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Comparative analysis on washability indices of anthracite coal by using MCDM methods

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Abstract: This paper presents the results of the Multi-Criteria Decision Making (MCDM) methods for determining the optimal separation density in the gravity separation of anthracite coal, and also suggests an appropriate MCDM model for this purpose. Five separation densities were selected as the output and seven coal washability indices were used as the input based on the washability results of coal. The SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods were used in this study. The evaluation indices of the five methods were averaged as a new index. Finally, alternative A4 (i.e., the optimal separation density is 1650 kg/m^3) was selected as the final separation density.

Keywords: coal, gravity, separation, density, MCDM

1. Introduction

Coal is the most widely used fossil fuel and energy source in the world. It is of crucial importance for economic and energy development and is becoming increasingly important for meeting our society's energy needs.

Coal continues to play an important role in the global economy as the primary energy source for the production of cement, steel, and electricity. In 2022, global electricity generation grew by around 2.3% to a total of 29 074 TWh (IEA, 2023). Coal accounted for 27% of primary energy consumption and 35% of electricity generation worldwide in 2022 (EI, 2023). According to forecasts by the IEA (2023), coal will remain the most important source of electricity in 2040, accounting for 22% of global electricity generation.

The raw coal must be processed effectively using various methods in or-der to produce high-quality coal for the domestic and export markets. Coal preparation, also known as coal beneficiation, involves crushing, screening and various methods of separating the coal. There are different methods depending on the physical and/or chemical differences between the coal and the associated minerals (Xia et al., 2021). In recent years, gravity separation, flotation, magnetic separation, electrostatic separation and leaching have been used in coal preparation (Min and Wheelock, 1977; Polat et al., 2003; Rao and Gouricharan, 2016; Luttrell and Honaker, 2020; Phengsaart et al., 2023).

Gravity separation is used for coarse and medium sized coal fractions, while flotation is used for fine coal fractions (Phengsaart et al., 2023; Laskowski, 2001; Vasumathi et al., 2018; Sokolović et al., 2023).

Gravity separation is often used as the primary method for cleaning coal due to its versatility and low cost. It is the most efficient method of coal preparation. Approximately 80% of the coal mined worldwide is processed by gravity separation (Phengsaart et al., 2023; Ito, 2018). A comprehensive review of gravity separation was published by Das and Sarkar (2018).

Various devices can be used for the gravity separation of coal. In this method, gravity or centrifugal force is used to separate coal in wet (suspension) or dry form. Both static and dynamic separation with dense media should be used for the separation of coarse coal particles. According to Phengsaart et al. (2023), conventional gravity separation is ineffective for the separation of fine (<0.1 mm) coal.

Dense media separation is used for coal processing due to its simplicity, low cost and high efficiency. Dense media separation produces high quality products, i.e., low ash content, with reasonable clean

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coal yields. The separation efficiency of dense media separation is largely influenced by the particle size distribution and the washability of the coal.

The washability of the coal is determined by a float-sink test. The washability data (float-sink test) can be used to calculate the theoretical yield and ash content of the clean coal at given specific densities.

The washability of coal is influenced by a variety of factors, including particle size, floating and sinking products, and the amount of near-gravity materials (Rao and Gouricharan, 2016). They are key factors for understanding and evaluating coal washability and gravity separation efficiency.

Many numerical indices have been proposed for determination of the washability of different coals. The degree of washing (N) (Sarkar and Das, 1974), Washability number (W) (Sarkar and Das, 1977), Index of Washability (IW) (Govindarajan and Rao, 1994), Near Gravity Material Index (NGMI) (Majumder and Barnwal, 2004) are very useful parameters for studying and comparing the washability of different coals.

The degree of washing (N) was introduced by Sarkar and Das (1974), and can be calculated as follows:

$$N = \frac{w\,(a-b)}{a} \tag{1}$$

where: a - the ash content of the run-of-mine coal (feed), b - the ash content of the clean coal at a given separation density, w - the yield of clean coal at a given separation density.

The washability number (W) is a useful measure for classifying coals based on their washability properties. This number can be expressed as the ratio between the Optimum Degree of Washability (ODW) and the clean coal ash (A) at this optimum level (Sarkar et al., 1974):

$$W = 10\left(\frac{ODW}{A}\right) \tag{2}$$

The optimal degree of washing (ODW) is calculated by plotting the degree of washing (N) against the separation density and determining the highest value. The values of this index lie between 0 and 100 and are generally in a straight line relationship to the washability index. A coal can be considered more washable if it has a higher ODW value and a lower clean coal ash.

The washability number (W) defines the boundary between the coal and the associated mineral impurities and the determination of the optimum separation density. It has values between 0 and 100. The higher the W, the easier the coal is to wash.

The near-gravity material (NGM) is one of the most useful indices. The amount of material with a specific gravity of ±0.1 at a given specific gravity is called the NGM. Bird (1931) proposed a classification based on near-gravity materials to indicate the degree of difficulty. Depending on the amount of near gravity material within the range of + 0.1 specific gravity, a coal can be classified into one of six different categories: simple (0-7), moderate (7-10), difficult (10-15), very difficult (15-20), exceedingly difficult (20-25), and formidable (over 25) (Holuszko and Grieve, 1990).

The index of washability (IW) was proposed for the first time by Govindarajan and Rao (1994). The values of this index range from 0 to 100 and do not depend on the total ash content of the raw coal or the clean coal ash content. If this index is low, the coal is difficult to wash and if the index is high, the coal is easy to wash.

Majumder and Barnwal (2004) proposed the new index known as near-gravity material index (NGMI). The methodology for calculating NGMI values at different specific weights is described in Majumder and Barnval, (2004) and Sokolović, et al., 2023.

From the recovery curves of coal shown in Fig. 1, the ADC and ABC curves represent the recovery curves for non-ash (R_n) and ash-forming (R_a) materials, respectively.

The near-gravity material index can be calculated from the following equation:

$$NGMI = \frac{\text{Area KLMN}}{\text{Area beetwen curves ADC and ABC}} = \frac{\frac{[6\,(a-p)\,X_1^2 + 4\,(b-q)\,X_1^3 + 3\,(c-r)\,X_1^4]}{12} - \frac{[6\,(a-p)\,X_2^2 + 4\,(b-q)\,X_2^3 + 3\,(c-r)\,X_2^4]}{12}}{\frac{6\,(a-p) + 4\,(b-q) + 3\,(c-r)}{12}} \tag{3}$$

These recovery curves are drawn by plotting the values of R_n and R_a against the cumulative fractional weight (X). The values of the constants (a, b, c, p, q, r) in Equation [3] are estimated by fitting these equations to the non-ash and ash recovery curves and using the least squares method. The values of NGMI vary from 0 to 1 for easy to difficult to wash (Majumder and Barnval, 2004).

