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ANALYSIS OF KINETICS MODELS OF BATCH FLOTATION

Received March 15, 2007; reviewed; accepted June 8, 2007

Flotation as a kinetic and thermodynamic phenomenon is a random process. The random variable determining the number of particles in flotation, attached permanently to the bubble surface, and consequently also the recovery, depend on time. Numerous models of flotation kinetics have been worked out to describe this dependence. Each of these models covers in fact a separate aspect of the problem but they complement each other. The paper presents a detailed analysis of the models based on the kinetics of chemical reactions and on the model of chemical absorption. It results from analysis of these models that in the case of flotation of the feed which is non-homogeneous with respect to flotation properties in the initial moments of the process, the particles undergoing flotation have the highest flotation properties according to the equation of zero order and, next, according to the equation of $1/2$ order. With time, the particles of decreasing ability to flotation undergo flotation and, simultaneously, the order of flotation kinetics increases. Narrow size-and-density coal fractions of intermediate floatability (type 33 of Polish classification) float according to the first order kinetic equation. From the theoretical point of view they can be assumed to be a homogenous material with respect to flotation properties.

Key words: flotation, chemical reaction, kinetics model, absorption model, flotation kinetics order

INTRODUCTION

Flotation as a process resulting from selective adhesion of mineral particles to air bubbles is both a thermodynamic and kinetic phenomenon. Kinetics, the course of the process in time, is affected by potential interactions of electromagnetic type between the mineral particle and the bubble. The interaction is necessary for adhesion. It is a random variable, and it also results from the successive inflow of the free surface in the form of air bubbles into the flotation system. The free surface is the place of adhesion of mineral particles and it limits, among others, the velocity of the process.

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The random character of interactions results from the fact that the particle surface properties determine the interaction and these properties change not only from one particle to another but they can also within the surface of the same particle.

To obtain a permanent adhesion of particle and bubble, the particle must overcome the energy barrier of the particle-bubble interaction. This phenomenon is of random character because of the random character of collision and the angle of collision of the particle with the bubble, as well as the random value of particle kinetic energy necessary to rupture of the thin film between particle and the air bubble. Apart from the above phenomena, the attached particle, especially the larger ones, can be detached from the bubble surface by external forces, present in the flotation chamber, the source of which originates from turbulences of the medium. This process is also of random character.

The process of bubble mineralization can be also considered as the adsorption process in which the role of adsorbent is played by the bubble surface while mineral particles fulfill the role of adsorbate. This is a selective adsorption whose selectivity depends on the interface energy, determined by surface properties of particles and the reagent system.

Due to the multi-topic character of problems determining the flotation course and affecting the process of mineralization of air bubbles, many authors dealt with the problem of flotation kinetics in the last several decades (Zuniga, 1935; Schuhmann, 1942; Sutherland, 1947; Pogorelyj, 1961ab, 1962, Melkich, 1963a, 1963b, 1964; Bogdanov et al., 1964; Volin and Swami, 1964; Bodziony, 1965; Zeidenberg et al., 1964; Harris and Rimmer, 1966; Loveday, 1966; Tille and Panou, 1968, Kapur and Mehrotra, 1973, 1974; Mehrotra and Kapur, 1974, 1975; Trahar and Warren, 1976; Collins and Jameson, 1976; Harris, 1978; Maksimov and Emelianov, 1983; Xu Changlian, 1985; Szatkowski and Freyberger, 1985a,b; Vanangamudi and Rao, 1986; Lasic and Calic, 2000; Brozek and Młynarczykowska, 2006). Many models of flotation kinetics were developed, from the deterministic and adsorption models to statistical and stochastic ones, out of which each dealt with another aspect.

This paper presents a critical analysis of the model based upon the model of kinetics of chemical reaction and the model of adsorption. This analysis has been supported by the example of flotation kinetics of narrow coal density fractions.

MODEL OF KINETICS OF CHEMICAL REACTION

The first kinetics model of batch flotation was proposed by Zuniga (1935). He applied the differential equation of kinetics of chemical reaction which, in its general form, can be written as:

$$\frac{dC}{dt} = -k C^n \quad (1)$$

where: $C(t)$ – concentration of floating particles remaining in the flotation chamber up to the moment t , k – flotation rate constant, n – constant characterizing the order character of the process (order of flotation kinetics).

