A sedimentation model for particulate suspensions in liquid–solid fluidized beds with inclined channels

Yanfeng Li 1, Ningbo Li 1, Xiangqian Qi 1, Wenjun Zhang 2, Rongtao Zhu 1

1 Key Laboratory of Coal Processing and Efficient Utilization of Ministry of Education, School of Chemical Engineering and Technology, China University of Mining and Technology, Xuzhou, Jiangsu 221116, People’s Republic of China
2 Key Laboratory of Coal-based CO2 Capture and Geological Storage, Jiangsu Province, China University of Mining and Technology, Xuzhou, Jiangsu 221116, People’s Republic of China

Corresponding authors: lyf3344@126.com (Yanfeng Li)

Abstract: The motion characteristics of mineral particles in a modified fluidized bed (mFB) with inclined plates have been studied both theoretically and experimentally. A particulate sedimentation model is built on the fluidization superficial velocity, terminal settling velocity, the device dimensions, the particle properties, and the volume fraction of the particulate suspensions in the inclined channel, which is to describe the motion behavior of particles in mono-disperse suspensions. The experimental particles are a mixture of silica and sand particles with the sizes in the range of 425 - 710 μm and 710 - 880 μm, respectively. Further, the model is extended to describe bi-disperse suspensions. The experimental system is established to be consistent with the theoretical arrangements, aiming to provide more accurate measurements. Specifically, the prediction results are in good agreement with the experimental data with the absolute deviation less than 11%. The results showed that the average solid volume fraction in the inclined channel fluctuates slightly for a given total solid inventory. The theoretical model is of certain practical significance for applications of this system to the classification, separation, and desliming of minerals.

Keywords: inclined channel, fluidized bed, inclined sedimentation, suspension, fluidization

1. Introduction

For the processing of coal and minerals, an inclined liquid–solid fluidized bed has been widely applied for classification and separation. The modified fluidized bed (mFB)—a relatively new system—is equipped with parallel inclined plates above a conventional liquid–solid fluidized bed. Considerable amount of efforts has been devoted to study the effects of inclined channels on the separation in a liquid fluidized bed. Unlike a conventional liquid–solid fluidized bed, it is found that mFB can achieve a wide range of concentrated suspensions at a certain fluidization velocity for the water in the inclined channels (Galvin and Nguyentranlam, 2002). Further, Galvin established a kinematic theoretical model based on the theories of Ponder and Nakamura and Kuroda to explain the maximum distance of mono-disperse particulate suspension (Ponder, 1926; Nakamura and Kuroda, 1937). On the basis of the previous studies, Doroodchi et al. (2004) developed the sedimentation theoretical model of Galvin and Nguyentranlam (2002) to describe the steady-state interaction between the fluidization of the particulate suspensions within the vertical channel and its following segregation within the inclined channels. The suppression of the effects of the particle size within the closely spaced inclined channels and the principles of gravity separation for the mFB at full scale were examined (Galvin, Zhou and Walton, 2010; Galvin et al., 2005). The continuity equation of the solid particles on the basis of the segregation of particles on an inclined channel was derived (Davis and Gecol, 1996). Iverson et al. (2014) researched the positive effects on the desliming of fine coal with particles ranging from 0.07 to 0.2 mm using the mFB.
Very recently, Li et al. (2017) developed a particulate sedimentation model based on the free terminal settling velocity and fluidization superficial velocity for extremely low concentrations in the mFB. Extensive work has been directed towards the correlation between the Reynolds number of the particles and n (Garside and Aldibouni, 1977). A generalized formula to predict the segregation efficiency and actual yield advantage of an RC has been established, and it was found that there was an asymptotic limit for the hydraulic capacity in inclined channels as the aspect ratio increased (Laskovski et al., 2006). The performance of inclined channels depending on the expansion characteristics of fine ash in a liquid–solid fluidized bed was researched (Li et al. 2014). Enhanced separation of mineral sands was achieved using the mFB with a large aspect ratio of about 200 (Zhou et al., 2006). It was found that the mFB was efficient at the desliming of coal with the size range from 2.0 to 0.25 mm by adding viscous glycerin to the suspensions (Hunter, Iveson, and Galvin 2014).