The Degree of Separability (DS) introduced by Ignjatović, (1983) is used to interpret separability data when studying the washability of coal. Equation (4) can be used to calculate the degree of separability at different specific densities:

$$DS = \frac{Rm}{Ra} \tag{4}$$

where R_m – theoretical clean coal yield or mass recovery (%), R_a – recovery for ash-forming materials (%), Higher values of DS indicate better separability of the coal.

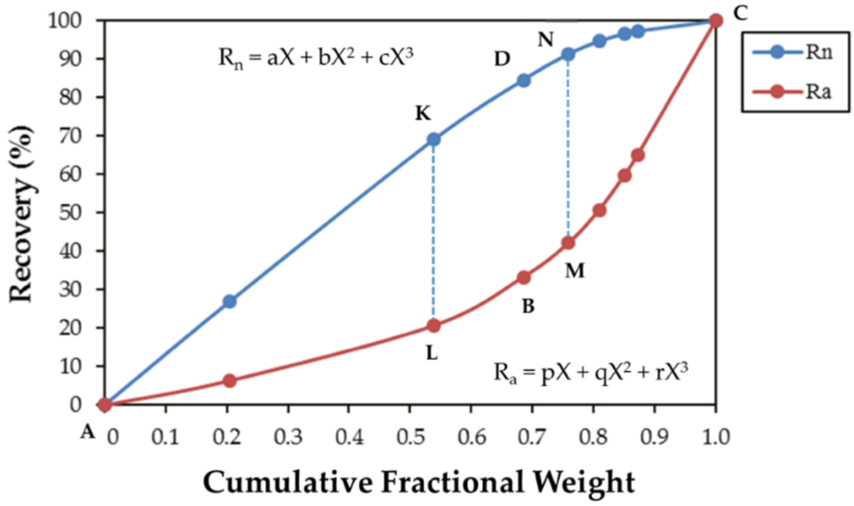

Fig. 1. Recovery curves for non-ash (Rn) and ash-forming (Ra) for a typical coal

Separation density or cut density is a key parameter in the gravity separation of coal. The results (yield and ash content of the clean coal) largely depend on the separation density. The optimal separation density can be selected from washability data and corresponding washability curves. Over the years, different indexes have been used to determine washability of coal. Decision making, particularly in laboratory tests, is not a straightforward procedure. In most cases, the right decisions and selection of optimal separation density are not easy and demand time.

The aim of this study is to determine the optimal separation density based on different coal washability indices, and also proposed a model for determining the optimal separation density based on the application of Multi-Criteria Decision Making (MCDM) methods.

It is known that MCDM methods allow evaluation based on a number of criteria, which usually have different weights, whereby the criteria can also have a different influence on the evaluation of the most appropriate alternative, that is, the criteria can be income, i.e. a higher value is better, and expense, that is, the lower value is better. Therefore, this research presents the results of a comparative study of the washability parameters of anthracite coal at different separation densities based on the use of several MCDM methods and also proposes an MCDM model for choosing the optimal separation density of coals

According to the available literature, there is no study in which MCDM methods were used to select a separation density. Therefore, the use of MCDM methods to determine the optimal separation density in the gravity separation of coal is a new approach in the scientific literature, since MCDM methods have not been used for this purpose before. Usability and effectiveness of the proposed approach are discussed in this study. Results obtained using complex MCDM procedures and, therefore, conclusions and successful ranking of the available alternatives are very often the basis for further decision.

2. Multi-criteria decision making (MCDM)

A method for evaluating or ranking options based on a set of conflicting criteria is called multi-criteria decision making or MCDM. These methods are a useful mathematical tool for choosing between multiple alternatives that appear similar and have many important variables to consider. One of the main advantages of MCDM is its adaptability, as it offers a range of techniques that can be adapted to different scenarios.

Since the introduction of the first MCDM methods, SAW and ELECTRE, in 1968, many MCDM methods have been developed and used in various scientific and industrial fields, in 1968 to make decisions where multiple criteria or factors need to be considered.

In recent decades, different MCDM methods have been developed, as well as different set theories, so that MCDM methods can be used in much larger and more complex scenarios for decision making. Table 1 summarizes some of the proposed MCDM techniques, while Table 2 lists some of the more popular set theories.

Table 1. An overview of the some of the proposed MCDM methods

Method	Author(s)	Year
SAW	MacCrimon	1968
ELECTRE	Roy	1968
AHP	Saaty	1977
TOPSIS	Hwang and Yoon	1981
PROMEHTEE	Brans	1982
COPRAS	Zavadskas et al.	1988
VIKOR	Opricovic and Tzeng	2004
ARAS	Zavadskas and Turskis	2010
MULTIMOORA	Brauers and Zavadskas	2010
WASPAS	Zavadskas et al.	2012
EDAS	Keshavarz Ghorabaee et al.	2015
CoCoSo	Yazdani et al.	2018
Simple WISP	Stanujkic et al.	2023

Table 2. An overview of the prominent set theories

Method	Author(s)	Year
Fuzzy set theory	Zadeh	1965
Rough set theory	Pawlak	1982
In intuitionistic fuzzy set theory	Attanasov	1986
Neutrosophic set theory	Smarandache	1998

MCDM methods can be helpful in determining the optimal approach to a variety of issues. It is widely used in mining and mineral processing due to its ability to evaluate complex issues (Popović et al, 2020). Sitorus et al. (2019) provide a complete overview of MCDM methods used in mining and mineral processing.

Many authors report that MCDM methods have been used in mineral processing for a range of objectives, e.g., for mineral processing plant site selection (Safari et al., 2010; Bakhtavar and Lotfian, 2017), flotation tailings dump (Stirbanovic et al., 2013), grinding circuit (Stanujkic et al., 2014; Stanujkic et al., 2019), and flotation circuit selection (Zavadskas et al., 2016; Magdalinowić et al., 2021), selection of different equipment such as crushers (Rahimdel and Ataei, 2014), and rougher flotation machines (Stirbanovic et al., 2019; Sitorus and Brito-Parada, 2020), collector selection in the flotation process (Kostovic and Gligoric, 2020; Stirbanović et al., 2023), and optimization of leaching parameters (Kursunoglu et al., 2021; Baral et al., 2014; Kursuncu et al., 2018). The results of MCDM methods and their application in mineral processing are summarized in Table 3.

In recent years, MCDM methods have been widely used in coal mining (Chakraborty and Chandra, 2005; Shamsuzzaman et al., 2013; We et al., 2016; Ooriad et al., 2018; Xu and Dong, 2019; Mohebali et al., 2020; Lipka and Szwed, 2021; Sivageerthi et al., 2022), as well as in wastewater treatment (Stirbanović et al., 2021), and recycling (Soković et al., 2021).

This paper presents the results of a comparative analysis of the washability indices of anthracite coal using the Multi-Criteria Decision Making (MCDM) methods, which combines the SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods and aims to provide an efficient selection of separation density for the gravity process of coal.

