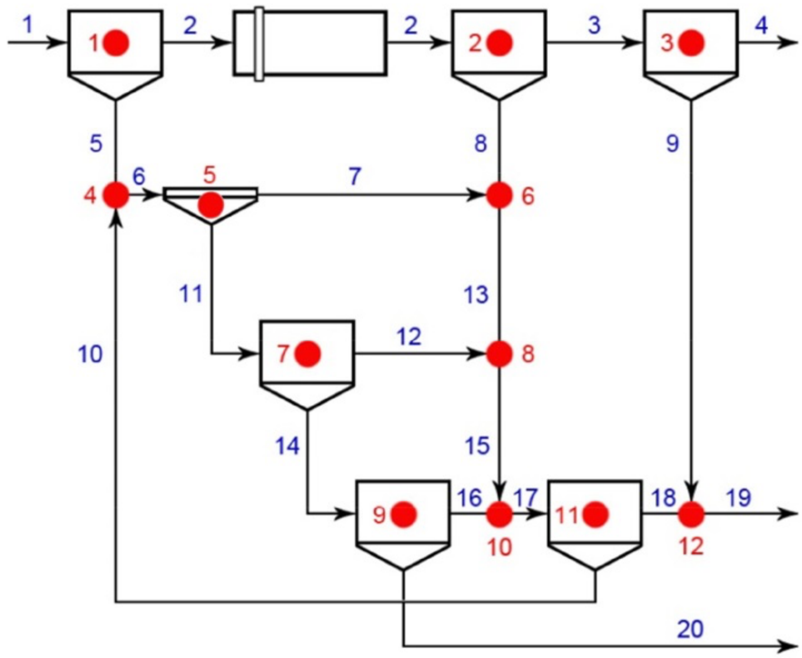

Fig. 3. A complicated flotation circuit matched to node-stream form (Khoshdast, 2019)

From the node-stream flowsheet of the flotation circuit shown in Fig. 3, there are 11 nodes (n) and 20 streams (m). The connection matrix has a 12×20 form as shown below,

$$C_{ij} = \begin{bmatrix}
+1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & -1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0
\end{bmatrix}$$

The connection tables are shown in Tables 3 and 4.

Table 3. Type of streams in the circuit demonstrated in Fig. 3

Column (stream)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sum	+1	0	0	–1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	–1
Type	F	I	I	P	I	I	I	I	I	I	I	I	I	I	I	I	I	I	P	P

Table 4. Connection table indicating the type of nodes in the circuit presented in Fig. 3

Row (node)	n_p	n_n	J	S
1	1	2	0	1
2	1	2	0	1
3	1	2	0	1
4	2	1	1	0
5	1	2	0	1
6	2	1	1	0
7	1	2	0	1
8	2	1	1	0
9	1	2	0	1
10	2	1	1	0
11	1	2	0	1
12	2	1	1	0
Sum			5	7

There are thus 7 simple separators, 5 simple junctions, and one feed stream; the minimum number of streams that must be sampled is,

$$N = 2(1+7)-1 = 15$$

Referring to the node-stream form of the circuit (Fig. 3), for 20 streams and 12 nodes the minimum number of sampling points is,

$$N = 2(m-n)-1 = 2(20-12)-1 = 15$$

As seen, by the new equation the need for developing the connection matrix is eliminated; therefore, it is easier and faster than the Smith and Frew equation.

4. Further simplification of the new equation

Current challenges in conventional methods include complexity of calculation process that obscures critical streams and dynamic variability in feed/material properties. Moreover, complex calculations may lead to data inaccuracies from sampling/measurement errors, and resource constraints balancing cost vs. precision. Generally, mathematically simplified models relied on real-world interactions can match actual fluctuating operations, risking incomplete or biased insights, hence, yield reliable mass balance outputs. To this purpose, Equation 7 was further simplified after some further ''mathematical'' substitutions. As mentioned earlier, each simple node (n_i) includes three streams ($m_{n,i}$), that is,

