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Prediction and visualization of charge shape and ball trajectory in tumbling mills: a python-based tool for liner design and operational optimizations

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Abstract: Tumbling mills are critical in mineral processing due to their high energy consumption and impact on downstream processes. The mining industry accounts for 1.7% of global energy consumption, with comminution responsible for approximately 25% of this usage. Mill performance is largely governed by charge shape and media trajectory, which are significantly influenced by liner design and wear conditions. However, existing tools provide limited capabilities for combined analysis of these critical parameters. This study introduces a Python-based tool that integrates the Morrell C model for charge shape prediction with Powell's model for media trajectory calculation, offering comprehensive visualization of mill dynamics. The tool's effectiveness was demonstrated through two case studies on an 8-meter SAG mill: first optimizing initial liner design parameters and then adapting operating parameters to compensate for liner wear over a six-month period. Results show how the tool enables proactive operational adjustments based on visualized trajectory changes, helping maintain optimal grinding efficiency throughout the liner lifecycle. This integrated approach to design and operational optimization contributes to improved energy efficiency, extended liner life, and more sustainable mineral processing practices.

Keywords: mineral processing, tumbling mill, charge shape, ball trajectory, mill liner design

1. Introduction

The mining industry is highly energy-intensive, accounting for 1.7% of total global energy consumption, and this is expected to increase significantly in the future (Aramendia et al., 2023). Comminution, which is responsible for approximately 25% of this energy usage (Allen, 2021), is known to be highly inefficient (Fuerstenau and Abouzeid, 2002), with maximum relative efficiency ranging between 3% and 26% (Tromans, 2008). A study conducted on three SAG-ball mill industrial circuits showed that only 9% of the supplied energy was used for breakage, while the rest was lost as heat. The slurry absorbed most of this energy, accounting for 79% of the total (Bouchard et al., 2019). However, there is significant potential to reduce this energy consumption (Napier-Munn, 2015). These efficiency challenges are further compounded by the broader environmental and economic impacts of wear in mining operations. Friction and wear in mineral mining alone contribute to 970 million tonnes of CO_2 emissions annually, accounting for 2.7% of global CO_2 emissions. The economic implications are equally significant, with annual losses reaching 210,000 million Euros, distributed across friction mitigation (40%), replacement parts and equipment (27%), maintenance work (26%), and lost production (7%) (Holmberg et al., 2017). These statistics underscore the critical importance of addressing efficiency and wear-related challenges in comminution processes.

To effectively address these efficiency and wear challenges, a fundamental understanding of grinding mechanisms in tumbling mills is essential, as these mills are predominantly used for fine grinding in mineral processing. The performance and operating cost of these mills is largely dependent on two interconnected factors: the shape of the charge (the mixture of ore and grinding media) and the trajectory of the grinding media. The shape of the charge influences how material is distributed within

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the mill, impacting the grinding mechanism. Meanwhile, the trajectory of the grinding media determines how effectively energy is transferred to the ore particles and influences wear patterns. Optimal grinding conditions occur when the media strike the charge at the toe, facilitating maximum energy transfer and effective particle breakage while minimizing unnecessary wear. However, if the media impacts too high on the charge, energy transfer becomes ineffective, leading to poor grinding performance and wasted energy. Conversely, if the media strike the mill liners directly, it can result in excessive wear and potential damage, further reducing efficiency and increasing maintenance costs. Fig. 1 illustrates these conditions: (a) shows media impacting too high on the charge, (b) depicts direct impact on the liners, and (c) represents the optimal scenario where media strikes the charge at the toe.