Equation 1 represents flotation kinetics of particles which are homogeneous from the point of view of surface properties, the so-called equally well floating particles which have the same value of the flotation rate constant.

The solution of Eq. 1, with the initial condition: $C(t=0) = C_o$, is as follows:

$$C(t) = \left[(n-1) k t + C_o^{(1-n)} \right]^{\frac{1}{1-n}} \quad \text{for } n \neq 1 \quad (2)$$

where C_o denotes the initial concentration of particles under flotation in the flotation chamber.

The recovery of the particles in the froth product ε after time t is, from the recovery definition, equal to

$$\varepsilon = \frac{C_o - C}{C_o} = 1 - \frac{C}{C_o} \quad (3)$$

According to Eqs. 2 and 3, the term C/C_o and recovery ε are expressed by the formulas:

$$\frac{C}{C_o} = \left[1 + (n-1) C_o^{n-1} k t \right]^{\frac{1}{1-n}} \quad \text{for } n \neq 1, \quad (4)$$

$$\varepsilon(t) = 1 - \left[1 + (n-1) C_o^{n-1} k t \right]^{\frac{1}{1-n}} \quad \text{for } n \neq 1. \quad (5)$$

The limit values $\varepsilon(t)$ for $n > 1$ are

$$\lim_{t \rightarrow \infty} \varepsilon(t) = 1. \quad (6)$$

It means that after a long time of flotation all particles will have undergone flotation. In the general case, especially for larger and more difficult-to-float particles, it does not have to be so because despite the adhesion process of particles to air bubbles, a reverse process occurs (of lower intensity), i.e. detachment of the particles from the surface of bubbles into the pulp (Mika and Fuerstenau, 1968; Schulze, 1977; Woodburn et al., 1971; Stachurski, 1970; Schulze, 1992; Maksimov and Emelianov, 1983). Accordingly, in the state of equilibrium, the value of recovery after a long time of flotation is smaller than 1 and equals ε_{∞} . After imposing the following condition upon Eq. 5

$$\lim_{t \rightarrow \infty} \varepsilon(t) = \varepsilon_{\infty} \quad (7)$$

the dependence of recovery on time is:

$$\varepsilon(t) = \varepsilon_{\infty} \left\{ 1 - \frac{1}{\left[1 + (n-1) C_o^{n-1} k t \right]^{\frac{1}{n-1}}} \right\} \quad \text{for } n > 1 . \quad (8)$$

Equations 5 and 8 represent a general formula of flotation kinetics of n order, obtained from the solution the differential equation of chemical reaction kinetics.

ADSORPTION MODEL OF FLOTATION KINETICS

From the physical point of view, the process of flotation is closed to the process of adsorption (Pogorelyj, 1962). The role of adsorbent is played by air bubbles (bubbles surface) while the role of adsorbate by mineral particles. Analogically, as in the process of adsorption (Oscik, 1973), the time of contact of particle with air bubble, leading to the stable connection of a particle with a bubble (induction time), is very short.

Adsorption occurs on the bubble surface, and therefore the number of mineral particles attached (adsorbed) to bubbles is proportional to the free, still not mineralized, surface of bubbles. For the material, homogeneous from the point of view of flotation properties, the mass of particle attached to the bubbles surface in time $d\tau$ is:

$$dm = \lambda s C d\tau \quad (9)$$

where: λ - mineralization (adsorption) rate constant (mass of particles adsorbed for a bubble surface unit in a unit of time), s – free surface of bubbles coming to the flotation chamber in the unit time, τ - time of remaining of a bubble in the flotation chamber, C – volume concentration of particle in the flotation chamber.