The present paper is the further work following the initial research of Li et al. (2017). The paper provides a relatively accurate description of the experimental system, accounting for the separation of particles in an inclined channel under different concentrations. The theoretical model without adjustable parameters is applied for describing the interaction of the bi-disperse suspensions. Our contribution lies in establishing an idealized theoretical arrangements, and offering further insight into the phenomena of fluidization and inclined settling. Compared to the approach of Li et al. (2017), the present paper has many advantages. Firstly, in this paper, the transition region is applied to connect vertical and inclined channel, where the fluidization velocity becomes higher than the vertical channel. Secondly, the average volume fraction of the inclined channel is applied in the sedimentation model in this paper, providing a relatively accurate and simple description of the segregation condition of concentrated suspensions. Thirdly, the sedimentation model is extended to describe the fluidization and settlement behavior of binary suspensions by varying the inventories of the particles and the fluidization velocity.

The purpose of this paper is to study the trajectory of mono-disperse particles in the modified fluidized bed (mFB) with inclined plates. The mechanisms of particle settlement are examined in the mFB. A novel particulate sedimentation model is built with incorporation of the hindered settling velocity of the particles with the influence of the fluidization superficial velocity, the volume fraction of suspensions, and the length of the particles through the inclined channels. Experiments are carried out for mono-disperse and bi-disperse suspensions to validate the newly developed model.

2. Theoretical methods

The solid particles in the suspension are subjected to many complicated forces such as the gravitational, drag, Saffman, and Magnus forces and an additional buoyancy. Amongst all these forces, two forces play the major roles, namely, gravity and fluid shear-induced lift force. In order to provide a relatively simple and accurate description of the novel model, in the present study, all main forces are considered by relating them to velocity vectors, which allows us to explore the motion characteristics of particles based on their velocity vectors. In the development of the theoretical model, the following simplifications are made (Acrivos and Herbolzheimer, 1979; Leung and Probstein, 2002):

1. No fragmentation and deformation occur during the movement of particles.
2. The movements of particles in the directions of \( \hat{X} \) and \( \hat{Y} \) in the inclined channel are independent.
3. The fluid velocity is uniform throughout the inclined channel.

The concentrated suspension passes upward in the vertical channel through a short transition region and then flows into the inclined zone at the position \( \hat{e} \). Heavy particles settle onto the inclined channel forming a sliding sediment, and slide back to the vertical fluidized bed where particles become fluidized and then continue to move along the inclined channel towards the overflow port. To account for the characteristics of particle settlement, a brief theoretical system is built, as shown in Fig. 1 and 2. The theoretical system developed in this study is operated using an equipment containing a single vertical channel with a width of \( w \) and smoothly connects to a single inclined channel with a width of \( k \) and an inclination angle of \( \theta \) with respect to the horizontal. The experimental system has a depth of \( z \) into the page.
2.1 Theoretical sedimentation model of mono-disperse suspensions in an inclined channel

The motion of particles in the inclined channel is analyzed, and hindered settling is also considered. The subscripts X and Y respectively denote the normal and tangential directions in the inclined channel. Richardson and Zaki established the following empirical equation to describe the relationship among mono-disperse particles (Richardson and Zaki, 1954):

\[ U_T = U_0(1 - \Phi_s)^n \]  

where \( U_0 \) is the free settling terminal velocity of a particle, \( U_T \) is the particulate terminal velocity for hindered settling, \( \Phi_s \) is the resulting volume fraction of the particles, \( (1 - \Phi_s)^n \) is the hindered settling factor, and the exponent \( n \) is only a function of the Reynolds number of the terminal particles with \( n \approx 4.6 \) for particles with a low Reynolds number. Acrivos et al. (1979) and Nir et al. (1990) have studied the profile of the fluidization velocity in an inclined channel. For simplicity, we assume that the fluidization superficial velocity is uniform across the inclined channel. Given the fluidization superficial velocity through the vertical channel \( U_L \), fluidization superficial velocity in the inclined channel is (Doroodchi et al. 2004; Iveson 2014):

\[ U_{LY} = U_L / \sin\theta \]  