$$m_{n,i} = 2n_i + 1 \tag{8}$$

Thus, the total number (m) of streams constructing a circuit with n nodes is,

$$m = \sum_1^n m_{n,i}$$

$$m = \sum_1^n (2n_i + 1)$$

$$m = 2n + 1 \tag{9}$$

A combination of Equations 9 and 7 leads to,

$$N = 2\left[(2n+1)-n\right]-1$$
$$N = 2(n+1)-1$$

and finally,

$$N = 2n+1 \tag{10}$$

where n is the total number of nodes included in the circuit. However, one should consider that n in Equation 10 is the node number in the original form of the circuit not in its node-stream form. That is, n is the total number of operating equipment employed in the circuit which can be considered as real separating nodes such as hydrocyclones, single concentrate flotation banks, etc. Therefore, single input-single output units such as crushers, storage bins, etc. cannot be considered as operating equipment. Considering the multi-input or multi-output units like conditioning tanks in calculations depends on the mass balance target; for example, if mass balance is done for water the tank would be an operating equipment, otherwise if solid mass balancing is run the tank should be eliminated from calculations.

5. Validation and discussion of simplified equation

Considering circuits shown in Figs. 2 and 3 to evaluate the validation of Equation 10, the minimum number of streams being sampled is determined as follows,

- Circuit 1 in Fig. 2 includes three flotation units with independent product streams and three classification units; therefore,
$$N = 2n+1 = 2(3+3)+1 = 13$$
- Circuit 2 in Fig. 3 has 6 independent flotation banks and a single deck fine screen, thus
$$N = 2n+1 = 2\times 7+1 = 15$$

As seen, Equation 10 can exactly determine the minimum number of sampling points as Equation 7 and the matrix base method. The state “independent unit” refers to the processing devices that act as a separating node in which the number of output streams is larger than the number of input streams.

In mineral processing, it is common to use a closed circuit to return below-criteria material as a circulating load to the head of the treatment loop for further processing prior to release. In addition, new feed streams are usually added to different stages of a processing circuit which charge solid and/or water to primary feed lines. When using Equation 10, one needs to consider the type of such non-operational nodes in the circuit. If the node is created in conjunction with a circulating load stream, the node should not be included in the calculations but if the node is created by a new feed stream, the node would be considered as an operational node and used in calculations. As an example, assuming a relatively complicated flotation circuit shown in Fig. 4.

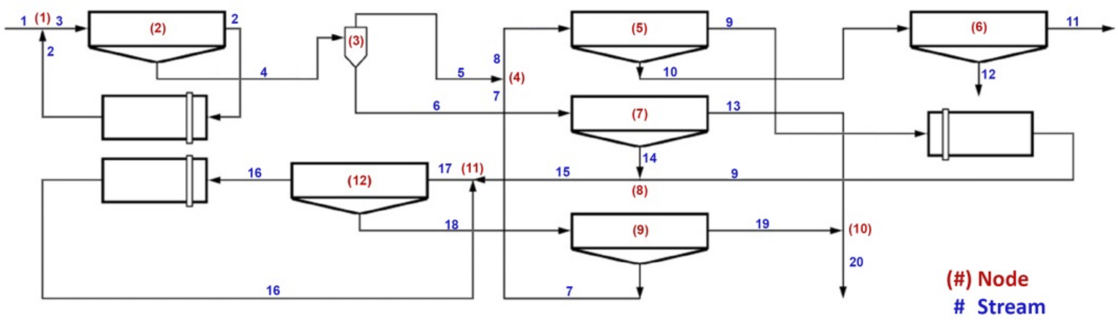

Fig. 4. A sample flotation circuit with a multiplicity of inter-streams (Khoshdast, 2019)

The circuit has 7 operational units including 6 independent flotation banks and an independent hydrocyclone unit. Therefore, the minimum number of sampling points can be determined by Equation 10,

$$N = 2n+1 = 2\times 7+1 = 15$$

The connection matrix of the circuit takes a 12×20 form (Appendix A). Referring to the connection table, the circuit has 7 simple separators (S), and 6 simple junctions, and one feed stream (F); the minimum number of streams that must be sampled is 15 as determined by Equation 10,

$$N = 2(1+7)-1 = 15$$

Assume that stream 11 is returned to flotation bank (6) as a circulating load stream. Then, the original flotation circuit in Fig. 4 gets the form shown in Fig. 5.