As a result of mineralization, the free (not mineralized) surface of bubble decreases. The decrease of this surface in a unit of time is equal to:

$$-ds = \frac{1}{\alpha} dm \quad (10)$$

where: α -mass of particles adsorbed by the bubble surface unit. From Eqs. 9 and 10 we obtain:

$$ds = -\frac{\lambda s C}{\alpha} d\tau \quad (11)$$

After solving Eq. 11, at the initial condition $s(0) = s_o$ we obtain the dependence:

$$s = s_o \exp\left(-\frac{\lambda C}{\alpha} \tau\right). \quad (12)$$

The mineralized surface in a unit of time will be expressed by the formula:

$$s_o - s = s_o \left[1 - \exp\left(-\frac{\lambda C}{\alpha} \tau\right)\right]. \quad (13)$$

According to the above, the decrease of mineral mass from the flotation pulp in time dt is:

$$-dm = \alpha s_o \left[1 - \exp\left(-\frac{\lambda C}{\alpha} \tau\right)\right] dt. \quad (14)$$

If the volume of the flotation chamber is V then $dm = V dC$ and from expression (14), the following equation is obtained:

$$\frac{dC}{1 - \exp\left(-\frac{\lambda \tau}{\alpha} C\right)} = -\frac{\alpha s_o}{V} dt. \quad (15)$$

The integer on the left side of expression (15) is equal to (Gradstein and Ryzik 1971):

$$\int \frac{dC}{1 - \exp\left(-\frac{\lambda \tau}{\alpha} C\right)} = -\frac{\alpha}{\lambda \tau} \ln \left[\frac{\exp\left(-\frac{\lambda \tau}{\alpha} C\right)}{1 - \exp\left(-\frac{\lambda \tau}{\alpha} C\right)} \right]. \quad (16)$$

After integrating the entire expression (15) and transforming it, the following dependence is obtained:

$$\frac{\exp\left(-\frac{\lambda \tau}{\alpha} C\right)}{1 - \exp\left(-\frac{\lambda \tau}{\alpha} C\right)} = D \exp\left(\frac{\lambda \tau s_o}{V} t\right) \quad (17)$$

where D is the constant of integrating, calculated from the initial condition $C(0) = C_o$.

After calculating constant D and substituting into Eq.17, the following equation is obtained:

$$\exp\left(\frac{\lambda \tau}{\alpha} C\right) = \left\{ \left[\exp\left(\frac{\lambda \tau C_o}{\alpha}\right) - 1 \right] \exp\left(-\frac{\lambda \tau s_o}{V} t\right) + 1 \right\}. \quad (18)$$

From Eq.18 we obtain the dependence of volume concentration of particles in the flotation chamber upon time:

$$C(t) = \frac{\alpha}{\lambda \tau} \ln \left\{ \left[\exp\left(\frac{\lambda \tau C_o}{\alpha}\right) - 1 \right] \exp\left(-\frac{\lambda \tau s_o}{V} t\right) + 1 \right\}. \quad (19)$$

The recovery of the floated mineral in the froth product after flotation time t , according to Eq. 3 will be:

$$\varepsilon(t) = 1 - \frac{\alpha}{\lambda \tau C_o} \ln \left\{ \left[\exp\left(\frac{\lambda \tau C_o}{\alpha}\right) - 1 \right] \exp\left(-\frac{\lambda \tau s_o}{V} t\right) + 1 \right\}. \quad (20)$$

It results from Eq. 20 that the recovery depends on the volume concentration of particle in the flotation chamber C_o , flotation properties of particle measured by value λ , size of free surface of bubbles s_o , and flotation time t .

ANALYSIS OF FLOTATION KINETICS MODELS

KINETICS EQUATIONS OF ZERO AND $\frac{1}{2}$ ORDER

Particular cases of kinetics equations of zero and $\frac{1}{2}$ orders, based on the model of chemical reaction, can be obtain by means of formal setting the order of kinetics in Eq. 5 or by a series expansion of the general equation of flotation kinetics.

From Eq. (5) for $n = 0$ we obtain the following dependence:

$$\varepsilon(t) = \frac{k t}{C_o}. \quad (21)$$

Therefore, it is a rectilinear dependence. The rate constant in this case is:

$$k = C_o \frac{d\varepsilon}{dt}. \quad (22)$$

Subsequently, with the increase of the initial volume concentration of particle, the rate constant of flotation increases because $\frac{d\varepsilon}{dt} = const$. It results from Eq. 21 that the

recovery reaches the maximum value of 1, theoretically after the finite flotation time equal to $t_m = \frac{C_o}{k}$. It means that if the flotation of equally floating particles occurs according to the equation of zero order, there is no equilibrium detachment from air bubbles.