The evaluation of the washability of anthracite coal was carried out through seven parameters such as degree of washing (N), washability number (W), near-gravity material (NGM), near-gravity material index (NGMI), degree of separability (DS), clean coal yield (Rm), and ash content of the clean coal (Ash). The paper discusses the usage of the Multi-Criteria Decision Making (MCDM) methods for the selection of the optimal separation density. In this study, five separation densities were selected as the output.

Table 3. Summary results of the application of MCDM in mineral processing

Selection		**Author(s)**	**Method(s)**
Location	Mineral processing plant	Safari et al. (2010)	AHP
		Bakhtavar and Lotfian (2017)	Fuzzy AHP and grey MCDM
	Flotation tailings dump	Štirbanovic et al. (2013)	Rough Set Theory
Grinding circuit		Stanujkic et al. (2014)	MOORA
		Stanujkic et al. (2019)	Interval-valued intuitionistic fuzzy sets
Flotation circuit	Lead-zinc ore	Zavadskas et al. (2016)	WASPAS and single-valued neutrosophic set
	Copper-pyrite ore	Magdalinović et al. (2021)	Preference Selection Index
Flotation collector	Lead-zinc ore	Kostovic and Gligoric (2015)	TOPSIS
	Porphyry copper ore	Štirbanovic et al. (2023)	VIKOR
Primary crusher		Rahimdel and Ataei (2014)	AHP
		Sitorus and Brito-Parada (2020)	Integrated Constrained Fuzzy Stochastic AHP
Flotation machine		Štirbanovic et al. (2019)	TOPSIS and VIKOR
Leaching acid type		Kursunoglu et al. (2021)	AHP
Leaching parameters	Rare earth metals	Baral et al. (2014)	TOPSIS, line graph and spider diagrams
	Copper	Kursuncu et al. (2018)	TOPSIS

From the large number of MCDM methods proposed so far, some characteristic ones have been selected. The SAW and TOPSIS methods have been proposed previously and have been used to solve many different decision-making problems. Moreover, the SAW method is based on a very simple calculation procedure, while the TOPSIS method ranks the alternatives based on their distances to the ideal and anti-ideal solutions in Euclidean space. In contrast, the WASPAS, CoCoSo and WISP methods can be described as later proposed MCDM methods. The calculation procedures for these methods are based on different approaches of integrating the weighted sum and the weighted product.

3. Materials and Methods

3.1. Materials

The samples for the study were taken from the coal separation plant in the anthracite coal mine "Vrška Čuka" in Serbia. Raw coal is sampled simultaneously from the "BSRI-1200" dense-medium separator, with a particle size range of 0.5–20 mm. This separation process makes it possible to obtain different types of clean coal with an ash quality of between 5 and 15%. The flotation process has been proposed for coal with a particle size below 0.5 mm (Soković et al., 2023). The flotation concentration process is not in operation, and this size fraction is deposited in settling basins.

The weight of the raw coal sample was approximately 300 kg. Then, the collected sample was mixed by coning, divided by quartering, and then sampled. Next, the sample was screened to prepare the representative samples for laboratory testing using the float-sink method.

3.2. Particle size analysis

Dry sieving on sieves: 19; 9.5; 4.75; 2.36; 0.6 mm was applied for determining the particle size distribution of anthracite coal sample. A Retsch AS200 laboratory shaker with an amplitude of 2 mm was used. The particle size fractions were analyzed for ash content according to SRPS ISO 1171: 2014.

3.3. Float–Sink Tests

The washability characteristics of anthracite coal by size fractions were investigated using float-sink tests performed with a zinc chloride medium at various specific densities from 1300 to 1850 kg/m^3. Separation with dense medium is used for the medium size fraction of 0.5 to 5 mm, and for coarser size fraction of 5 to 20 mm in a separation plant for hard coal from anthracite mine. Therefore, the following narrow fractions obtained from screening, (-19+9.5) mm, (-9.5+4.75) mm, (-4.75+2.36) mm, and (-2.36+0.6) mm, were used for the float-sink tests.

All products were washed and dried at room temperature. They were then weighed and ground to below 106 μm to determine the ash content. Based on the obtained results, the cumulative yield and ash values were calculated.

The washability data obtained in the study were used to calculate the washability data of the reconstructed size fraction (-19+0.6) mm (gravity separation feed) of anthracite coal. Following the methods of Sarkar and Das (1974), Sarkar et al. (1977), Majumder and Barnwal (2004), and Ignjatović (1983), washability parameters such as degree of washing (N), washability number (W), near-gravity material (NGM), near-gravity material index (NGMI), and degree of separability (DS) were calculated. These indices are very useful parameters for the interpretation of the washability of coal (Holuszko and Grieve, 1990).

A comparison of coal washability, and theoretical values of yield and ash content was carried out using the MCDM methods for different separation densities. The comparison was carried out using the SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods. The procedure for evaluating alternatives using these methods is explained in the following methodology.

The main objective of this study was to determine the optimum separation density for the coal gravity process using MCDM methods. Therefore, the most important alternatives in this selection are the separation density values, namely: 1350 kg/m^3 (A_1), 1450 kg/m^3 (A_2), 1550 kg/m^3 (A_3), 1650 kg/m^3 (A_4), and 1750 kg/m^3 (A_5). The alternative with the highest evaluation score will be determined as the best option for the final selection.

3.4. MCDM methodology

As mentioned above, many MCDM methods have been proposed so far, which differ to some extent in the methodology applied to evaluate alternatives, i.e., these methods use different normalization and aggregation procedures. Therefore, this section presents some characteristic MCDM methods that were used in later calculations.

3.4.1. SAW Method

The Simple Additive Weighting (SAW) method is one of the earliest proposed and commonly used MCDM methods with a relatively simple aggregation procedure. Another characteristic of this method is that it can be used with different normalization procedures. The procedure for solving an MCDM problem in which m alternatives are evaluated based on n criteria is shown in Fig. 2.

In Fig. 2, x_{ij} represents the evaluation of alternative i according to criterion j, Ω_{max} and Ω_{min} represent the set of criteria of type max and min, respectively, and w_j represents the weight or importance of criterion j.

3.4.2. TOPSIS Method

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method can also be described as one of the best known and most frequently used MCDM method. This method is based on the determination of the relative distance to the ideal and anti-ideal solution or to the ideal point, usually in Euclidean space, and on the application of the square root normalization method.

The procedure of the TOPSIS method is shown in Fig. 3.

3.4.3. WASPAS Method

The Weighted Aggregates Sum Product Assessment (WASPAS) method integrates the Weighted Sum (WS) and Weighted Product (WP) approaches, more specifically the WS and Power-Weighted Product (P-WP) approaches, to determine the most appropriate alternative. This method is based on the use of the Max normalization procedure.