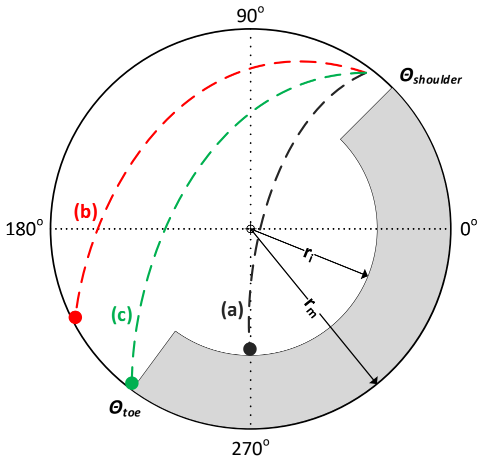

Fig. 1. Charge shape and different media trajectory

To achieve and maintain the optimal grinding conditions described above, proper liner design and ongoing wear management are crucial. Liner design has been extensively studied with numerous investigations demonstrating its significant influence on grinding mechanisms, power draw, and overall mill performance (Bian et al., 2017; Hlungwani et al., 2003; Makokha and Moys, 2006; Maleki Moghaddam et al., 2019; Mishra and Rajamani, 1993; Powell and Vermeulen, 1994; Safa and Aissat, 2023; Yahyaei et al., 2009). Similarly, the impact of liner wear on operational efficiency has been well-documented through various studies, showing its effects on throughput, breakage rates, and economic outcomes (Cleary and Franke, 2012; Powell et al., 2011; Toor et al., 2011; Yahyaei and Banisi, 2010). Within this context, liner geometry, along with operating conditions, plays a crucial role in controlling the trajectory of the grinding media and maintaining the optimal charge shape (Powell, 1991). Well-designed liners guide the media towards effective collision points with the charge, enhancing overall grinding efficiency. However, as liners wear over time, their ability to control media trajectory diminishes. This wear necessitates regular monitoring and timely adjustments to operating conditions such as mill speed and charge volume. Without these adjustments, grinding efficiency declines, leading to increased energy consumption and higher operational costs due to more frequent maintenance and unscheduled downtime (Powell et al., 2006).

Given the significant impact of charge shape and media trajectory on mill performance and wear patterns, accurately predicting and monitoring these factors is of paramount importance. Recent studies have developed control and optimization strategies utilizing these parameters (Chen et al., 2023; Fangrong et al., 2023; Malkhuuz et al., 2023; Medina et al., 2023; Olmedo and Rodriguez, 2023; Pax and Bevan, 2023; Pedersen et al., 2023), as they directly influence grinding efficiency and operational costs. Additionally, several investigations have specifically addressed liner wear prediction (Ahmadzadeh and Lundberg, 2013; Boemer and Ponthot, 2017; Franke et al., 2015; Pural et al., 2024), as understanding wear progression enables better maintenance planning and operational adjustments. Together, these capabilities can lead to enhanced grinding efficiency, extended liner life, and overall improvements in the sustainability and cost-effectiveness of grinding operations.

To address this prediction and monitoring needs, we have developed a visual tool that employs the Morrell C model for charge shape prediction and Powell's model for trajectory calculation. Implemented through Python programming, this tool offers a practical and efficient solution for optimizing mill design and operation. It improves upon the widely-used MillTraj software by additionally computing and visualizing charge shape, offering a more comprehensive analysis ("MillTraj Liner Design Software," n.d.). It enables engineers to simulate and visualize different operating conditions and lifter configurations, providing valuable insights for decision-making. Through two detailed hypothetical case studies - one focusing on liner design optimization and another on operational parameter adjustment throughout liner wear progression - we demonstrate how this tool can be applied to real-world scenarios, showcasing its potential to significantly improve tumbling mill performance across various operational parameters.

2. Models and methodology

2.1. Morrell C model for charge shape prediction

(Morrell, 1993) developed a series of empirical equations to predict the charge shape inside a tumbling mill. The model uses mill speed and filling fraction to determine the toe angle (θ_T), shoulder angle (θ_S), and the inner radius of the charge (r_i). Toe angle is calculated using the Equation 1 in radians:

$$\theta_T = 2.5307 \cdot (1.2796 - J_t) \cdot \left(1 - \exp\left(-19.42 \cdot (\Phi_c - \Phi)\right)\right) + \frac{\pi}{2} \tag{1}$$

where J_t is the filling fraction, Φ denotes the fraction of the critical speed at which the mill operates, Φ_c indicates the experimentally determined fraction of the theoretical critical speed required to achieve complete centrifuging and can be determined based on the following conditions:

$$\Phi_c = \begin{cases} \Phi, & \Phi > 0.35 \cdot (3.364 - J_t) \\ 0.35 \cdot (3.364 - J_t), & \Phi \leq 0.35 \cdot (3.364 - J_t) \end{cases} \tag{2}$$

The shoulder angle (θ_S) in radians, is then calculated by Equation 3:

$$\Phi_c = \begin{cases} \Phi, & \Phi > 0.35 \cdot (3.364 - J_t) \\ 0.35 \cdot (3.364 - J_t), & \Phi \leq 0.35 \cdot (3.364 - J_t) \end{cases} \tag{3}$$

