Physicochem. Probl. Miner. Process., 54(3), 2018, 847-857

http://www.journalssystem.com/ppmp

Physicochemical Problems of Mineral Processing

ISSN 1643-1049

Received October 16, 2017; reviewed; accepted February 14, 2018

A new approach in separation process evaluation. Efficiency ratio and upgrading curves

Mehdi Irannajad, Omid Salmani Nuri, Ebrahim Allahkarami

Amirkabir University of Technology, Department of Mining & Metallurgical Engineering, 424 Hafez Avenue, 1591634311, Tehran, Iran

Corresponding authors: iranajad@aut.ac.ir (Mehdi Irannajad)

Abstract: In mineral processing separation efficiency (*SE*), operation efficiency (*OE*), selectivity index (*SI*) and other indices have been used to evaluate the separation process. Up to now, no study has been conducted on the relationship between the *SE, OE* and *SI* indices. In this research, two upgrading curves are proposed based on the above indices for process and selectivity evaluation. The first upgrading curve is based on recovery *R, SE,* and *OE* as a function of concentrate grade. This curve has three background lines, including no upgrading line, ideal upgrading line and the ideal mixing line. The proposed upgrading curve is applicable not only for process evaluation by specification of *OE* and *SE*, but also for selectivity evaluation with the lowest difference between *SE* and *OE*. The curve showed that the recovery value is always greater than the *SE* and *OE* values. The parameters of *OE*, *SE* and *R* were used for plotting the upgrading curve as a function of concentrate grade taking into consideration all of them at a time. A new selectivity indicator, namely Efficiency Ratio (*ER*) as the selectivity parameter, is proposed as the ratio of *OE* to *SE*. The *ER* values fluctuate between 1 and ∞. It can be presented as a function of concentrate and tailing grades ($ER = [c\cdot(1-t)]/[1\cdot(c-t)]$). The results showed that *ER* is insensitive to the feed grade and has the inverse relationship with *SI*. To measure the separation selectivity, another upgrading curve is proposed based on *ER* and *SI* parameters. This curve is divided into seven separation classes for evaluation the class of a separation process from ideal class to no separation one. The results of this research can be useful for separation process evaluation.

Keywords: efficiency ratio, upgrading curve, separation efficiency, operation efficiency, mineral processing, process evaluation, separation process

1. Introduction

Mineral processing, regardless of the process type, has always a unique goal, which is to separate the valuable minerals into the concentrate and gangue minerals into the tailing. Separation processes provide products differing in quantity and in most cases in quality (Drzymala, 2007). The quality of a product is expressed by grade, that is the content of a valuable component in the product. The recovery is the qualitative and quantitative parameter of the product in which C and c are the weight and grade of concentrate while respectively F and f are the weight and grade of the feed. Recovery is the percentage of the total mineral or metal contained in the ore, that is recovered in the concentrate. The recovery is given by $R=C\cdot c/F\cdot F$. Drzymala expressed the recovery as ($\varepsilon=\gamma\cdot\beta/\alpha$) where γ is the yield, β is the grade of metal or mineral in the concentrate, and α is the grade of metal or mineral in the feed (Drzymala-I, 2007; Drzymala-II, 2006). The grade and recovery are the most widely accepted process noneconomical indices, which are used in mineral processing to evaluate the process (Wills and Napier-Munn, 2006). So far, many other indices have been suggested by different scholars. Some of them have been characterized by Drzymala (2006, 2007a, 2007b, 2008). It seems that the most applicable indices used for evaluation of mineral processing processes are the separation efficiency *SE*, operation efficiency *OE* and selectivity index *SI*. In this paper a new upgrading curve based on recovery, *OE* and *SE* is proposed. In

http://dx.doi.org/10.5277/ppmp1886

the second part of the paper, a selectivity indicator and new upgrading curve, which is based on this indicator is offered. Furthermore, the applications and background lines of the new upgrading curves as well as the application of new indicator are explained in detail.

1.1 Technological parameters

1.1.1 Separation efficiency (SE)

It was shown that grade and recovery can be combined to form other indices including SE. The SE has been defined by Hancock as the difference between the recovery of valuable mineral to the concentrate and recovery of the gangue mineral to the concentrate (Schulz, 1970; Sztaba, 1993; Kelly and Spottiswood, 1982):

$$SE = (R_{v,C} - R_{g,C}) \cdot 100 \tag{1}$$

$$SE = \left(\frac{C \cdot c}{F \cdot f} - \frac{C \cdot (1-c)}{F \cdot (1-f)}\right) \cdot 100 \tag{2}$$

where $R_{v,C}$, $R_{g,C}$ are recoveries of valuable and gangue minerals into the concentrate and tailing (as a part of unity), respectively. When the recovery and grade of two products are different, the index can be useful, and sometimes has a better application in the selection of the best upgrading process. It is apparent that by increasing $R_{v,C}$ and decreasing $R_{g,C}$, the SE will be increased. The values of SE are between 0 and 100. This index has been frequently used to technically evaluate separation processes.