The procedure of the WASPAS method is shown in Fig. 4. In Eq. (16), λ stands for a coefficient and$\lambda \in [0,1]$. In many calculations, a simplified form of Eq. (16) is used, which is known as Eq. (17).

Step 1. Calculate normalized decision-making matrix $R = [r_{ij}]_{m\,x\,n}$,

$$r_{ij} = \begin{cases} \frac{x_{ij}}{max_i x_{ij}}, & j \in \Omega_{max} \\ \frac{max_i x_{ij}}{x_{ij}}, & j \in \Omega_{min} \end{cases} \quad (5)$$

Step 2. Calculate the performance score S_i of each alternative,

$$S_i = \sum_{j=1}^{n} r_{ij}\, w_j \quad (6)$$

Step 3. Rank the alternatives in the descending order and select the best most appropriate. In the SAW method the alternative with the higher S_i is more preferable.

Fig. 2. The procedure of the SAW method (Source: Authors' elaboration)

Step 1. Calculate normalized decision-making matrix $R = [r_{ij}]_{m\,x\,n}$,

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{i=1}^{m} x_{ij}^2\right)^{1/2}}. \quad (7)$$

Step 2. Determine the ideal A^+ and anti-ideal A^- solution

$$A^+ = \{r_1^+, r_2^+, \ldots, r_n^+\} = \{(max_i r_{ij} | j \in \Omega_{max}), (min_i r_{ij} | j \in \Omega_{min})\} \quad (8)$$

$$A^- = \{r_1^-, r_2^-, \ldots, r_n^-\} = \{(min_i r_{ij} | j \in \Omega_{max}), (max_i r_{ij} | j \in \Omega_{min})\} \quad (9)$$

Step 3. Calculate separation measures of each alternative from the ideal d_i^+ and anti-ideal d_i^- solution

$$d_i^+ = \left\{\sum_{j=1}^{n} \left[w_j \left(r_{ij} - r_j^+\right)^2\right]\right\}^{1/2} \quad (10)$$

$$d_i^- = \left\{\sum_{j=1}^{n} \left[w_j \left(r_{ij} - r_j^-\right)^2\right]\right\}^{1/2} \quad (11)$$

Step 4. Calculate the closeness coefficient C_i

$$C_i = \frac{d_i^-}{d_i^- - d_i^+} \quad (12)$$

Step 5. Rank the alternatives and select the best one. That means that the alternative with the higher value of C_i is more preferable.

Fig. 3. The procedure of the TOPSIS method (Source: Authors' elaboration)

3.4.4. WISP method

The Simple Weighted Sum Product (WISP) method also integrates the WS and WP approaches, but instead of the P-WP approach, the WISP method uses the "simple" Weighted Product (S-WP) approach used in the MULTIMOORA method (Stanujkic, 2022).

The WISP method integrates four utility measures that represent the relationships between the effects of the maximum and minimum criteria to determine the overall utility of an alternative.

The procedure of the WISP method is shown in Fig. 5.

Step 1. Calculate normalized decision-making matrix $R = [r_{ij}]_{m \times n}$,

$$r_{ij} = \begin{cases} \frac{x_{ij}}{max_i x_{ij}}, & j \in \Omega_{max} \\ \frac{max_i x_{ij}}{x_{ij}}, & j \in \Omega_{min} \end{cases} \quad (13)$$

↓

Step 2. Compute the relative importance $Q_i^{(1)}$

$$Q_i^{(1)} = \sum_{j=1}^{n} r_{ij}\, w_j \quad (14)$$

↓

Step 3. Compute the relative importance $Q_i^{(2)}$

$$Q_i^{(2)} = \prod_{j=1}^{n} r_{ij}^{\,w_j} \quad (15)$$

↓

Step 4. Compute total relative importance Q_i

$$Q_i = \lambda\, Q_i^{(1)} + (1-\lambda) Q_i^{(2)} = \lambda \sum_{j=1}^{n} r_{ij}\, w_j + (1-\lambda) \prod_{j=1}^{n} (r_{ij})^{w_j} \quad (16)$$

$$Q_i = 0.5\, Q_i^{(1)} + 0.5 Q_i^{(2)} \quad (17)$$

↓

Step 5. Rank the alternatives and select the best one. That means that the alternative with the higher value of Q_i is more preferable.

Fig. 4. The procedure of the WASPAS method (Source: Authors' elaboration)

Step 1. Calculate normalized decision-making matrix $R = [r_{ij}]_{m \times n}$,

$$r_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \quad (18)$$

↓

Step 2. Calculate four utility measures: u_i^{sd}, u_i^{pd}, u_i^{sr} and u_i^{pr}

$$u_i^{sd} = \sum_{j\in\Omega_{\max}} r_{ij} w_j - \sum_{j\in\Omega_{\min}} r_{ij} w_j \quad (19)$$

$$u_i^{pd} = \prod_{j\in\Omega_{\max}} r_{ij} w_j - \prod_{j\in\Omega_{\min}} r_{ij} w_j \quad (20)$$

$$u_i^{sr} = \frac{\sum_{j\in\Omega_{\max}} r_{ij} w_j}{\sum_{j\in\Omega_{\min}} r_{ij} w_j} \quad (21)$$

$$u_i^{pr} = \frac{\prod_{j\in\Omega_{\max}} r_{ij} w_j}{\prod_{j\in\Omega_{\min}} r_{ij} w_j} \quad (22)$$

↓

Step 3. Calculate recalculated values of four utility measures $\bar{u}_i^{sd}$, $\bar{u}_i^{pd}$ $\bar{u}_i^{sr}$ and $\bar{u}_i^{pr}$

$$\bar{u}_i^{sd} = \frac{1+u_i^{sd}}{1+max_i\, u_i^{sd}} \quad (23)$$

$$\bar{u}_i^{pd} = \frac{1+u_i^{pd}}{1+max_i\, u_i^{pd}} \quad (24)$$

$$\bar{u}_i^{sr} = \frac{1+u_i^{sr}}{1+max_i\, u_i^{sr}} \quad (25)$$

$$\bar{u}_i^{pr} = \frac{1+u_i^{pr}}{1+max_i\, u_i^{pr}} \quad (26)$$

↓

Step 4. Determine the overall utility u_i of each alternative

$$u_i = \frac{1}{4}(\bar{u}_i^{sd} + \bar{u}_i^{pd} + \bar{u}_i^{sr} + \bar{u}_i^{pr}). \quad (27)$$

↓

Step 5. Rank the alternatives and select the most appropriate one. In this approach the alternative with the highest value of u_i is the most suitable one.

Fig. 5. The procedure of the WISP method (Source: Authors' elaboration)

3.4.5. Combined Compromise Solution (CoCoSo) Method

The Combined Compromise Solution (CoCoSo) method was proposed by Yazdani et al., (2018). The CoCoSo method is based on the integration of weighted sum method and the exponentially weighted product method.