Finally, the inner radius of the charge is calculated using the following Equation 4:

$$r_i = r_m \cdot \left(1 - \frac{2\pi \cdot \beta \cdot J_t}{2\pi \cdot \theta_s - \theta_t}\right)^{0.5} \tag{4}$$

where β represent the active part of the charge expressed as a fraction of the total charge, determined as:

$$\beta = \frac{t_c}{t_c + t_f} \tag{5}$$

where t_c is the mean time spent by the media in contact with the charge, and t_f is the mean time spent in free flight. The parameters t_c and t_f are calculated using the following equations:

$$t_c = \frac{2\pi - \theta_T - \theta_S}{2\pi \bar{N}} \tag{6}$$

where $\bar{N}$ represents the mean rotational rate:

$$\bar{N} = \frac{N_m}{2} \tag{7}$$

with N_m being mill rotational speed.

The free flight time t_f is calculated as:

$$t_f = \left[\frac{2\bar{r}}{g} \cdot (\sin\theta_S - \sin\theta_T)\right]^{0.5} \tag{8}$$

where $\bar{r}$ is the mean radial position of the charge:

$$\bar{r} = \left[1 + \left(1 - \frac{2\pi J_t}{2\pi + \theta_S - \theta_T}\right)^{0.5}\right] \tag{9}$$

these equations complete the set of parameters needed to calculate β in Equation 5, which in turn allows for the determination of the inner radius of the charge through Equation 4.

2.2. Powell's model for trajectory calculation

Powell's first principal model (Powell, 1991) provides a detailed approach to understand the trajectory of grinding media within a mill. The model uses two key parameters to represent the liner geometry:

lifter height and lifter face angle. Additionally, the lifter base width has been incorporated into the software, enabling further calculations and visualizations. These aspects are illustrated in Fig. 2.

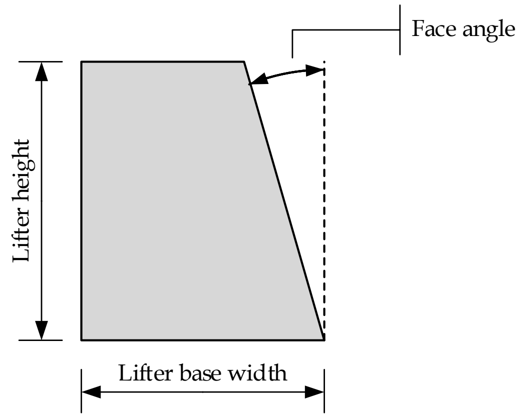

Fig. 2. Visualization of lifter height, lifter face angle, and lifter base width parameters.

The model describes the ball motion in three distinct phases: rolling, sliding, and free flight. Calculations begin by determining the point of equilibrium on the lifter bar, where the forces acting on the grinding media balance each other. After the ball passes this equilibrium point, it starts to roll under the influence of static friction between the ball and lifter bar. The model calculates the ball's displacement and velocity over time, considering initial conditions and angular position. When the static friction can no longer sustain the motion, the ball transitions to sliding motion under kinetic friction.

Once sliding begins, the model continues to track the ball's motion until it reaches the tip of the lifter bar. At this stage, the ball starts a free flight phase. The model provides equations to describe the ball's position and velocity during free flight until it impacts the mill shell. The impact velocity and angle are then calculated based on the final position.

2.3. Python implementation

Python was chosen for its flexibility, and extensive ecosystem of scientific computing libraries, making it ideal for developing interactive engineering tools. The implementation architecture consists of three main components: user interface, computation engine, and visualization module.

The user interface is built using the *IPyWidgets* library, which enables interactive parameter adjustment through sliders, input fields, and dropdown menus. This allows engineers to modify mill parameters such as speed, filling degree, and liner dimensions in real-time. An example of the data input interface is shown in Fig. 3.