When more particles are added to the system, the effects of the particle–particle interactions in a highly concentrated suspension are often studied using the empirical hindered settling function in the Eq. 1.
The velocities of the particles in the X and Y directions are expressed as:

\[ U_{TX} = U_T \cdot \cos \theta \]
\[ U_{TY} = U_T \cdot \sin \theta \]

where the subscripts X and Y denote the components along the X and Y directions, respectively. These velocities can be rewritten with Eq. 1 as:

\[ U_X = U_{TX} = U_0(1 - \Phi_s)^n \cdot \cos \theta \]
\[ U_Y = U_{TY} - U_T / \sin \theta = U_0(1 - \Phi_s)^n \cdot \sin \theta . \]

Suppose one particle enters the inclined channel at the point \( f \) (\( ef = k \)). \( U_X \) and \( U_Y \) are relative to the vessel; thus, the time required for a particle travels a distance \( k \) in the X direction is equal to the time required for the particle travels a distance \( L_p \) in the Y direction. Therefore:

\[ k / U_X = L_p / U_Y \]
\[ k = w \cdot \sin \theta . \]

The substitution of Eqs. 5 and 6–8 into Eq. 7 gives the settling distance along the inclined channel:

\[ L_p = \frac{w}{\cos \theta} \left( \frac{U_1}{U_0(1 - \Phi_s)^n} - \sin \theta^2 \right). \]

Strictly, the fluidization superficial velocity is greater than particle’s terminal velocity for hindered settling. Supposing that \( U_i / U_0 = a(a \geq 1) \), Eq. 9 is expressed as:

\[ L_p = \frac{w}{\cos \theta} \left( \frac{a}{(1 - \Phi_s)^n} - \sin \theta^2 \right). \]

Generally, suppose a particle enters the inclined channel from the middle part of line \( ef \) for a mono-disperse suspension is:

\[ 0 \leq L_p \leq \frac{w}{\cos \theta} \left( \frac{a}{(1 - \Phi_s)^n} - \sin \theta^2 \right). \]

It is clear that the fluidization superficial velocity and the motion trajectory of the particles in the inclined channel vary with the vertical channel parameters, resulting in different settling distances along the inclined channel. The settlement length \( L_p \) is directly affected by \( U_i / U_0, \Phi_s \), and the structural parameters of the vessel; thus, the feeding rate and particle concentration are closely related to \( L_p \). Furthermore, when the feed rate and fluidization superficial velocity are given, the particle velocities in the X and Y directions are constant, which means that the particles move substantially along a straight line in the inclined channel rather than along a parabolic curve.

### 2.2 Theoretical sedimentation model of bi-disperse suspensions in an inclined channel

In this section, the sedimentation model for mono-disperse particle suspensions is extended to a bi-disperse particle suspension system. A concentrated suspension consists of two particle species with different terminal velocities, which results in particle segregation. Two sedimentation zones are established in the inclined channel. The first zone is a suspension of slower settling species. The second zone is a bi-disperse suspension including both faster and slower settling species, and the upper zone includes only lighter species. A concentrated sediment is formed by the deposition of the particles at the end of the inclined channel, which then slides and returns to the fluidized zone below the inclined channel.

Supposing that the fully mixed particles enter the inclined channel from points \( A_0, B_0 \), and \( C_0 \) and that the heavier particles settle onto the corresponding points \( A_1, B_1, \) and \( C_1 \), the length of the sediment is \( L_{p1} \). Further, the lighter particles settle onto points \( A_2, B_2, \) and \( C_2 \), and the length of the sediment is \( L_{p2} \). \( L_{p1} \) and \( L_{p2} \) are constant for a given fluidization superficial velocity. The sedimentation zone of lighter particles, \( L_{po} \), is formed between points \( C_1 \) and \( C_2 \); \( L_{po} \) is the length of the settling zone for the slower particles in the inclined section, and it is related to the properties of the lighter particles. \( L_{po} \) can be obtained by \( L_{p2} \) and \( L_{p1} \), as follows:

\[ L_{po} = L_{p2} - L_{p1}. \]