The calculation procedure of the CoCoSo method is shown in Fig. 6. Based on the above considerations, the main objective of this article is to determine the optimal separation density for the gravity preparation of coal using the SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods. The most important alternatives in this choice are therefore the values for the separation density, namely: 1350 kg/m^3 (A_1), 1450 kg/m^3 (A_2), 1550 kg/m^3 (A_3), 1650 kg/m^3 (A_4), and 1750 kg/m^3 (A_5). The alternative with the highest evaluation score will be determined as the best option for the final selection.

Step 1. Calculate normalized decision-making matrix $R = [r_{ij}]_{m\,x\,n}$, as follows:

$$r_{ij} = \begin{cases} \frac{x_{ij} - min_i x_{ij}}{max_i x_{ij} - min_i x_{ij}} & j \in \Omega_{max} \\ \frac{max_i x_{ij} - x_{ij}}{max_i x_{ij} - min_i x_{ij}} & j \in \Omega_{min} \end{cases} \quad (28)$$

Step 2. Calculate the sum of weighted comparability sequence S_i and power-weighted comparability sequences P_i of each alternative as follows:

$$S_i = \sum_{j=1}^{n} r_{ij} w_j \quad , \text{ and} \quad (29)$$

$$P_i = \sum_{j=1}^{n} r_{ij}^{w_j} \ . \quad (30)$$

Step 3. Calculate three aggregated appraisal scores k_{ia}, k_{ib} and k_{ic} as follows:

$$k_{ia} = \frac{S_i + P_i}{\sum_{i=1}^{m}(S_i + P_i)} \ , \quad (31)$$

$$k_{ib} = \frac{S_i}{\min_i S_i} + \frac{P_i}{\min_i P_i} \ , \quad (32)$$

$$k_{ic} = \frac{\lambda S_i + (1-\lambda) P_i}{\lambda \max_i Si + (1-\lambda) \max_i P_i} \ , \quad (33)$$

where: λ is coefficient, $\lambda \in [0,1]$ and it is often set to λ = 0.5.

Step 4. Calculate relative performance score k_i, for each alternative, as follows:

$$k_i = \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) + (k_{ia} k_{ib} k_{ic})^{\frac{1}{3}} \ . \quad (33)$$

Step 5. Rank the alternatives, sorting by the value k_i in ascending order. The alternative with the maximum value of k_i is the most appropriate alternative.

Fig. 6. The procedure of the CoCoSo method (Source: Authors' elaboration)

4. Results and discussion

4.1. Particle size analysis

The particle size distribution of anthracite coal is given in Table 4. The results given in Table 4 show that the average ash content was around 20%. It can also be seen that the content of particles below 0.6 mm was about 29%, with the average ash content being approximately 16%. The content of smaller particle size fractions (0.6–2.36 mm) was 30.92%, with 13.16% of ash. The results have shown that the participation of particles ranging from 2.36 to 4.75 mm was about 11%, while the participation of particles coarser than 9.5 mm was approx. 14% with an ash content of approx. 10%.

Table 4. Particle size analysis and ash contents of anthracite coal sample

Grain size	**Internal values**		**Cumulative passing value**	
(mm)	Weight (%)	Ash (%)	Weight (%)	Ash (%)
-19+9.5	14.32	10.09	100.00	20.07
-9.5+4.75	11.24	55.35	85.68	21.74
-4.75+2.36	14.91	25.73	74.44	16.67
-2.36+0.6	30.92	13.16	59.53	14.40
-0.6+0	28.61	15.74	28.61	15.74

4.2. Float–Sink Test

The results of the float-sink test by size fraction of anthracite coal are shown in Table 5. Based on the results of the float-sink test by size fraction of anthracite coal, which are shown in Table 5 and based on the results of the particle size analysis, shown in Table 4, the input for the separation process of the size fraction (-19+0.6) mm was reconstructed. Using the float-sink data shown in Table 6 and Fig. 7, the washability curves of anthracite coal were plotted. The parameters were calculated according to the methods described above and shown in Table 7.

Table 5. Float-sink analysis by size fraction of anthracite coal

Specific gravity range kg/m³	**Average specific gravity kg/m³**	**(-19+9.5) mm**		**(-9.5+4.75) mm**		**(-4.75+2.36) mm**		**(-2.36+0.6) mm**	
		Weight (%)	Ash (%)	Weight (%)	Ash (%)	Weight (%)	Ash (%)	Weight (%)	Ash (%)
Float at 1300	1250	0.00	0.00	0.27	4.80	0.03	2.06	8.02	1.96
-1400 + 1300	1350	57.91	3.42	19.38	5.71	57.46	3.80	64.11	7.87
-1500 + 1400	1450	18.06	7.28	6.64	14.01	13.60	15.28	12.48	12.45
-1600 + 1500	1550	6.41	14.23	5.71	24.59	4.08	33.52	2.40	20.68
-1700 + 1600	1650	3.79	20.20	2.53	33.33	2.21	42.63	1.08	27.40
-1800 + 1700	1750	2.29	30.17	2.64	49.63	1.36	47.17	0.76	33.34
-1850 + 180C	1825	0.90	35.88	1.57	60.74	0.53	52.78	0.34	35.19
Sink at 1850	1925	10.64	48.10	61.26	80.45	20.73	87.38	10.81	54.17
∑		100.00		100.00		100.00		100.00	

Based on the washability data, it can be concluded that the tested anthracite coal is very easy to separate. Table 46 shows that the weight of the clean coal product is high and lies between 60 and 80% depending on the separation density. The fractions with the lowest density, 1300 and 1400 kg/m³, provide the best quality products with an ash content of less than 6%.

Table 6 and Fig. 7 show that, depending on the yield and quality of the clean coal product, the best separation results were achieved at specific densities between 1350 and 1750 kg/m³. At specific densities of 1350, 1450, 1550, 1650 and 1750 kg/m³, the coal yield values were 30.74, 64.41, 72.91, 75.99, and 77.78% respectively, while for ash values of 5.45, 6.27, 7.20, 7.91, and 8.81% were measured at these corresponding densities.