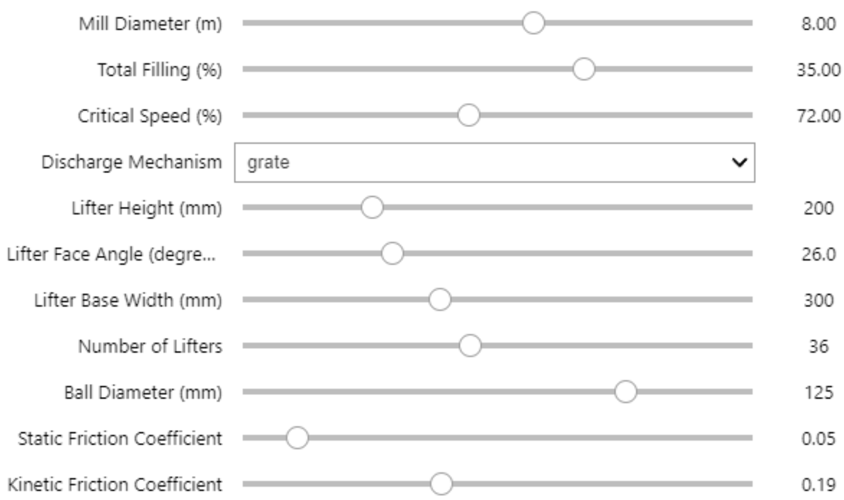

Fig. 3. Software interface for model parameters input

The computation engine utilizes *NumPy* for efficient array operations and matrix manipulations, essential for trajectory calculations and charge shape predictions. The *SciPy* library's *optimize* module solves the complex equations of motion in Powell's model, particularly during the transition between rolling and sliding phases. This combination ensures both accuracy and computational efficiency.

Results visualization is handled through the *Matplotlib* library, providing dynamic updates of charge shape and ball trajectory as parameters change. The visualization includes color-coded trajectory paths, charge boundary representation, and impact point indicators, enabling clear interpretation of simulation results.

The accuracy of our implementation has been verified through validation against published data. When run under conditions specified by Yahyaei and Banisi, (2010) the program produced an impact point of 217.2° compared to the reported 215.7°. Similarly, for the benchmark case in the MetDynamic documentation ("Tumbling Mill (Media Trajectory)," n.d.), our model yielded 232.2° versus the reference 231.2°. These minimal differences (less than 1%) demonstrate the reliability of our computational approach, with slight variations likely attributable to numerical solution methods and rounding procedures.

3. Results and discussion: hypothetical case studies

3.1. Case study 1: liner design optimization

To evaluate the effectiveness of the developed Python-based tool, a hypothetical case study was designed involving a SAG mill equipped with a variable speed drive operating under typical conditions. The mill has an internal diameter of 8 meters and is set to operate at 72% of its critical speed, with a charge filling fraction of 27%. Additionally, there are 35 rows of bolting on the mill shell, based on the formula (4/3) * Diameter (ft). The grinding media consists of steel balls with a maximum size of 120 mm.

For simulation purposes, reasonable estimates for the coefficients of friction are 0 or 0.05 for static friction and 0.19 for kinetic friction. as recommended in the MetDynamic case documentation ("Tumbling Mill (Media Trajectory)," n.d.). Powell, (1991) discusses that while these coefficients can be determined experimentally, they inherently contain some uncertainty and, importantly, notes that under typical operating conditions, small variations in friction coefficients do not significantly alter the overall trajectory predictions or charge behavior patterns. This observation supports the use of these standard values for most practical applications.

One of the most important factors is the spacing between the lifter bars and the spacing-to-height (S/H) ratio. If the spacing between the lifter bars is too wide, slippage against the shell plate may occur, leading to excessive wear. Conversely, if the spacing is too short relative to the height of the lifter bar, pockets of material may form between the bars. This can create a relatively smooth lining, resulting in accelerated wear on the tops of the lifter bars (Moller and Brough, 1989). Additionally, the S/H ratio has a significant impact on mill throughput (Yahyaei and Banisi, 2010). The S/H ratio is not fixed but varies depending on the mill's critical speed.

The liner design procedure employed in this study follows the methodology established by (Powell et al., 2006), which provides a systematic approach to optimizing liner configurations for grinding efficiency and wear resistance.

Based on the given mill geometry, an appropriate lifter bar height will first be selected to match the ideal S/H ratio. For the specified critical speed, the ideal S/H ratio is approximately 3.6. Considering that the lifter bar will wear down over time, it is advisable to achieve the ideal S/H ratio when the lifter bar is at approximately half of its lifespan. Using these parameters in the tool the lifter height is determined to be 135 mm. The lifter height should be allowed to decrease to a minimum of 60 mm, based on the 120 mm grinding media size. If the height decreases further, the lifters will no longer be able to effectively lift the balls, resulting in excessive wear due to increased slippage. Given this minimum height and the accelerated wear rate as the lifter height decreases, it would be appropriate to design the new lifters with an initial height of 200 mm. When the lifter base width is set to 200 mm, the resulting charge shape and media trajectory under these conditions are shown in Fig. 4. The default value for the lifter face angle in the software is 15 degrees. As can be seen, at this angle, the media directly impacts the liner, which is an undesirable outcome.