1.1.2 Efficiency of operation (OE)

OE is the another index which was proposed by Fomienko as shown by Eqs (3) and (4) (Fomienko, 1957; Abouzeid, 1990). This index is expressed as the recovery of valuable minerals into concentrates multiplied by the recovery of gangue minerals into the tailing. This index is somewhat similar to SE except that in this equation the recovery of gangue mineral into the tailing is considered. So, it is necessary to simultaneously measure the grades of feed and concentrate as well as the grade of tailing. It can be seen that the OE value directly depends on the recovery of valuable and gangue minerals into the concentrate and tailing, respectively. The values of OE are between 0 and 100. Thus:

$$OE = (R_{v,C} - R_{g,T}) \cdot 100 \tag{3}$$

$$OE = \frac{C \cdot c}{F \cdot f} \times \frac{T \cdot (1-t)}{F \cdot (1-f)} \cdot 100 \tag{4}$$

where $R_{g,T}$ denotes the recovery of gangue minerals into the tailing as a part of unity.

1.1.3 Metallurgical efficiency (M.E)

ME was proposed by Diamond (1928). The index is defined as the arithmetical average of the recoveries for the main component of each product, even in the tailing (Taggart, 1945; Diamond, 1928). The index given in Eq. 5, where $R_{n,N}$ denotes the recovery of component n in the N product.

$$E = \frac{\sum R_{nN}}{n}. \tag{5}$$

ME is the simplest index for evaluating various processes. For example, in a separation of magnetite, ilmenite and quartz, if the Fe recovery in magnetite concentrate is 90%, the Ti recovery in ilmenite concentrate is 80% and the Si recovery in tailing is 95%, ME is (90+80+95)/3=88.33%. It is noteworthy that this index is not considering the effect of main component losses in other products. In the above example, the grade and recovery of Ti in magnetite and tailing fractions are not taken into account.

1.1.4 Selectivity index (SI)

Gaudin (1939) proposed the selectivity index as the convenient measure of two-product separation. Selectivity index is a geometrical mean of the relative rejections and relative recoveries of two components (minerals, metals, or groups of minerals or metals). The index is given in Eqs (6) and (7) (Gaudin, 1939; Taggart, 1945):

$$SI = \sqrt{\frac{R_{v,C} \cdot R_{g,T}}{(1-R_{v,C}) \cdot (1-R_{g,T})}} \tag{6}$$

$$SI = \sqrt{\frac{c\cdot(1-t)}{t\cdot(1-c)}}. \tag{7}$$

The *SI* is used by some scholars to show how the separation of minerals by flotation was successful (Irannajad, 2014; Irannajad and Mehdilo, 2016; Salmani Nuri et al., 2016). Until now, no study has been conducted on the relationship between the three important indices. In this regard, the aim of this work is to define a new indicator relating *OE* to *SE* and *SI*. This relation is applicable not only to simultaneously evaluate the quality of a separation process for different *SE* and *OE* values, but also to determine the class of separation.

1.2 Economical parameter (*NSR*)

The evaluation of process economics is of great importance in mineral processing, because the grade-recovery relationship is a key factor in determining the best combination of grade and recovery as a point of process economics. However, in mineral processing the aim is to attain the highest financial return per ton (megagram) of a processed ore in the plant. One of the proposed parameters for evaluating the economics of mining and milling operations is the net return from the smelter (*NSR*), which can be considered as the difference between income (payment for metal content) and costs (smelter charges and transportation costs). It is obvious that the recovery and grade of concentrate play the vital role in determining the *NSR* in which the more the metal grade in concentrate the less the smelter charges and transportation costs. However, the variations in metal price result in changing the *NSR* value versus concentrate-grade relationship.

2. Comparison of *SE* and *OE*

According to the definition, *SE* is the difference between the recovery of valuable mineral to the concentrate and recovery of the gangue mineral to the concentrate. The *OE* is the recovery of valuable mineral to the concentrate multiplied the recovery of gangue mineral to the tailing. By replacing $R_{g,T}$ = 1- $R_{g,C}$ in relation (3) and comparing with Eq. (1), it can be found that the *OE* is greater than *SE*. Since the subtracted value in relation with *OE* (i.e. $R_{v,C} \times R_{g,C}$) is smaller than that part in *SE* (i.e. $R_{g,C}$), it is acceptable that the value of the *OE* should be greater than *SE*. It should be pointed out that separation diagrams and mathematical relations are used for assessment of separation results. Figure 1 shows the relationship between *SE* and *OE* via recovery-time curve.