Table 6. Results of the float-sink tests of anthracite coal

Specific gravity	Elementary curve			Cumulative curves							
				Cumulative floats				**Cumulative sinks**			
kg/m³	**Wt %**	**Ash %**	**Ash product**	**Cum Wt %**	**Cum Ash product**	**Ash %**	**Ra %**	**Cum Wt %**	**Cum Ash product**	**Ash %**	**Ra %**
Floats 1300	3.52	1.99	7.00	3.52	7.00	1.99	0.31	100.00	2233.61	22.34	100.00
-1400+1300	54.43	5.90	321.14	57.95	328.14	5.66	14.69	96.48	2226.60	23.08	99.69
-1500+1400	12.91	11.75	151.69	70.86	479.83	6.77	21.48	42.05	1905.47	45.31	85.31
-1600+1500	4.08	22.19	90.54	74.94	570.37	7.61	25.54	29.14	1753.77	60.18	78.52
-1700+1600	2.09	29.28	61.20	77.03	631.56	8.20	28.28	25.06	1663.24	66.37	74.46
-1800+1700	1.49	39.55	58.93	78.52	690.49	8.79	30.91	22.97	1602.04	69.75	71.72
-1850+1800	0.69	47.42	32.72	79.21	723.21	9.13	32.38	21.48	1543.11	71.84	69.09
1850 Sinks	20.79	72.65	1510.39	100.00	2233.61	22.34	100.00	20.79	1510.39	72.65	67.62

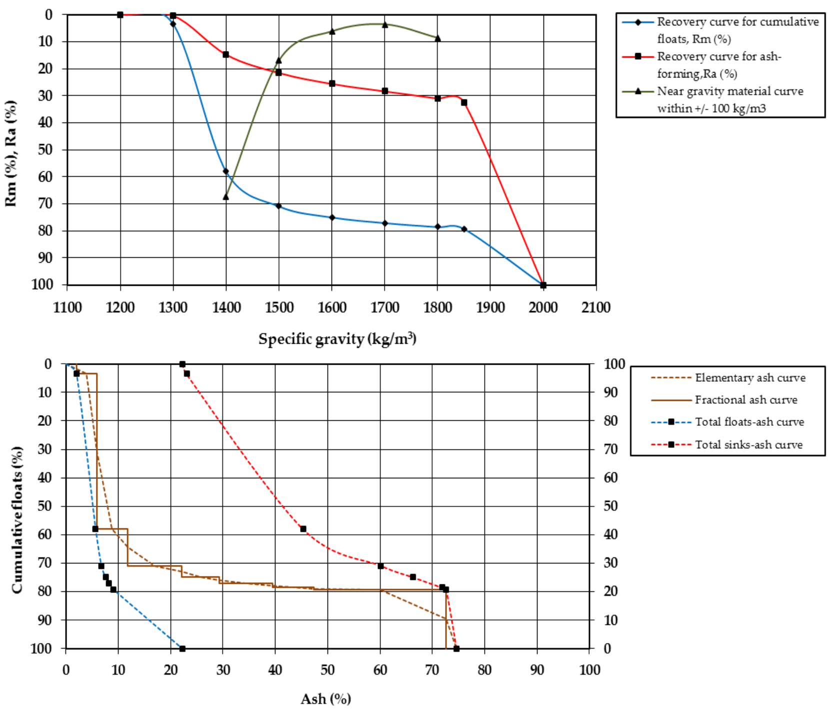

Fig. 7. Washability curves of anthracite coal

The results have shown that the optimal separation density is in the range of 1350 to 1750 kg/m³ and its choice depends strongly on the value of the washability index. Using the various parameters and methods described above, the values for N, W, NGM, NGMI, and DS were calculated. In addition, the values for the clean coal yield (Rm) and for the clean coal (Ash) at specific density were calculated.

Table 7 shows the comparative values for the washability indexes, the clean coal yield and the ash content of the clean coal.

Table 7. Comparative values of washability indexes, the clean coal yield and ash content of the clean coal

Specific density (kg/m³)	**Degree of Washing (N)**	**Washability number (W)**	**Near-Gravity Material (NGM)**	**Near-Gravity Material Index (NGMI)**	**Degree of Separability (DS)**	***Coal coal Yield Rm(%)***	***Ash content of the clean coal Ash(%)***
1350	23.23	42.60	62.65	0.59	4.10	*30.74*	*5.45*
1450	46.32	73.83	42.17	0.35	3.56	*64.41*	*6.27*
1550	49.40	68.57	11.58	0.13	3.10	*72.91*	*7.20*
1650	49.09	62.06	4.87	0.07	2.82	*75.99*	*7.91*
1750	44.98	51.07	3.22	0.04	2.63	*77.78*	*8.50*

As shown in Table 7, based on the washability data obtained from the float-sink tests and the calculation of the washability parameters, it can be concluded that anthracite coal has good washability properties.

The analysis shows that the values for the degree of washing (N) ranged from 23.23 (1350 kg/m³) to 49.40 (14550 kg/m³). It was found that the N values increased with the increase in specific density from 1350 to 1550 kg/m³. The slightly lowest valuesof N were achieved at densities of 1650, and 1750 kg/m³. The highest N value was therefore achieved at 1550 kg/m³.

Table 7 shows that the washability number (W) varies with specific densities and was quite high (73.83) at a density of 1450 kg/m³. The lowest value was 42.40 at 1350 kg/m³.

The values of NGMI were 0.59, 0.35, 0.13, 0.07 and 0.04 at densities of 1350, 1450, 1550, 1650, and 1750 kg/m³, respectively. It was found that the minimum NGMI values were obtained at densities of 1650 and 1750 kg/m³, which could represent the optimal separation density according to this index.

Table 7 shows the variation in the percentage of near-gravity material (NGM) at different specific gravities for anthracite coal. The low NGM value indicates the suitability of the gravity process for handling the coal. Based on the data resulting from the density fractions of ±0.1, it can be concluded that the 1650 or 1750 kg/m³ density fractions may be the best choice. However, the densities 1650 or 1750 kg/m³ have the lowest DS values of 2.82 and 2.63, respectively. A low DS value indicates the poorest separability of the coal.

In the density range of 1350 to 1750 kg/m3, the yield of clean coal is between 30.74 and 77.78%, and the ash content in the clean coal is between 5.45 and 8.50%, as shown in Table 7. Lower separation densities can lead to a cleaner coal product, but also to a lower yield as more impurities are separated. Higher separation densities can increase the yield, but can also lead to poorer coal quality.

From the results of the float-sink tests, it can be concluded that the separation density influences the efficiency of the gravity separation of coal. The results of this study show that anthracite coal is easy to wash. Based on the data resulting from the density fractions of ±0.1, it can be concluded that the density fractions 1650 or 1750 kg/m³ could be the optimal choice. At densities of 1650 and 1750 kg/m³, a clean coal product with an ash content of about 8% is obtained. A clean coal product of this quality can be used in many ways as an energy source for steel production and/or cement production.

It was found that it is very difficult to determine the optimal separation density based on the determined parameter values. The aim of this study is therefore to determine the optimal separation density for the gravity separation of anthracite coal using MCDM methods.

4.3. Evaluation of separation densities

Table 8 shows the evaluation of five separation densities of coal-water mixtures based on five criteria. The main criteria used in the separation of coal were Degree of washing (C_1), washability number (C_2), near-gravity material (C_3), near-gravity material index (C_4), degree of separability (C_5), the yield of clean coal (C_6), and the ash content of clean coal (C_7).