To ensure that the grinding media lands at the charge toe, the lifter face angle was increased to 24 degrees. The trajectory in this scenario is illustrated in Fig. 5, where the impact point and the toe point are approximately the same, indicating optimal media behavior.

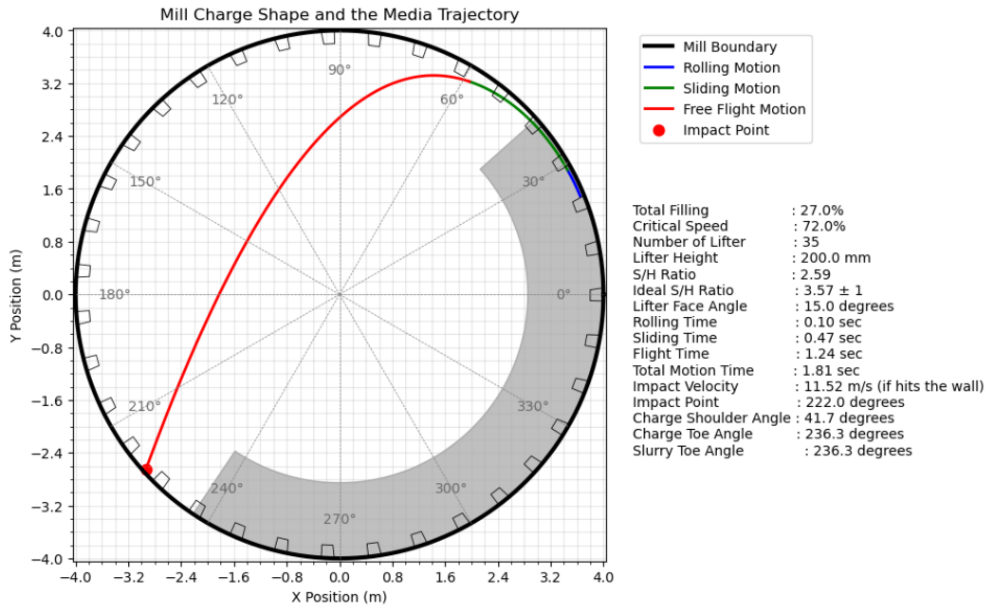

Fig. 4. Charge shape and trajectory with 200 mm lifter base width and 15° lifter face angle.

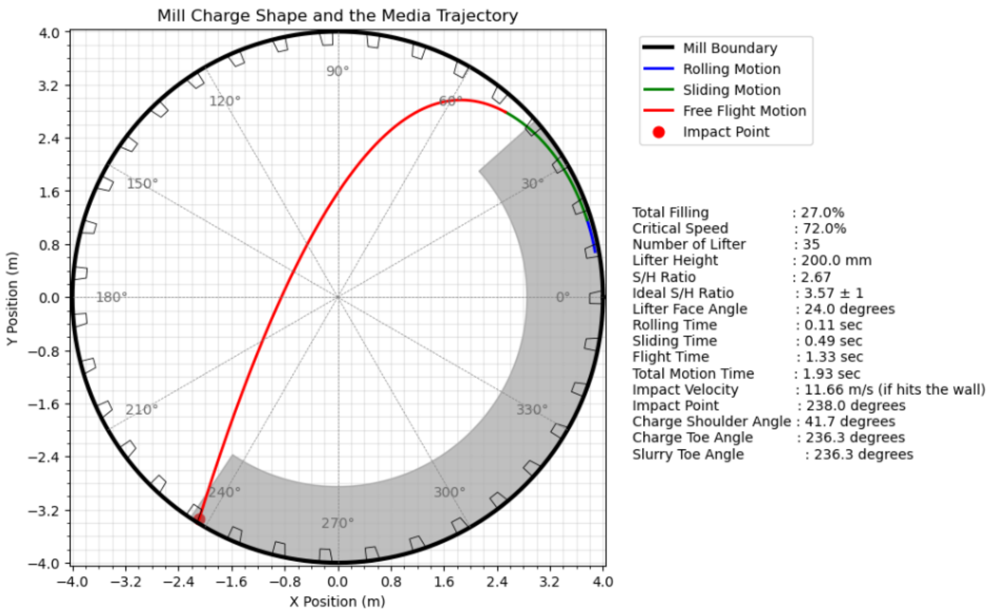

Fig. 5. Charge shape and trajectory with 200 mm lifter base width and 24° lifter face angle.