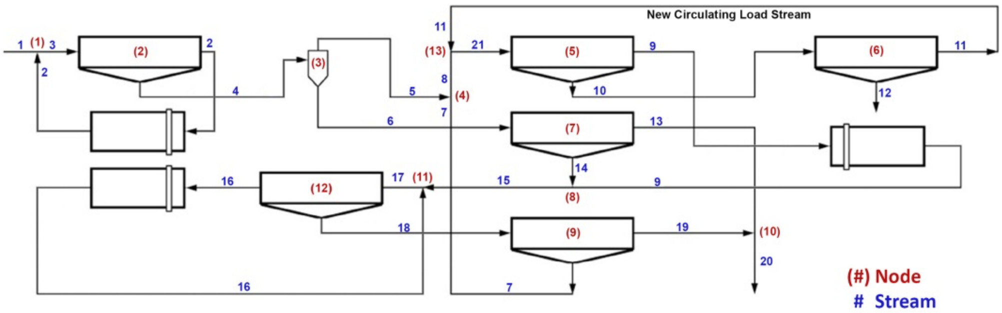

Fig. 5. Flotation circuit of Fig. 4 with a new circulating load stream

As seen in Fig. 5, the number of nodes and streams increases to 13 and 21, respectively. Using connection data (Appendix B), the circuit is constructed by 7 simple separators (*S*), 6 simple junctions, and one feed stream (*F*); the minimum number of streams that must be sampled is 15 as determined by equation 10,

$$N = 2(1+7)-1 = 15$$

Similar to the original circuit, the new circuit has 7 operational units including 6 independent flotation banks and one independent hydrocyclone. Therefore, the minimum number of sampling points can be determined using Equation 10 as follows,

$$N = 2n+1 = 2\times 7+1 = 15$$

Now, presuming the circuit in Fig. 6 showing the original circuit (Fig. 4) with a new feed stream added to the flotation bank (5). The circuit now includes 13 nodes and 22 streams. The minimum number of sampling points can be calculated using Equation 10 considering the circuit has one new feed stream in addition to 6 independent flotation banks and one independent hydrocyclone. Therefore,

$$N = 2n+1 = 2\times 8+1 = 17$$

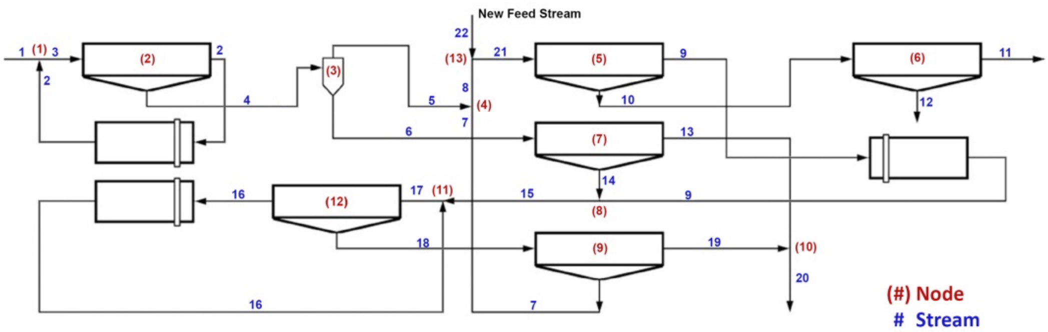

Fig. 6. Flotation circuit of Fig. 4 with a new feed stream

The circuit has 7 simple separators, 6 simple junctions, and two feed streams (Appendix C); the minimum number of streams that must be sampled is 17 which is equal to the value determined by Equation 10,

$$N = 2(2+7)-1 = 17$$

The developed model serves as the mathematical foundation for the mass balance program, making it inherently scalable for application across various operational sizes. Industrial examples drawn from real-world case studies in this article further validate the model's effectiveness at an industrial scale, bridging theory and practice. By combining theoretical rigor with empirical validation, the approach ensures reliability and adaptability, confirming its utility in both conceptual frameworks and large-scale mineral processing systems. Generally speaking, the proposed method prioritizes usability, computational efficiency, and broad applicability, making it more practical for engineers in diverse mineral processing scenarios. Table 5 lists the advantages of new method over conventional ones.