Table 8 shows evaluation criteria with optimization directions and criteria weights. The weights of the criteria were determined by direct assignment. The Criteria C_1-C_5, which refer to different washability indexes, and the criteria C_6-C_7, which refer to the clean coal yield (C6), and the ash content of the clean coal at specific density were assigned the same weights (1/7=0.1428570), as it was not possible to determine which criterion was more important than the other.

The most important alternatives in this selection are the separation density values, namely: 1350 kg/m^3 (A_1), 1450 kg/m^3 (A_2), 1550 kg/m^3 (A_3), 1650 kg/m^3 (A_4), and 1750 kg/m^3 (A_5). The evaluations of the considered alternatives, i.e., separation densities, in relation to the criteria in Table 7 are shown in Table 9. Based on these values, the following five MCDM methods were used to rank the five separation densities such as: SAW, TOPSIS, WASPAS, WISP, and CoCoSo

Table 8. Evaluation criteria

	Criterion	**Optimisation**	**Weight**
C_1	Degree of washing (N)	max	0.142857
C_2	Washability number (W)	max	0.142857
C_3	Near-Gravity Material (NGM)	min	0.142857
C_4	Near-Gravity Material Index (NGMI)	min	0.142857
C_5	Degree of Separability (DS)	max	0.142857
C_6	Clean coal yield (Rm)	max	0.142857
C_7	Ash content of the clean coal (Ash)	min	0.142857

Table 9. Initial decision-making matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	23.23	42.60	62.65	0.59	4.10	30.74	5.45
A_2	46.32	73.83	42.17	0.35	3.56	64.41	6.27
A_3	49.40	68.57	11.58	0.13	3.10	72.91	7.20
A_4	49.09	62.06	4.87	0.07	2.82	75.99	7.91
A_5	44.98	51.07	3.22	0.04	2.63	77.78	8.50

4.3.1. Evaluation of separation densities by using the SAW method

The details of the calculations performed using the SAW method, i.e., Eq. (5) and Eq. (6), are summarized in Table 10. As can be seen from the above table, the most acceptable alternative, based on all the considered criteria, is the alternative A_5, and the three best-ranked alternatives are ranked as follows: $A_5 \succ A_4 \succ A_3$.

Table 10. Ranking order of alternatives obtained using the SAW method

	S_i	*Rank*
A_1	0.472	5
A_2	0.529	4
A_3	0.617	3
A_4	0.749	2
A_5	**0.920**	**1**

4.3.2. Evaluation of separation densities by using the TOPSIS method

The details of the calculations obtained by applying the TOPSIS method, i.e., Eqs.(7) - (12), are summarized in Table 11. As can be observed from the table above, alternative A_4 is more acceptable, and the three top-rated alternatives are ranked as follows: $A_4 \succ A_5 \succ A_3$, i.e., opposite to the ranking resulting from the application of the TOPSIS method. The table above also shows that the difference in the C_i values between alternatives A_3 and A_5 is quite small, so that even small variations in the weight of the criteria can lead to a change in the ranking of the alternatives.

Table 11. Ranking order of alternatives obtained using the TOPSIS method

	d_i^-	d_i^+	C_i	*Rank*
A_1	0.045	0.167	0.212	5
A_2	0.080	0.103	0.438	4
A_3	0.143	0.048	0.749	3
A_4	**0.159**	**0.046**	**0.776**	**1**
A_5	0.163	0.051	0.760	2

4.3.3. Evaluation of separation densities by using the WASPAS method

The details of the calculations obtained by applying the WASPAS method, i.e., using Eqs.(13)–(17), are summarized in Table 12. As it can be observed from Table 12, alternative A_5 is also more acceptable, and the three top-rated alternatives are ranked as follows: $A_5 \succ A_4 \succ A_3$, which is the same as when using the SAW method.

Table 12. Ranking order of alternatives obtained using the WASPAS method

	$Q_i^{(1)}$	$Q_i^{(2)}$	Q_i	*Rank*
A_1	0.434	0.252	0.343	5
A_2	0.601	0.377	0.489	4
A_3	0.659	0.579	0.619	3
A_4	0.755	0.742	0.748	2
A_5	**0.849**	**0.834**	**0.841**	**1**

4.3.4. Evaluation of separation densities by using the WISP method

The details of the calculations using the WASPAS method, i.e., Eqs.(18)–(27), are summarized in Table 13. As can be seen from Table 13, alternative A_4 is also more acceptable and the ranking of the top three alternatives is as follows: $A_4 \succ A_3 \succ A_5$.

Table 13. Calculation details obtained using the WISP method

	u_i^{sd}	u_i^{pd}	u_i^{sr}	u_i^{pr}	$\bar{u}_i^{sd}$	$\bar{u}_i^{pd}$	$\bar{u}_i^{sr}$	$\bar{u}_i^{pr}$	u_i	*Rank*
A_1	-0.03	0.004	0.92	2.962	0.726	0.985	0.470	0.003	0.546	5
A_2	0.23	0.018	1.82	22.402	0.921	0.999	0.688	0.020	0.657	4
A_3	0.34	0.019	2.90	191.015	1.000	1.000	0.952	0.162	0.779	2
A_4	**0.34**	**0.013**	**3.09**	**1181.118**	**1.000**	**0.994**	**1.000**	**1.000**	**0.998**	**1**
A_5	0.30	0.000	2.89	0.000	0.973	0.982	0.949	0.001	0.726	3
max	0.34	0.019	3.09	1181.118						

4.3.5. Evaluation of separation densities by using the CoCoSo method

The details of the calculations obtained by applying the CoCoSo method, i.e., using Eqs. (28)–(34), are summarized in Table 14. CoCoSo method has the following ranking orders of alternatives: $A_3 > A_2 > A_4 > A_5 > A_1$. As it can be observed from Table 14, the alternative denoted as A3 is also more acceptable.

Table 14. Ranking order of alternatives obtained using the CoCoSo method

	k_{ia}	k_{ib}	k_{ic}	k_i	*Rank*
A_1	0.078	2.000	0.309	1.159	5
A_2	0.246	5.663	0.982	3.408	2
A_3	**0.251**	**5.912**	**1.000**	**3.528**	**1**
A_4	0.242	5.634	0.967	3.378	3
A_5	0.183	4.453	0.730	2.630	4

4.3.6. Comparative analysis

The selection alternatives of five MCDM methods such as: SAW, TOPSIS, WASPAS, WISP and CoCoSo were compared. The results of the previous calculations are summarized in Table 15.