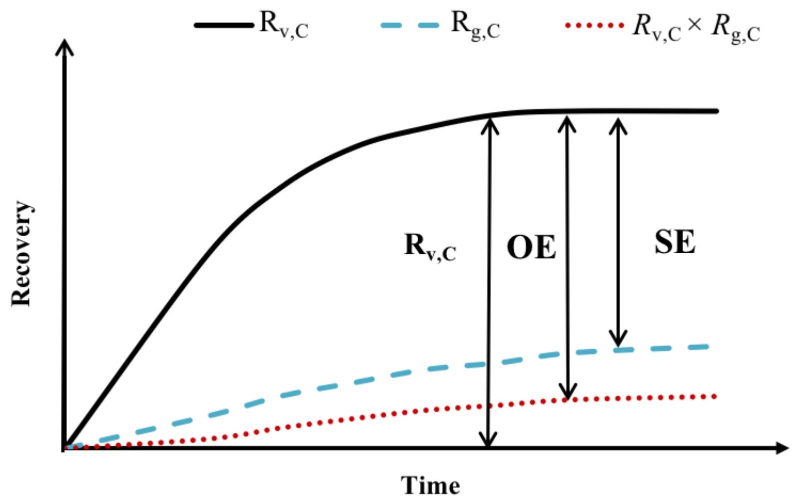

Fig. 1. Relationship between *SE* and *OE* with recovery-time curve

To clarify the issue, an example of process parameters for flotation is presented in Table 1. This process consists of four flotation stages concentrating galena with the feed grade of 8.968% Pb (Fig. 2). Using the recovery and grade of each concentrate, the *SE*, *OE* and *SI* values for each stage were calculated from the aforementioned relations. It can be found from Table 1, that the *OE* values of each process are greater than the *SE* ones.

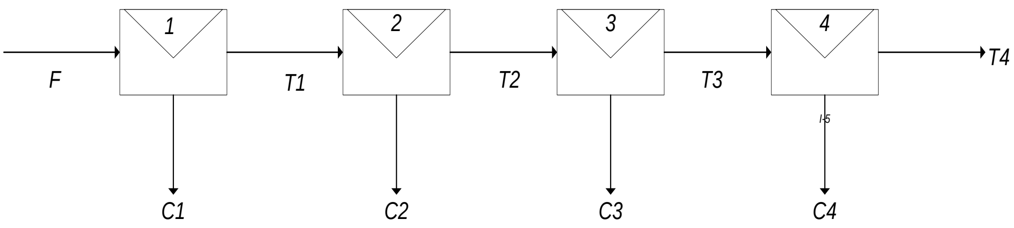

Fig. 2. Flowsheet of galena flotation process

Table 1. Process parameters for galena flotation

Stage	Concentrate (%)			Tailing (%)			SE	OE
	W	G	$R_{v,C}$	W	G	$R_{g,C}$		
1	6.398	85.46	60.97	93.602	3.739	1.02	59.95	60.35
2	4.238	43.41	52.57	89.364	1.858	2.66	49.99	51.16
3	4.958	15.78	47.12	84.406	1.04	4.763	42.35	44.87
4	8.038	5.02	45.96	76.36	0.62	9.14	36.82	41.76

3. Determining the best process condition

3.1 Mathematical evaluation

To technically determine the best process condition with respect to relations (1) and (3), *SE* and *OE* should be simultaneously maximized and equal. From the equality of *SE* and *OE*, and replacing $R_{g,T} = 1 - R_{g,C}$, one can obtained the $R_{v,C} - R_{g,C} = R_{v,C} \cdot (1 - R_{g,C})$ term. After simplification, the term $R_{g,C} \cdot (1 - R_{v,C}) = 0$ is obtained. Thus, for reaching the best process condition, one of two conditions including $R_{g,C} = 0$ or $R_{v,C} = 1$ should be satisfied. Under these circumstances, *SE* and *OE* will be equal to each other as well as $R_{v,C}$. It can be concluded that under condition of $R_{g,C} = 0$, the values of concentrate grade and tailing grade are 1 and 0, respectively. So, the $R_{v,C}$, *OE* and *SE* values will be 1. When $R_{v,C} = 1$, the values of concentrate and tailing grades are equal to the feed grade and 0, respectively. In this case, the value of $R_{g,C} = 1$, and the *OE* and *SE* ones will be 0. The summary of these conditions is presented in Table 2.

Table 2. Values of process performance under *SE* = OE conditions

Condition	$R_{v,C}$ (%)	$R_{g,C}$ (%)	OE (%)	SE (%)	Description
c=100; t=0	$C{\cdot}100/F{\cdot}f$	0	$C{\cdot}100/F{\cdot}f$	$C{\cdot}100/F{\cdot}f$	Ideal upgrading
c=f; t=0	100	100	0	0	No upgrading
c=f=t	100	100	0	0	Ideal mixing