Table 5. Comparative analysis of the proposed method against conventional ones

Criteria	New method	Conventional methods
Understandability	Intuitive for engineers; clear logic flow	Steeper learning curve; complex terminology
Calculation Simplicity	Simplified equations; fewer variables	Requires iterative/complex computations
Coding Effort	Minimal code for soft-calculation	Lengthy, nested code structures
Scalability	Seamless compatibility across all scales	Limited to specific scales or configurations
Circuit Versatility	Adaptable to all circuit types (e.g., open/closed)	Restricted to specialized circuits

6. Conclusions

Mass balancing is a vital procedure in mineral processing plants, essential for optimizing plant performance and ensuring efficient resource utilization. Traditional methods for mass balancing require extensive data from a complete sampling program, which is often impractical and economically burdensome. The complexity of mineral processing circuits further complicates the task, as conventional approaches rely on constructing connection matrices to determine the necessary number of sampling points. These matrices, which represent the relationships between streams and nodes, are challenging to create for intricate circuits, demanding significant effort and computational resources. This study addressed these challenges by introducing a novel method to calculate the minimum number of sampling points required for comprehensive mass balancing. The proposed method simplifies the process by using straightforward equations that leverage the total number of streams and nodes within the circuit. This approach eliminates the need for constructing connection matrices and relevant tables, streamlining the mass balancing process. The effectiveness of the new method was validated through comparisons with the conventional matrix-based approach across various flotation circuits. The results unequivocally demonstrated the superiority of the new method in terms of simplicity and speed. By reducing the complexity and time required for mass balancing, this method provides a practical and efficient solution for mineral processing plants, especially those dealing with complicated circuits. One of the key advantages of the proposed method is its accessibility. Plant operators and engineers can quickly determine the minimum number of sampling points without the need for specialized software or detailed knowledge of matrix construction. This ease of use can significantly enhance the operational efficiency of mineral processing plants, allowing for more frequent and accurate mass balancing assessments. Furthermore, the elimination of the connection matrix requirement reduces potential sources of error and improves the robustness of the mass balancing process. The new method ensures that mass balance constraints are satisfied with fewer assumptions and less data manipulation, leading to more reliable and consistent results. In practical terms, the implementation of this new method can lead to substantial cost savings. By minimizing the number of samples required, plants can reduce labor and analytical expenses while maintaining or even improving the accuracy of their mass balance calculations. This efficiency gain can be particularly beneficial in large-scale operations where extensive sampling programs would otherwise be prohibitive. In conclusion, this study has introduced a significant advancement in the field of mineral processing by providing a simpler, faster, and more reliable method for mass balancing. The proposed equations for determining the minimum number of sampling points represent a practical alternative to the conventional matrix-based approach, offering clear advantages in terms of ease of use, accuracy, and cost-effectiveness. Future research could further refine these equations and explore their applicability to even more complex processing circuits, potentially expanding their utility across a broader range of mineral processing applications. This innovative method stands to greatly enhance the efficiency and effectiveness of mass balancing in mineral processing plants, contributing to more optimized and sustainable operations.

Appendix A

Connection matrix and the connection table for the simple flotation circuit with a multiplicity of inter-streams (Figure 4).

Connection Matrix

$$C_{ij} = \begin{bmatrix}
+1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & {=}1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & +1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1 & -1 & 0 & 0
\end{bmatrix}$$

Connection Table

Row (node)	n_p	n_n	J	S
1	2	1	1	0
2	1	2	0	1
3	1	2	0	1
4	2	1	1	0
5	1	2	0	1
6	1	2	0	1
7	1	2	0	1
8	2	1	1	0
9	1	2	0	1
10	2	1	1	0
11	2	1	1	0
12	1	2	0	1
Sum			5	7

Appendix B

Connection matrix and the connection table for the simple flotation circuit with a with a new circulating load stream (Figure 5).

Connection Matrix

$$C_{ij} = \begin{bmatrix}
+1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}$$

Connection Table

Row (node)	n_p	n_n	J	S
1	2	1	1	0
2	1	2	0	1
3	1	2	0	1
4	2	1	1	0
5	1	2	0	1
6	1	2	0	1
7	1	2	0	1
8	2	1	1	0
9	1	2	0	1
10	2	1	1	0
11	2	1	1	0
12	1	2	0	1
13	2	1	1	0
Sum			6	7

Appendix C

Connection matrix and the connection table for the simple flotation circuit with a new feed stream (Figure 6).

Connection Matrix

$$C_{ij} = \begin{bmatrix}
+1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & +1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & +1
\end{bmatrix}$$

Connection Table

Row (node)	n_p	n_n	J	S
1	2	1	1	0
2	1	2	0	1
3	1	2	0	1
4	2	1	1	0
5	1	2	0	1
6	1	2	0	1
7	1	2	0	1
8	2	1	1	0
9	1	2	0	1
10	1	2	1	0
11	2	1	1	0
12	2	1	0	1
13	2	1	1	0
Sum			6	7

References

BAKALARZ, A., DUCHNOWSKA, M., 2024. *Analysis of the possibility of copper recovery from flotation stratiform copper ore tailings*. Mineral Processing and Extractive Metallurgy Review, 45(8), 943–949.