Table 15. Summary results of calculations

	Values					*Ranks*				
	SAW	TOPSIS	WASPAS	WISP	CoCoSo	SAW	TOPSIS	WASPAS	WISP	CoCoSo
	S_i	C_i	Q_i	u_i	k_i					
A_1	0.472	0.212	0.418	0.546	1.049	5	5	5	5	5
A_2	0.529	0.438	0.574	0.657	4.232	4	4	4	4	3
A_3	0.617	0.749	0.679	0.779	**4.563**	3	3	3	2	**1**
A_4	0.749	**0.776**	0.770	**0.998**	4.391	2	**1**	2	**1**	2
A_5	**0.920**	0.760	**0.833**	0.726	3.645	**1**	2	**1**	3	4

Table 15 shows that alternative A_5 is ranked best, that is, it appears twice in first position (SAW and WASPAS), but after TOPSIS, WISP, and CoCoSo, it is ranked second, third and fourth, respectively. Alternative A_4 also appears three times in second position, but after the TOPISIS and WISP methods it is ranked best. Alternative A_3 is the ranked best according to the CoCoSo method, but according to the other methods it is ranked third three times and second once.

Table 15 also shows that the differences between the first three alternatives in the coefficients (S_i, C_i, Q_i, u_i, and k_i) are very small for all five methods. However, the final ranking of alternatives, i.e., the optimal separation density, could not be chosen based on performed analyses, as almost all applied MCDM methods resulted in different ranking of the alternatives. Therefore, the additional analysis was performed.

Based on the theory of dominance, used in the MULTIMOORA method to select the most suitable alternative, it can be concluded that alternatives A_5 and A_4 are the most acceptable. To establish the final ranking of the alternatives, it is proposed to recalculate the utility of the alternatives $\overline{U}_i$, as the mean of the values of S_i, C_i, Q_i, u_i, and k_i as follows:

$$\overline{U}_i = \frac{1}{5}\left(\bar{S}_i + \bar{C}_i + \bar{Q}_i + \bar{u}_i + \bar{k}_i\right) = \frac{1}{5}\left(\frac{S_i}{max_i\, S_i} + \frac{C_i}{max_i\, C_i} + \frac{Q_i}{max_i\, Q_i} + \frac{u_i}{max_i\, u_i} + \frac{k_i}{max_i\, k_i}\right), \quad (35)$$

where: u_i^w denote relative importance i obtained using WISP method and u_i^{ws} denote the degree of utility of alternative i obtained using WISP-S method.

As previously mentioned, different MCDM methods use different normalization and aggregation procedures that are applied to select the optimal solution, i.e. the most appropriate alternative, which in some cases can cause differences in the ranks of the most appropriate alternative, obtained by applying different MCDM methods. In some cases, this problem can be solved by slightly varying the weight of some criteria, but this is not the primary goal of this research. Therefore, a compromise model that should harmonize possible disagreements in the selection of the most suitable alternative that can be achieved in some cases by applying different MCDM methods is proposed using Eq. (35). The details of the calculations obtained by Eq. (35) are presented in Table 16.

Fig. 8 presents ranking of the alternatives obtained by SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods and the average rank (final rank).

Table 16. The final ranking of alternatives

	SAW	TOPSIS	WASPAS	WISP	CoCoSo		
	$\bar{S}_i$	$\bar{C}_i$	$\bar{Q}_i$	u_i	k_i	$\bar{U}_i$	*Rank*
A_1	0.513	0.273	0.502	0.547	0.230	0.413	5
A_2	0.575	0.564	0.689	0.658	0.927	0.683	4
A_3	0.671	0.966	0.815	0.780	**1.000**	0.846	3
A_4	0.815	**1.000**	0.925	**1.000**	0.962	**0.940**	**1**
A_5	**1.000**	0.979	**1.000**	0.727	0.799	0.901	2

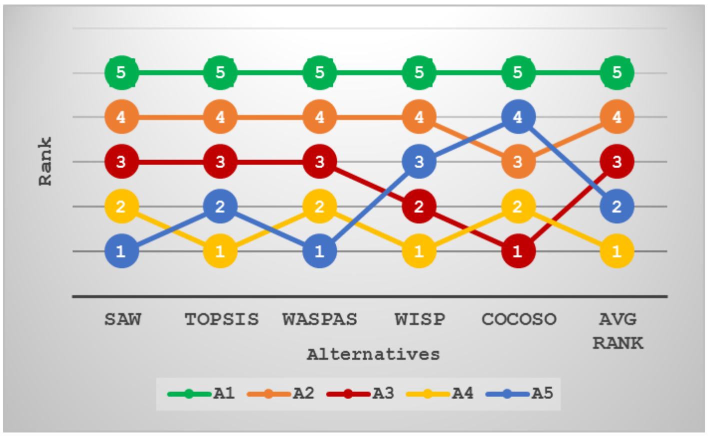

Fig. 8. Ranks of the alternatives

Finally, the evaluation indexes of the five methods were averaged to a new index and A4 (i.e., the optimal separation density is 1650 kg/m³) was selected as the final separation density. As it can be seen from Fig. 8, alternative A_5, which represents a separation density of 1750 kg/m³ was the second best.

The final ranking of the alternatives was as follows: $A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$. These results are consistent with the results of the study by Soković et al. (2023), which shows that the separation density (1650 kg/m³) is the optimum density for washing anthracite coal of wider grain size fractions.

From the detailed analysis, it can be concluded that the obtained results fully justify the application of MCDM methods for the choice of separation density in the gravity separation of coal based on different washability indices. Thus, it can be used as an efficient multi-objective optimization tool for solving parametric optimization problems in gravity separation.

5. Conclusions

Gravity separation of coal is usually used to process the coarse fractions. It is a very complex process that involves various input and output parameters. The separation density plays an important role in the efficiency of the gravity process, so the selection of the optimal separation density is crucial in the gravity separation of coal.

In this paper, the Multi-Criteria Decision Making (MCDM) methods were used to determine the optimal separation density in the gravity separation of coal. The study was conducted on anthracite coal, which is processed in the Vrska Cuka coal mine in Serbia.

The SAW, TOPSIS, WASPAS, WISP, and CoCoSo methods were used in this study. Based on the washability data obtained from the Float-Sink test, five separation densities values namely: 1350 kg/m^3 (A1), 1450 kg/m^3 (A2), 1550 kg/m^3 (A3), 1650 kg/m^3 (A4), and 1750 kg/m^3 (A5) were selected as alternatives (output). Seven indices such as Degree of washing (N), Washability number (W), Near-Gravity Material (NGM), Near-Gravity Material Index (NGMI), Degree of Separability (DS), the Clean coal yield (Rm), and Ash content of the clean coal (Ash) were considered as criteria (input). Based on the value of the new index for the ranking of alternatives, alternative A4 (i.e., the optimal separation density is 1650 kg/m3) was selected as the final separation density.

This paper has shown that MCDM methods are applicable for the selection of separation density based on the comparative analysis on washability indices of anthracite coal. It should be noted that this method represents a new direction of application of MCDM methods in gravity separation of coal. As a further direction for the future development of the proposed approach, this methodology could be applied to other types of coal. According to the obtained results on anthracite coal, it is feasible.

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