3.2 New upgrading curve based on recovery, *SE* and *OE*

In order to find the optimum condition in no economical terms but process performance, graphical analysis is one of the best methods for process evaluation. Different upgrading curves are available for evaluation of separation processes. All of them offer the same information but in different geometrical forms. Drzymala reviewed varous upgrading curves and classified them into three categories: A (feed grade-insensitive), B (feed grade-sensitive), and C (feed grade-insensitive but covering a limited range of variables). The Hancock upgrading curve belongs to the B (feed grade-sensitive) category which presents the *SE* values as a function of yield (Drzymala-I, 2007; Drzymala-II, 2006). The Fomienko curve belongs to the A (feed grade-insensitive) category which is plotted in two forms: *OE* as a function of recovery and *OE* as a function of *SE* (Fomienko, 1957). One of the useful upgrading curves, which is well accepted for evaluation of mineral processing systems, is the Halbich curve. The curve belongs to the B category (feed grade-sensitive) (Drzymala, 2006, 2007). However, these upgrading curves separately present useful results based on the needs of the users. The proposed upgrading curve has the same features as the Halbich curve that belongs to the B category (feed grade -sensitive) with the square area for plotting. The upgrading curve has three key regions, which are the same as in the case of the Halbich curve. This curve simultaneously presents three parameters: recovery, *OE* and *SE* as a function of concentrate grade. Plotting the recovery, *OE*, and *SE* curves as a function of concentrate grade is useful to compare different processes as a point of metallurgical efficiency. It is clear that the *OE* values are greater than *SE* ones, and also their values are below the recovery at the whole range of grades (i.e. $R_{v,C} > OE > SE$). So, these curves do not cross each other except the no upgrading and ideal upgrading lines. One of the advantages of this curve is determination of the best process between several ones. Hence, the process which has high *OE* or *SE* values is selected as the best one. It can be also found that for other points of the recovery curve, the difference between the recovery and *SE* curves is specified by the $R_{g,C}$ values. Also, the difference between the recovery and *OE* curves is specified by $R_{g,C} \cdot R_{v,C}$ values. It means that for fixed values of $R_{v,C}$, decreasing the $R_{g,C}$ value resultes in increasing the *OE* and *SE* values. To select the best process among two processes, it is necessary that both the *SE* and *OE* indices have the highest values. In Fig. 3 the background line of no upgrading corresponds to the feed grade when there is no separation. This line represents $R_{v,C}$=100% and $R_{g,C}$ = 0% as well as *SE*= 0 and *OE*=0. When the separation process begins, the recovery of valuable minerals is gradually decreasing and the concentrate grade and recovery of gangue minerals in the concentrate is simultaneously increased. Consequently, the *OE* and *SE* values go up and continue until the best process condition is achieved. The ideal upgrading line is located in the last region of this plot in which the grade of concentrate is at maximum. When the valuable component is a mineral, then grade is equal to 100%. This line presents the value of $R_{g,C}$=0% and the values of $R_{v,C}=SE=OE=100C/F\cdot f$. Under ideal mixing condition the value of $R_{v,C}$ and $R_{g,C}$ is 100 (the top horizontal line in Fig. 3) as well as *SE*= 0 and *OE*=0 (the bottom horizontal line in Fig. 3).

As seen from Fig. 3, the recovery-grade curve can be divided into three regions. In the first region, the recovery of valuable mineral was high due to the presence of gangue minerals in the concentrate. This phenomenon was attributed to the relationship between the liberation degree and the recovery (Drzymala et al., 2013). The most industrial processes in mineral processing are located in the central region. The break point (the maximum curvature of separation curve) is an important point because industrial separation processes work above that point. This point can be also specified on the *SE* and *OE* curves. In the third region, a rich concentrate with low recovery is obtained. In this region the valuable minerals are liberated from gangue minerals and the separation process is selectively performed. So, the $R_{g,C}$ value is decreasing until reaching the minimum value.

Different parameters affect the shifting of the grade-recovery curve as well as *SE* and *OE* including operational (chemical reagent, pH, time, solid percentage, and etc.), feed characteristics (grade and liberation degree), and machinery design. Fixing other variables is necessary to obtain changes of the recovery, *OE* and *SE* curves as a function of each variable. For instance, Bradshaw (2014) investigated the effect of liberation degree on the recovery-grade curve and found that the presence of gangue minerals in the concentrate for any reason (entrainment, entrapment, activated gangue mineral, and etc.) resulted in dilution and down-shifting the recovery-grade curve. In other cases, the locked minerals, fine liberated minerals, and surface coatings on valuable minerals can result in losing the valuable mineral as well as decreasing the grade of concentrate. Then, the recovery-grade curve is shifted to the left side. By changing the location of the recovery-grade curve, the difference between recovery curve and both of *OE* and *SE* will change. It means that the dilution and losses phenomena lead to the increase of the difference between recovery, *SE* and *OE* curves. On the other hand, optimizing the process, by controlling the dilution and losses phenomena, leads to a decrease of the difference between recovery, *OE* and *SE* curves.

4. New selectivity index (efficiency ratio)

The new proposed index has been defined as the ratio of operation efficiency to separation efficiency, that is

$$ER = \frac{OE}{SE}. \tag{8}$$

The *ER* value is always greater than 1, except under ideal conditions for which is equal to one. This simplification is useful for graphical determination and presentation of *SI* as a function of *ER* by replacing Eqs (2) and (4) in relation (5) and simplification of the terms. Then, the dependency of efficiency ratio and the grades of concentrate and tailing is obtained as:

$$ER = \frac{c\cdot(1-t)}{1\cdot(c-t)}. \tag{9}$$

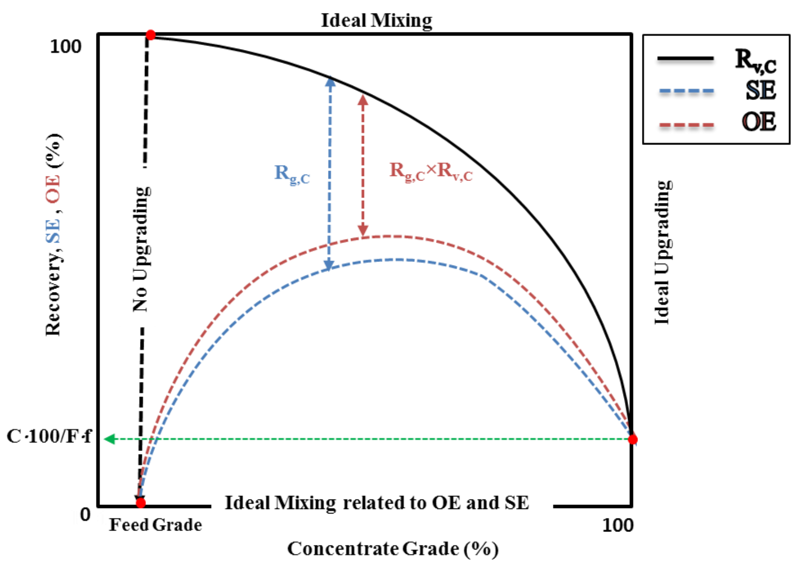

Fig. 3. New upgrading curve based on R, SE and OE as a function of concentrate grade