The design phase has highlighted the tool's benefits for optimizing mill performance. Furthermore, it can be employed to adjust operating conditions based on liner wear in a functioning mill, ensuring continued efficiency and effectiveness throughout its lifecycle.

3.2. Case study 2: operating conditions optimization

To demonstrate the application's capability in optimizing operating conditions based on liner wear, a series of simulations were conducted using the following baseline parameters. The simulations were performed on a SAG mill with an inner diameter of 8m, equipped with 35 rows of lifter bars having a height of 200 mm and a face angle of 24 degrees. The grinding media consisted of 120 mm balls with static and kinetic friction coefficients of 0.05 and 0.2 respectively.

The wear progression was simulated over a 6-month period:

- Initial conditions with new liners showed optimal ball trajectory landing at the toe point

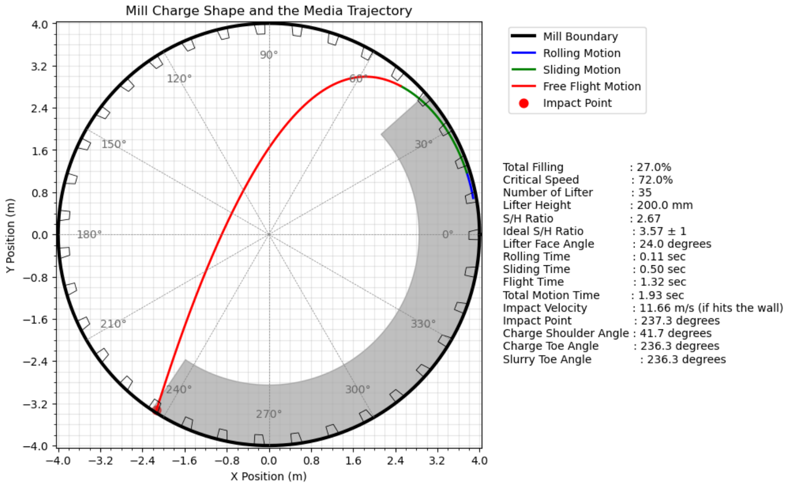

Fig. 6. Simulated charge motion and ball trajectory in a hypothetical 8m SAG mill with new liner

- After 2 months: lifter height reduced to 170 mm, face angle increased to 29 degrees

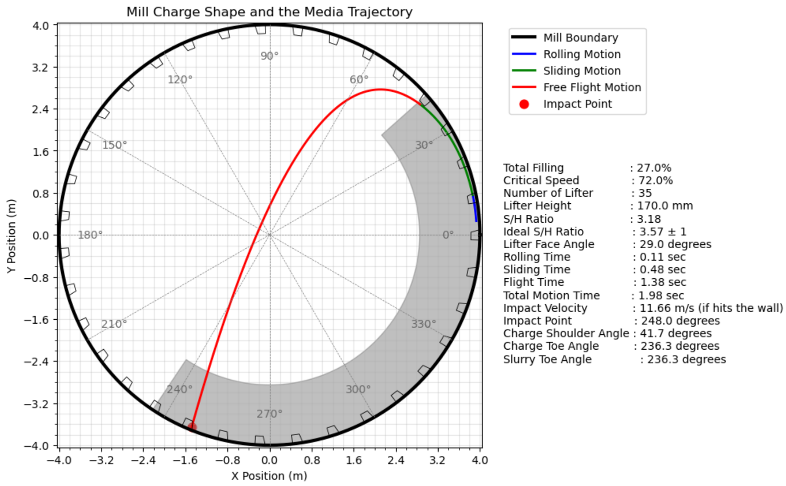

Fig. 7. Simulated charge motion and ball trajectory after 2 months of liner wear in the hypothetical mill

- After 4 months: lifter height reduced to 130 mm, face angle increased to 34 degrees