Equation 21 can be obtained from Eq. 5 in a different way. The expression in the square brackets of Eq. 5 is a binomial in relation to variable t . For small values of flotation time t , the second component of the binomial is small. The extension of binomial into a power series for $|x| < 1$ is as follows (Leja 1971):

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots + \frac{m(m-1)\dots(m-k+1)}{k!}x^k + \dots \quad (23)$$

Leaving two terms of extension in Eq. 5 we obtain:

$$\varepsilon(t) \cong C_o^{n-1} k t. \quad (24)$$

It is therefore possible, according to Eq. 24, to express an opinion that at initial moments of flotation, it always runs due to the rectilinear dependence. This equation for $n = 0$ is transformed into Eq. 24. Substituting the value $\frac{1}{2}$ for n in Eq.5 we obtain the following dependence of recovery on time:

$$\varepsilon(t) = \frac{k t}{\sqrt{C_o}} - \frac{k^2 t^2}{4 C_o}. \quad (25)$$

The dependence $\varepsilon(t)$, according to Eq. 25 is presented by a section of a parabola. Recovery reaches the maximum value after time $t_m = \frac{2\sqrt{C_o}}{k}$, equal $\varepsilon_{max} = \varepsilon(t_m) = 1$. Therefore, similarly to the kinetics of zero order, there is no detachment of particles in the sense of a stochastic process. The equation of kinetics of $\frac{1}{2}$ order, analogically to the former case, can be obtained from a general Eq. 5 by a series expansion. Expanding the expression included in the square bracket of Eq. 5 and considering three terms of expansion, the following dependence for $\varepsilon(t)$ is obtained:

$$\varepsilon(t) \cong C_o^{n-1} k t - \frac{1}{2} n C_o^{2(n-1)} k^2 t^2 \quad (26)$$

which for $n = 0.5$ can be transformed into dependence given by Eq. 25. It can be therefore said that for each flotation process there is a time range in which the recovery depends on time, according to the $\frac{1}{2}$ order kinetics equation. If, for obtaining

Eq. 26, the three term of expansion should be taken into consideration to get the required accuracy, it means that the interval of process duration in which flotation occurs according to the kinetics equation of $\frac{1}{2}$ order must be longer than in the kinetics of zero order. Hence, it can be concluded that after flotation according to the kinetics equation of zero order the process runs according to the equation of $\frac{1}{2}$ order.

Successively, the analysis of the kinetics model based on the process of adsorption leads to the following observations. In the initial moments of flotation, i.e. small value of time t , high volume concentration of particle in the flotation chamber C , not very much different from C_o , high value of λ since, first of all, the particles of the highest floatability are floating and at low of the surface accessible for adsorption in the time range τ , because the bubbles are quickly mineralized by the easily floating particles, Eq. 20 can be transformed as follows. For small values of s_o and t , $\exp\left(-\frac{\lambda \tau s_o}{V} t\right) \cong 1 - \frac{\lambda \tau s_o}{V} t$, and Eq. (20) transforms into the form:

$$\varepsilon(t) = 1 - \frac{\alpha}{\lambda \tau C_o} \left[\frac{\lambda \tau C_o}{\alpha} + \ln\left(1 - \frac{\lambda \tau s_o}{V} t\right) \right] \quad (27)$$

For small values s_o and t $\ln\left(1 - \frac{\lambda \tau s_o}{V} t\right) \cong -\frac{\lambda \tau s_o}{V} t$ (Leja, 1971) and the dependence of recovery upon the time of flotation is:

$$\varepsilon(t) = \frac{\alpha s_o}{V C_o} t = \frac{k t}{C_o} \quad (28)$$

where: $k = \frac{\alpha s_o}{V}$. Therefore we obtain the kinetics equation of zero order, analogical to Eq. 21. It can be said that according to the kinetics equation of zero order the particles which undergo flotation are the most easily floatable (of the highest λ values), which mineralize quickly the surface of the bubbles and consequently, the free surface of the bubble, accessible for the adsorption of consecutive particle in the time range τ , i.e. the time of the bubble remaining in the flotation chamber is small at large values of λ and C , according to Eq.12.