4.1 Selectivity index vs. *ER*, *OE*, and *SE*

Selectivity index can also be written as in Eq. (10):

$$SI = \sqrt{\frac{ER}{ER-1}}. \tag{10}$$

By placing Eq. (8) in Eq. (10) and simplification of the terms, the selectivity index is obtained as the relation:

$$SI = \sqrt{\frac{OE}{OE-SE}}. \tag{11}$$

By using a back calculation and placing Eqs. (2) and (4) in (11), validity of Eq. (7) is proved because $SI = \sqrt{\frac{OE}{OE-SE}} = \sqrt{\frac{c\cdot(1-t)}{t\cdot(1-c)}}$).

Basing on Eq. 11 it can be concluded from that the *SI* index is simultaneously related to both *SE* and *OE* indices. Thus, the index can be used to select the most accurate measures. Also, if the two indices are specified, the relation can be used to directly determine the third index. With respect to the validation of Eq. (11), it can be seen that the *OE* value should be greater than the *SE* one. This relation is in a good agreement with the result presented in sections 2 and 3.

4.2 Interpretation and application of *ER* and *SI* indices

If the results of two processes show different values of *SE* and *OE* (higher *SE* and *OE* than those of another process or vice versa), the choice is not obvious. Then, the *SI* and *ER* parameters can be used for evaluation of process selectivity. Sometimes, both *SE* and *OE* of one (C) process are greater than another (D). From the standpoint of selectivity of separation process C should not be selected as more selective process. In this case, the process that has the smallest value of *ER* should be selected as more efficient. So, to get the best value in terms of selective separation, the necessary condition (not sufficient) is that both *SE* and *OE* should be as high as possible, and the sufficient condition is that the *ER* value simultaneously should be the lowest. For example, Table 3 presents the results of upgrading by flotation of an iron ore.

Table 3. Upgrading parameters of the considered flotation processes

Process	f (%)	c (%)	t (%)	$R_{v,C}$ (%)	$R_{g,C}$ (%)	$R_{g,T}$ (%)	SE (%)	OE (%)	SI	ER
A	-	f=t	-	50	50	50	0	25	1	∞
B	4.455	25.69	0.83	84.018	11.335	88.664	72.683	74.495	6.412	1.02492
C	2.529	23.98	0.31	88.691	7.302	92.697	81.388	82.214	9.977	1.01014
D	2.004	24.05	0.29	86.546	5.591	94.408	80.955	81.707	10.421	1.00929
E	1.852	24.16	0.24	87.901	5.210	94.789	82.691	83.322	11.497	1.00762
F	-	100	-	100	0	100	100	100	∞	1

According to Table 3, processes A and F exhibit no separation and ideal separation, respectively. The *SI* and ER values of process A is 1 and ∞, respectively. On the other hand, the grade of concentrate of process F is equal to 100, and the *SI* and *ER* values are ∞ and 1, respectively. Among of the B, C, D and E processes, process B is the worst due to low values of *OE*, *SE*, and *SI*. In the case of process E, the *SI* and *ER* values are the greatest and the lowest, respectively. This process can be selected as selective. It is noteworthy that the *SE* and *OE* values of process C are slightly better than process D, but the *SI* value of process C is lower than that of D. So, the selective process is the one with the lowest value of *ER*. Thus, the importance of *ER* value becomes clear. The best separation process in terms of *SI* and *ER* is shown in Figs. 4 and 5.