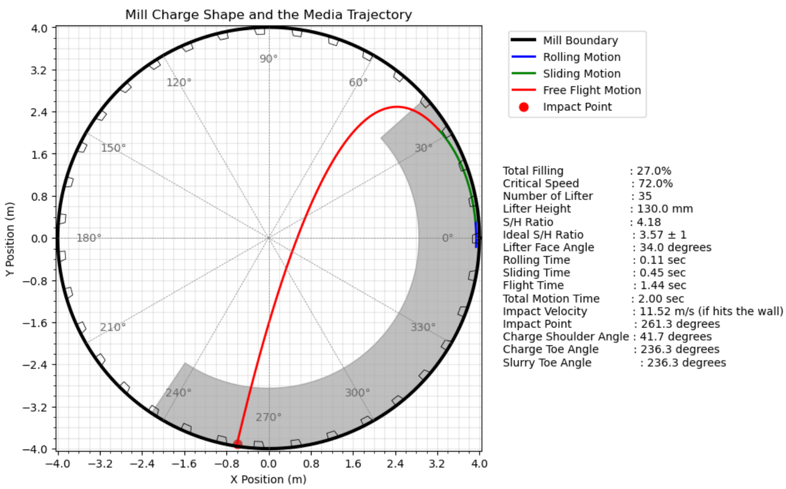

Fig. 8. Simulated charge motion and ball trajectory after 4 months of liner wear in the hypothetical mill

- After 6 months: lifter height reduced to 70 mm, face angle increased to 40 degrees

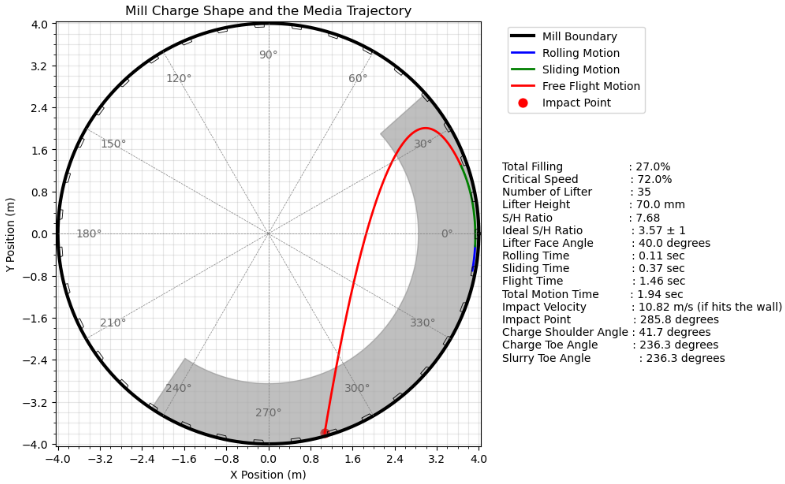

Fig. 9. Simulated charge motion and ball trajectory after 6 months of liner wear in the hypothetical mill

Analysis of these cases revealed a gradual shift in ball trajectory from the toe point to higher impact points as wear progressed. This shift results in decreased impact energy and reduced mill efficiency. These findings align with comprehensive DEM studies that have statistically modeled the impact of liner wear on key performance indicators, as demonstrated by (Cleary and Owen, 2018). Based on these simulation results, mill operators can proactively adjust operating parameters such as mill speed or charge volume to compensate for liner wear effects, rather than waiting for performance degradation to become evident through reduced throughput or increased power consumption. When making these adjustments, it is crucial to monitor the mill power draw to ensure it remains within design specifications while optimizing the grinding efficiency. This integrated approach to mill operation enables maintaining optimal grinding performance throughout the liner lifecycle while potentially extending the service life of the liner system.

4. Conclusions

This study presents a comprehensive Python-based tool integrating the Morrell C and Powell's models for optimizing tumbling mill performance. Through two case studies, we demonstrated the tool's capabilities in: (1) optimizing initial liner design parameters for ideal trajectory, and (2) guiding operational adjustments throughout liner wear progression. The tool's key innovation lies in its combined visualization of charge shape and ball trajectory, enabling engineers to make proactive decisions about mill operation and maintenance. This approach contributes to both operational efficiency and environmental sustainability by optimizing energy usage and reducing wear-related losses. Future development could enhance the tool's capabilities through incorporation of rod trajectory calculations, wear prediction models, and real-time monitoring features.

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