In accordance with the monosized motion model in the foregoing section, the particle–particle interactions in a highly concentrated suspension affect both the lighter and heavier particles. The
volume fractions of the lighter and heavier particles are the total volume fractions of the particles in the inclined section; therefore, the settling distances of the lighter and heavier particles are:

\[ L_{p1} = \frac{w}{\cos \theta} \left( \frac{u_L}{u_{01}(1-\phi_1)^n} - \sin \theta^2 \right) \]

\[ L_{p2} = \frac{w}{\cos \theta} \left( \frac{u_L}{u_{02}(1-\phi_2)^n} - \sin \theta^2 \right). \]

Therefore, \( L_{po} \) is expressed as:

\[ L_{po} = \frac{w}{\cos \theta} \left[ \left( \frac{u_L}{u_{02}(1-\phi_2)^n} - \sin \theta^2 \right) - \left( \frac{u_L}{u_{01}(1-\phi_1)^n} - \sin \theta^2 \right) \right] \]

\[ L_{po} = \frac{w \cdot u_L}{\cos \theta (1-\phi_2)^n} \left( \frac{1}{u_{02}} - \frac{1}{u_{01}} \right). \]

3. Experimental methods

In order to verify the theoretical model, an experimental system was established, as shown schematically in Fig. 3. The vessel is made of Perspex. It consists of two channels, where the vertical channel has a cross-sectional area of 30 mm x 50 mm, a width \( w=50 \) mm, and the inclined channel has a height of 600 mm with an inclination angle of 72° (Zhou et al., 2006; Gorjikandi et al., 2016). The inclined channel is 840 mm long. The overflow flowed into the overflow tank from the top of the inclined channel, while the underflow was discharged from the bottom of the vertical channel. All of the fluidization water was delivered by a pump through a distributor at the base of fluidized bed; the distributor was specially designed in the shape of an inverted pyramid with 80 x 0.5 mm holes in a triangular lattice layout. The fluidized bed, inclined section, and distributor were connected with four flanges and 6 x 40 mm M10 bolts with a rubber gasket between the flanges to prevent leakage. The particulate suspensions from the continuously stirred tank with a height of 0.6 m and a diameter of 0.4 m were fed into the vertical channel through a feed inlet 400 mm above the flange. The particles used in the experiment were silica sands, and the particle properties are listed in Table 1. The sand particles were sieved, with two ranges of size of 425 μm – 710 μm and 710 μm – 880 μm, respectively used in the experiment. The fluidization water pump is opened when the device is filled with water, and the fluidization superficial velocity is slightly higher than the terminal velocity of particles, which prevents particles from sinking into the holes of the distributor and hence failing to reach the inclined channel. The concentrations of the feed particles should be sufficiently uniform to reduce experimental errors. The fluidized water velocity was gradually increased to move the particles along the inclined channel, and particles settled on the surface of the inclined channel. Then, we adjusted the fluidization superficial velocity to obtain values of \( a \left( a = \frac{U_L}{U_0} \right) \) of 1.2, 1.4, 1.6, 1.8, and 2.0. \( L_p \) and the corresponding fluidization superficial velocities were recorded when the experimental system reached a steady state. A flow meter was installed in the line of the fluidization water pump, to calculate the rate of fluidizing water by means of the cross-sectional area of the device.

**Fig. 3. Schematic diagram of experimental system**
Table 1. Particle properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m³)</th>
<th>Size (μm)</th>
<th>Average (μm)</th>
<th>Terminal velocity (m/s)</th>
<th>Richardson-Zaki exponent (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica sands</td>
<td>2600</td>
<td>425-710</td>
<td>568</td>
<td>0.0802</td>
<td>4.6</td>
</tr>
<tr>
<td>Silica sands</td>
<td>2600</td>
<td>710-880</td>
<td>795</td>
<td>0.1126</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Samples of the suspension at the vertical section were obtained after reaching a steady state. The sample outlets and pressure sensors were evenly distributed in the vertical channel. Overall pressure gradient data was obtained for deriving and calculating the entire volume fraction of the particles. The average of five measurements was calculated to reduce error. The reliability of the result was checked by actual concentration sampling data. Generally, the concentration gradients throughout the vertical channel remains constant, which indicates little separation in this section. During the experiment, the appropriate water velocity helps the particles to maintain a stable trajectory of conveyance and sedimentation in the inclined channel. Clear silica sands with particle sizes in the range of 0.425–0.71 mm was applied in the experiments for the mono-disperse particulate suspensions. In the experiment of the bi-disperse particulate suspension system, mixtures of silica particles with different sizes in the ranges of 0.425–0.71 mm and 0.71–0.88 mm were used. The ratio of particles with the former size to particles with the second size was 1:1 based on the mass.