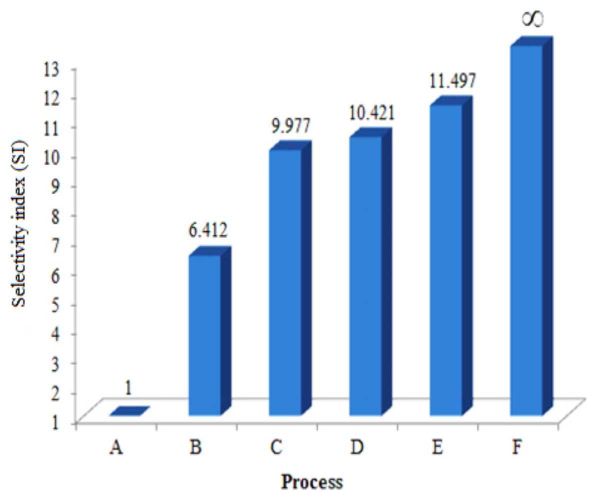

Fig. 4. Comparison of upgrading process by means of selectivity index

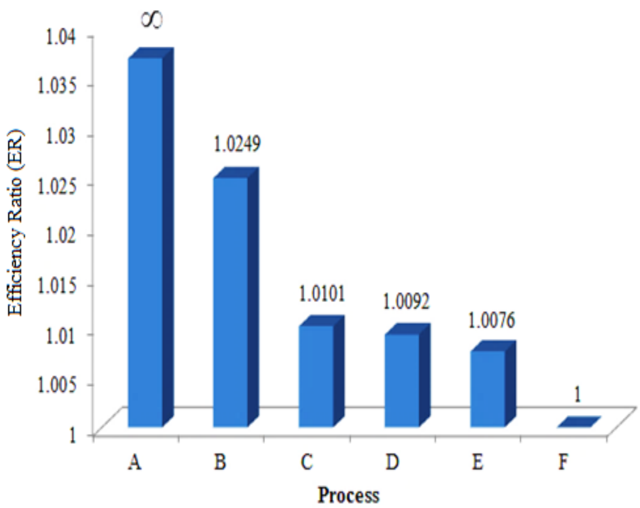

Fig. 5. Comparison of upgrading process by means of efficiency ratio

Table 4. Process, selectivity and economic parameters of a tin upgrading process

Grade	Process parameters								Selectivity parameters		Economic parameter
	$R_{v,C}$	c	t	C	T	$R_{g,C}$	SE	OE	ER	SI	NSR
High	62	63	0.3838	0.0098	0.9902	0.3678	61.6322	61.7720	1.0023	21.0230	47.88
Medium	72	42	0.2849	0.0171	0.9829	1.0043	70.9957	71.2769	1.0040	15.9205	52.8
Low	78	21	0.2285	0.0371	0.9629	2.9639	75.0361	75.6881	1.0087	10.7738	48.1

The process and economic parameters of the concentrating process of tin are presented in Table 4. The parameters of recovery and grade of concentrate and *NSR* are adapted from Wills and Finch (2016). The parameters of *SE*, *OE*, *ER* and *SI* are calculated based on the aforementioned relations. As shown in this table, the process, selectivity and economic parameters are different:

I) *SE*, *OE* and *ER*: low grade > medium grade > high grade

II) *SI*: high grade > medium grade > low grade

III) *NSR*: medium grade > low Grade > high grade.

Therefore, the selection of parameters is highly depending on the aim of the separation process. When the goal is to obtain a maximum economic return, the *NSR* can be used to choose the best process. In some cases, the selection of a process among two or more processes with different concentrates grade and recovery is vague. Hereon, the *SE* and *OE* parameters can be used to choose the best one. Finally, if the goal is to determine the separation quality of two minerals from each other's (selectivity) and to achieve a high-grade product via different separation process methods, different reagent types and etc., the *ER* factor can be used to choose the best process. In other words, if the process objective is the elimination of the harmful elements in the concentrate (for example, arsenic in the lead and zinc concentrates or sulfur in the iron concentrate) and also no valuable components in the tailing (for example, gold and silver in tailing), the *ER* is the best parameter that can be used to select the separation method, machinery type, chemical reagent type and etc. in terms of selectivity aspect.

5. New upgrading curve based on *ER* and *SI*

In this section, the different classes of separation are proposed with respect to *SI* and *ER*. As can be seen in Table 5, the efficiency ratio of the separation process is divided into seven sub-levels. The *ER* index is the ratio of *OE* and *SE* indices and they are local selectivity parameters. These parameters usually change in a different manner for different separation processes. As presented in Table 2, these parameters change with the yield of the concentrate. Therefore, the *ER* index evaluates a process as a local selectivity parameter and it cannot be, in most cases, used for comparison of separation data. However, to overcome this limitation, the authors presented the upgrading curve based on *ER* and *SI*. In other word, this curve can be used for comparison of different separation processes. Thus, the meaningful upgrading curve, plotted with the background lines, can be used as a global selectivity measure to compare different separation processes.

The efficiency ratio curve can be used not only for assessing the upgrading results of different processes, but also for comparing the quality of the separation process. Figure 6 presents the *ER* curve versus the *SI* value under various conditions of separation. It is apparent that the real separation curve has two asymptotic curves, that is the vertical and horizontal asymptotes *ER*=1 and *SI*=1, respectively. The background lines (i.e. no and ideal separation lines) are the important parts of upgrading curves for meaningful and simple use. Under operational conditions, the range of *SI* is usually between 4 and 40 (Taggart, 1945). With respect to these values, the $ER = \frac{OE}{SE}$ values corresponding to the operational *SI* were calculated from Eq. (10). The *ER* values are fluctuating from 1.066666 to 1.000625. This part of curve is shown as the operational condition. The part with *SI* greater than 40 and 1 < *ER* < 1.000625 is best. Eventually, the process with *ER* greater than 1.06666 and 1 < *SI* < 4 is the worst.

The advantages of this upgrading curve over other ones include simple use, interpretation, and classification of separation processes into seven sub-levels based on selectivity. Also, a comparison of several process in terms of separation method, machinery type, chemical reagent type etc., can be made, because this curve is insensitive to feed grade ($ER = \frac{c(1-t)}{1(c-t)}$).