BERNEDER, I., LEHNER, M., CAPO-TOUS, F., ENENGEL, M., 2017. *Mass balance study of a multistage process for the carbonation of mineral resources*. Energy Procedia, 114, 7162–7170.

FREW, J.A., 1983. *Computer-aided design of sampling schemes*. International Journal of Mineral Processing, 11, 255–265.

FREW, J., 1986. *Procedures for automating the construction of connection matrices in mineral processing*. Minerals Engineering, 3(4), 245–253.

GAMA, T.S.M., DE CARVALHO, R.D., TAVARES, L.M., 2025. *Comparison of approaches to predict grinding in an industrial rod mill operating at low fillings*. Mineral Processing and Extractive Metallurgy Review, 46(2), 240–253.

GHARAEI, R.S., GHOLAMI, A., KHOSHDAST, H., BIGLARI, H., 2024. *A multi-method machine learning framework for simulating power dynamics in an iron ore cone crusher plant*. Mineral Processing and Extractive Metallurgy Review, Doi: 10.1080/08827508.2024.2395828.

GHOLAMI, A., KHOSHDAST, H., 2025. *Using artificial neural networks for the intelligent estimation of selectivity index and metallurgical responses of a sample coal bioflotation by rhamnolipid biosurfactants*. Energy Sources, Part A: Recovery, Utilization, and Environmental Effects, 47(1), 3713-3731.

GHOLAMI, A., ASGARI, K., KHOSHDAST, H., HASSANZADEH, A., 2022. *A hybrid geometallurgical study using coupled Historical Data (HD) and Deep Learning (DL) techniques on a copper ore mine*. Physicochemical Problems of Mineral Processing, 58 (3), 147841.

HODOUIN, D., 2010. *Methods for automatic control, observation, and optimization in mineral processing plants*. Journal of Process Control, 20(7), 741–750.

JARKANI, S.A., KHOSHDAST, H., SHARIAT, E., SAM, A., 2014. *Modeling the effects of mechanical parameters on the hydrodynamic behavior of vertical current classifiers*. International Journal of Mining Science and Technology, 24(1), 123–127.

KHOSHDAST, H., 2019. *Practical problems in froth flotation*. Hormozgan University Press, Tehran, Iran

KHOSHDAST, H., SHOJAEI, V., KHOSHDAST, H., 2017. *Combined application of computational fluid dynamics (CFD) and design of experiments (DOE) to hydrodynamic simulation of a coal classifier*. International Journal of Mining and Geo-Engineering, 51(1), 9–24.

LAPLANTE, A.R., CLEARY, J.G., 1986. *Techniques for automated sampling and mass balancing*. Minerals Engineering, 1(2), 163–174.

LING, H., PERIN, M., BLANPAIN, B., GUO, M., MALFLIET, A., 2023. *Process flowsheet development for selective arsenic removal, lead and antimony recovery from lead softening slag*. Mineral Processing and Extractive Metallurgy Review, 45(4), 343–355.

SADEGHI, M., HODOUIN, D., BAZIN, C., 2018. *Mineral processing plant data reconciliation including mineral mass balance constraints*. Minerals Engineering, 123, 117–127.

SALAMA, A.I.A., 1999. *Sampling schemes for mass balance of flowsheets with multi-component streams*. International Journal of Mineral Processing, 55, 219–229.

SALAMA, A.I.A., 2003. *A technique for interpolation and smoothing of mass–size–density data*. International Journal of Mineral Processing, 70, 123–146.

SILVA, A.C., SILVA, E.M.S., REZENDE, R.A., 2014. *Circulating load calculation in grinding circuits*. REM: Revista Escola de Minas, 67(1), 101–106.

SMITH, H.W., FREW, J.A., 1983. *Design and analysis of sampling experiments–a sensitivity approach*. International Journal of Mineral Processing, 11, 267–284.

SMITH, L.K., FREW, J., 1984. *Mass balancing in mineral processing*. Minerals Engineering, 2(3), 231–239.

WILLS, B.A., FINCH, J., 2015. *Wills' mineral processing technology: An introduction to the practical aspects of ore treatment and mineral recovery*. 8th ed., Butterworth-Heinemann, UK.