4. Results and discussion
4.1 Mono-disperse suspension tests

In this part, the experimental data of mono-disperse particulate suspensions is collected and used to validate the novel model. The experimental results for the model indicate that the average volume fraction of the inclined section can be calculated using trapezium rule (Doroodchi et al. 2004). It can be concluded that the settling length of monosized particles at a certain terminal velocity along the inclined channel differs with the entry position. Moreover, the maximum settling length is fixed and calculated using Eq. 12. We find that the experimental results are in good agreement with theoretical values.

Fig. 4 shows the changes in $L_p$ for monosized particles with different fluidized water velocities and the distance of the particulate suspensions of the incline is proportional to the fluidization superficial velocity. It is evident that $L_p$ increases with the increase in $U_L/U_0$. The open symbols represent the experimental results, and the solid curves represent the model predictions results. The maximum settling length has a one-to-one correspondence with the volume fraction in the inclined section at a certain fluidization superficial velocity. A group of fluidization superficial velocities are used to obtain $U_L/U_0$ at 1.2, 1.4, 1.6, 1.8, and 2.0.

![Fig. 4. The change value of settling length, L_p.](image-url)
Generally, the experimental data for $L_p$ is in good agreement with the predicted values derived from the model, especially for moderate volume fractions. In general, the prediction results of the model agree well with the experimental data with a maximum deviation of 11%, and the absolute deviation between the predicted values and experimental data are within 0.02 m. The deviation could be attributed to the axial movement of particles in the inclined channel. When the volume concentration in the inclined channel decreases, the interaction of the particles decreases, therefore, the axial motion of the particles is enhanced.

Fig. 5 shows the effects of $U_L$ on $\Phi_s$ in the inclined section of the monosized suspension of silica sands. Again, the open squares, open triangles, and open circles correspond to particle inventories of 350 g, 550 g, and 750 g, respectively. The average volume fractions of the inclined channel are represented by the open symbols. It is evident that the average solid volume fraction in the inclined channel fluctuates slightly for a given total solid inventory.

Fig. 6 shows that the relative deviation $L_p^*/L_p$, where $L_p^*$ represents the experimental settling length, and $L_p$ represents the theoretical settling length. The real line denotes $L_p^* = L_p$. It is evident that the relative deviation is in the range of 0.9–1.1.

4.2 Bi-disperse suspensions tests

Experiments were conducted with the bi-disperse suspension of silica sands with diameters of ~0.56 and ~0.80 mm to validate the bi-disperse particle model. The theoretical model can predict the settling
lengths $L_{p1}$ and $L_{p2}$ at a certain fluidized water velocity. Moreover, the utilization ratio $\delta$ could be obtained.

Figs. 7 and 8 shows $L_{p1}$ and $L_{p2}$ for different inventories of solid particles and fluidization superficial velocities, respectively. The inventories of solid particles were 350 g, 550 g, and 750 g. $L_{p1}$ contains all of the heavy particles and some light particles, and the difference between $L_{p2}$ and $L_{p1}$ is based on a mono-disperse area. It is demonstrated that the experimental results are in good agreement with the sedimentation model. And the relative deviations are typically much lower than 11%.