Table 5. Relative scale for separation class of a process based on efficiency ratio (*ER*) and selectivity index (*SI*)

Separation region	Intervals		Class of separation
	Selectivity index (SI)	Efficiency Ratio (ER)	
I	1	∞	Lack of separation
II	4-8	1.06666 - 1.01587	Negligible separation
III	8-16	1.01587 - 1.00392	Weak separation
IV	16-24	1.00392 - 1.00174	Medium separation
V	24-32	1.00174 - 1.00098	Good separation
VI	32-40	1.00098 - 1.00062	Very good separation
VII	∞	1	Ideal separation

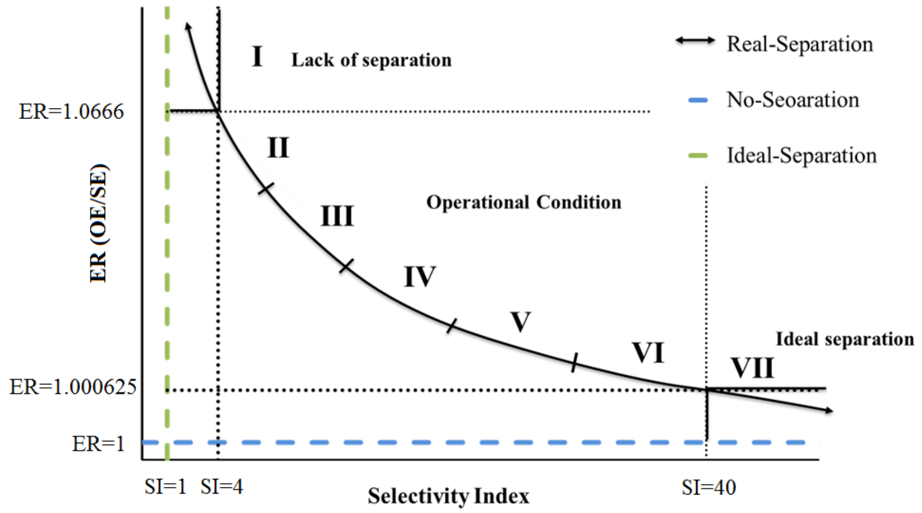

Fig. 6. New upgrading curve based on efficiency ratio (*ER*) as a function of selectivity index (*SI*)

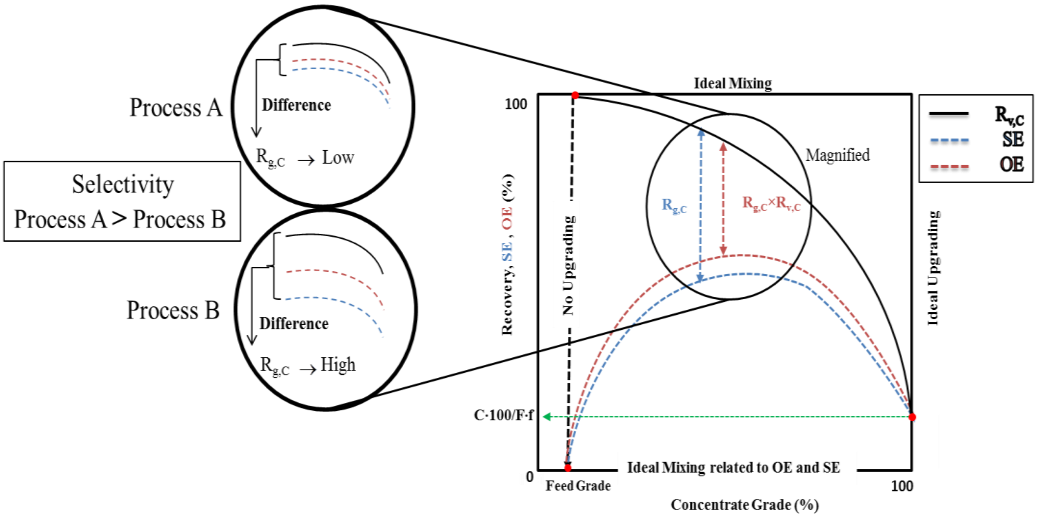

Fig 7. Determination of process selectivity via new upgrading curve and *ER* index

It is noteworthy that the selectivity upgrading curve can be related to Fig. 3. Since, *ER* is defined as the ratio of *OE* to *SE*, therefore, specifying *OE* and *SE* from Fig. 3 leads to determination of classes of separation selectivity by the upgrading curve ($ER = OE/SE$). Finally, this upgrading curve helps users to determine process selectivity based on process parameters. The importance of *ER* application is clearer when users and engineers want to simultaneously determine the selectivity of process via two, *SE* and *OE*, parameters. *ER* will lead to the selectivity of process without spending time and without using the complicated formula of *SI* ($SI = \sqrt{\frac{R_{v,C} \cdot R_{g,T}}{(1-R_{v,C}) \cdot (1-R_{g,T})}}$).

As mentioned in section 3.2, the difference between *R* and *SE* is $R_{g,C}$ and the difference between *R* and *OE* is $R_{v,C} \cdot R_{g,C}$. To select the best selective process among several others, it is necessary to minimize as much as possible the difference between the *OE*, *SE*, and *R*. It can be found from Fig. 7, that the selective process is a process with the smallest recovery of gangue minerals in the concentrate $R_{g,C}$. To sum up, Fig. 3 not only can be used to determine the best process, but also it can be used to determine the best selective process.