Fig. 7. The change value of settling length, $L_{p1}$. Settling length of $L_{p1}$ under different inventories of solid particles of 350 g, 550 g and 750 g was investigated, with the value of the fluidization superficial velocity and terminal settling velocity 1.10, 1.22, 1.34, 1.43, and 1.55. The open symbols such as squares, triangles, and circles correspond to measured settling lengths at different inventories of solid particles, respectively. The continuous curves represent the predictions of the new sedimentation model

Fig. 8. The change value of settling length, $L_{p2}$. Settling length of $L_{p2}$ under different inventories of solid particles of 350 g, 550 g and 750 g was investigated, with the value of the fluidization superficial velocity and terminal settling velocity 1.48, 1.61, 1.74, 1.87, and 1.99. Again, the open symbols such as squares, triangles, and circles correspond to measured settling lengths at different inventories of solid particles, respectively. The continuous curves represent the predictions of the new sedimentation model

4.3 Discussion

The theoretical model is built on the hindered settling terminal velocity and the volume fraction in the inclined channel and demonstrates a reasonable description of the sedimentation of particles for mono-disperse and bi-disperse suspensions in liquid–solid fluidized beds with an inclined channel. The
experimental results and the values derived from the theoretical model are in good agreement, which suggests that the model for particulate sedimentation provides a good description of the actual motion of particles.

The volume fraction of the inclined channel is completely non-uniform. This could be easily overcome by averaging the measured volume fraction of the inclined channel based on the trapezium rule. When the average volume fraction is relatively higher, the predicted value of the settling length using the model is higher than the actual settling length, especially in the case of mono-disperse particles with an inventory of solid particles of 750 g, as can be seen in Fig. 4. At a higher volume fraction, the number of sliding particles is also higher. As a result, the fluidization superficial velocity may be reduced owing to the sliding sediment, and more particles may be entrained along with the downward sediment. Finally, this results in less suspension entering the inclined channel, more particles sliding down along the channel, and a shorter settling length.

At a smaller inventory of solid particles, there is a trend for the predicted value of the model with respect to the actual settling lengths. For instance, with a 350 g inventory of mono-disperse solid particles, and the theoretical results are lower than the predicted results. This phenomenon may result from the fact that the fluidized water velocity is higher than the terminal velocity along with the emergence of higher shear caused by resuspension of particles. This shear will impose a “transport effect” on the particles and cause larger settling distances, which would become significant as the water velocity increases. This effect also occurs for larger inventories of solid particles but may be offset by the influence of entrainment.

A study of particulate sedimentation within inclined channels by Li et al. (2017) indicated that particle motion is uniformly linear in the inclined channel at extremely low concentrations, which can hardly explain the actual motion of particles at high concentrations. In this study, the volume fraction of the inclined channel is applied to the theoretical model, which provides a relatively simple and accurate description of the experimental system, explaining steady-state particulate sedimentation under the fluidized suspensions and the following separation in an inclined channel. Moreover, a transition region is applied to describe the connection between the inclined channel and the vertical channel, where the fluidization superficial velocity becomes higher. The theoretical framework established over the transition region for describing particulate sedimentation for mono-disperse and binary particulate suspensions in the inclined channel is relatively accurate.

In general, this theoretical model provides a relatively accurate description of steady-state interaction of fluidization and the subsequent segregation in the inclined channel. It is noted that the theoretical model is suitable for the fluidized bed with an inclined channel at the laboratory level. Furthermore, by applying multi-channel industrial scale of different aspect ratio over the vertical fluidized bed housing, it is possible to separate different particles into multiple size fractions, and the arrangement has the potential to benefit a broad range of process systems.

5. Conclusions

(1) In this paper, the movement characteristics of both mono-disperse and binary-disperse suspension particles were comprehensively studied for different fluidization superficial velocities and inventories in an inclined channel.

(2) The theoretical model of the settling length of particles along the inclined channel is

$$L_p = \frac{w}{\cos \theta \left( \frac{a}{(1-\phi_s)^n} - \sin \theta^2 \right)}$$

and the kinematic model is a function of the fluidization superficial velocity. Excluding the adjustable parameters, it generally gives a simple and accurate description of the overall system.

(3) The kinematic model is relatively accurate for the prediction of the actual motion of particles compared with the experimental data, especially when the inventory of solid particles is 550 g, the experimental data with a maximum deviation of 11%. The absolute deviation between the predicted values and experimental data are within 0.03 m. The predicted settling length in the inclined channel plays a role in the separation of minerals based on the particle size and density.
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References