6. Conclusions

In this study, an attempt was carried out to define a new selectivity index based on process parameters such as *SE* and *OE*. The efficiency ratio was defined as the ratio of operation efficiency to separation efficiency: $SI = \sqrt{\frac{OE}{OE-SE}}$. Also, two new upgrading curves were proposed to evaluate a separation process in terms of process and selectivity aspects. The results of this research are presented as follows.

1. A new upgrading curve was proposed based on the curves of recovery, *SE*, and *OE* as a function of concentrate grade. This curve has four background lines, including no upgrading, ideal upgrading and two lines of ideal mixing. The curve belongs to the B category (feed grade sensitive) with the square area available for plotting.
2. Mathematical and graphical analysis showed that the *OE* values are greater than *SE* ones, and also their values are lower than the recovery at the whole range of grade (i.e. $R_{v,C} > OE > SE$).
3. The new index of efficiency ratio *ER* was defined as a ratio of *OE* to *SE*. The index combines two important process parameters.
4. Contrary to selectivity index *SI*, the *ER* can be conveniently used to evaluate and choose the upgrading process as a point of separation selectivity. In this regard, the separation method, machinery type, chemical reagent type etc. can be selected by this index in terms of selectivity.
5. The quality of separation selectivity can be directly determined via efficiency ratio *ER*. $ER = OE/SE$, $ER = \frac{c(1-t)}{1(c-t)}$ and $ER = \frac{SI^2}{SI^2-1}$
6. The new upgrading curve and efficiency ratio *ER* values as a function of *SI* help users to determine the separation class in terms of selectivity.
7. Selectivity of a separation process is divided into seven sub-levels via the efficiency ratio from the lack of separation to ideal separation.

References

DRZYMALA J., 2007. *Mineral Processing, Foundations of theory and practice of minerallurgy*. 1st English edition. Wroclaw University of Technology.

DRZYMALA J., 2006. *Atlas of Upgrading Curves Used in Separation and Mineral Science and Technology*. Part I. Physicochemical Problems of Mineral Processing, 40, 19-29.

WILLS B.A. and Napier-Munn, T.j., 2006, *Mineral Processing Technology*. 7th edition. Elsevier Science & Technology Books.

DRZYMALA J., 2007. *Atlas of Upgrading Curves Used in Separation and Mineral Science and Technology*. Part II. Physicochemical Problems of Mineral Processing, 41, 27-35.

DRZYMALA J., 2008. *Atlas of Upgrading Curves Used in Separation and Mineral Science and Technology*. Part III. Physicochemical Problems of Mineral Processing, 42, 75-84.

SCHULZ N.F., 1970. *Separation efficiency*. Trans. Soc. Min.Eng. AIME, 247, 81–87.

SZTABA K., 1993. *Przesiewanie (Sieving),* Śląskie Wydawnictwo Techniczne, Katowice.

KELLY E.G. and SPOTTISWOOD D.J., 1982, *Introduction to mineral processing*. New York : Wiley.

TAGGART A.F., 1945. *Handbook of Mineral Dressing: Ores and Industrial Minerals*. 1st edition. John Wiley & Sons Inc.

FOMENKO T. G., 1957. *Determination of optimal indices of upgrading*, USSR Magadanskij NII 1, chapter IV, Upgrading and metallurgy, 24, Severostoc-zoloto. (in Russian).

ABOUZEID A.Z.M., 1990. *Mineral Processing Laboratory Manual*. Volume 9. Series on mining engineering. Trans Tech Publications.

DIAMOND R.W., 1928. *Ore concentration practice of the Consolidated Mining and Smelting Co. of Canada*, Ltd, Trans. A.I.M.M.E, 79, 95-106.

GAUDIN A.M., 1939. *Principles of Mineral Dressing*. First Edition edition. McGraw-Hill Inc. US.

IRANNAJAD M., MEHDILO A. and SALMANI NURI O., 2014. *Influence of microwave irradiation on ilmenite flotation behavior in the presence of different gangue minerals*, Separation and Purification Technology, 132, 401-412.

IRANNAJAD M. and MEHDILO A., 2016. *Comparison of microwave irradiation and oxidation roasting as pretreatment methods for modification of ilmenite physicochemical properties*. Journal of Industrial and Engineering Chemistry, 33, 59–72.

SALMANI NURI O., ALLAHKARAMI E., IRANNAJAD M. and ABDOLLAHZADEH A., 2016. *Estimation of selectivity index and separation efficiency of copper flotation process using ANN model*. Geosystem Engineering, 20(1), 41-50.

DRZYMALA J., KOWALCZUK P. B., OTENG-PEPRAH M., FOSZCZ D., MUSZER A., HENC T and LUSZCZKIEWICZ A., 2013. *Application of the grade-recovery curve in the batch flotation of Polish copper ore*. Minerals Engineering, 49, 17-23.

BRADSHAW, D., 2014. *The role of process mineralogy in improving the process performance of complex sulphide ores*. In: Proceedings of the XXVII International Mineral Processing Congress, pp. 1-23.

WILLS B.A., FINCH J., 2016. *Mineral Processing Technology*. 8th edition. Elsevier Science & Technology Books. 12.