KINETICS EQUATION OF THE FIRST ORDER

The kinetics equation of the first order will be obtain from Eq. 8 by means of the transition with the order of equation n to 1. Denoting $\frac{1}{1-n} = m$, Eq.8 can be written as the following function sequence:

$$\varepsilon_m(t) = \varepsilon_\infty \left\{ 1 - \left[1 - C_o^{-1/m} \frac{k t}{m} \right]^m \right\}. \quad (29)$$

If n heads for 1, then m heads for infinity. Hence, the limit of the function sequence $\varepsilon_m(t)$ is:

$$\lim_{m \rightarrow \infty} \varepsilon_m(t) = \varepsilon(t) = \varepsilon_\infty (1 - e^{-k t}) \quad (30)$$

because the expression with square brackets in Eq. 29 heads for $e^{-k t}$ ($C_o^{-1/m} \rightarrow 1$). This is the form of the kinetics equation which is most often applied for investigation of flotation kinetics. As it can be seen from Eqs.8 and 30, only in the case of flotation kinetics of the 1st order the course of dependence of recovery upon time does not depend on the initial concentration of particles in the flotation chamber. This is a criterion stating that for a given raw material, under given physicochemical conditions, flotation occurs according to the kinetics equation of the first order (Loveday, 1966). It results from the adsorption model of flotation kinetics that after a long flotation time t , i.e. low volume concentration of particles and low value of λ , the free surface, accessible for adsorption, is larger according to Eq.12. Under these conditions $\exp\left(\frac{\lambda \tau C_o}{\alpha}\right) \cong 1 + \frac{\lambda \tau C_o}{\alpha}$ and Eq. 20 assumes the form:

$$\varepsilon(t) = 1 - \frac{\alpha}{\lambda \tau C_o} \ln \left[1 + \frac{\lambda \tau C_o}{\alpha} \exp\left(-\frac{\lambda \tau s_o}{V} t\right) \right] \quad (31)$$

Expanding the logarithmic expression into a series and considering the 1st component of expansion, Eq. 31 simplifies to the form:

$$\varepsilon(t) \cong 1 - \exp\left(-\frac{\lambda \tau s_o}{V} t\right). \quad (32)$$

The equation of kinetics of the first order is obtained. Therefore flotation according to the kinetics equation of the first order runs after a longer flotation time in the conditions of lower concentration of particles in the flotation chamber, larger free surface of the bubble in the time τ and lower values of λ .

EQUATIONS OF KINETICS OF HIGHER ORDERS

Higher orders of kinetics equations are obtained from the basic equation (Eq. 8) after substituting a proper value for n .

a) kinetics equation of the 1.5th order:

$$\varepsilon(t) = \varepsilon_{\infty} \left[1 - \frac{1}{\left(\frac{1}{2} \sqrt{C_o} k t + 1 \right)^2} \right] \quad (33)$$

b) kinetics equation of the 2nd order:

$$\varepsilon(t) = \varepsilon_{\infty} \frac{C_o k t}{1 + C_o k t} \quad (34)$$

c) kinetics equation of the 3rd order:

$$\varepsilon(t) = \varepsilon_{\infty} \left[1 - \frac{1}{\sqrt{1 + 2 C_o^2 k t}} \right] \quad (35)$$

d) kinetics equation of the 4th order:

$$\varepsilon(t) = \varepsilon_{\infty} \left[1 - \frac{1}{\sqrt[3]{1 + 3 C_o^3 k t}} \right] \quad (36)$$

e) kinetics equation of the 5th order:

$$\varepsilon(t) = \varepsilon_{\infty} \left[1 - \frac{1}{\sqrt[4]{1 + 4 C_o^4 k t}} \right] \quad (37)$$

f) kinetics equation of the 6th order:

$$\varepsilon(t) = \varepsilon_{\infty} \left[1 - \frac{1}{\sqrt[5]{1 + 5 C_o^5 k t}} \right] \quad (38)$$

Pictorial $\varepsilon(t)$ dependence for the above models are shown in Fig. 1.

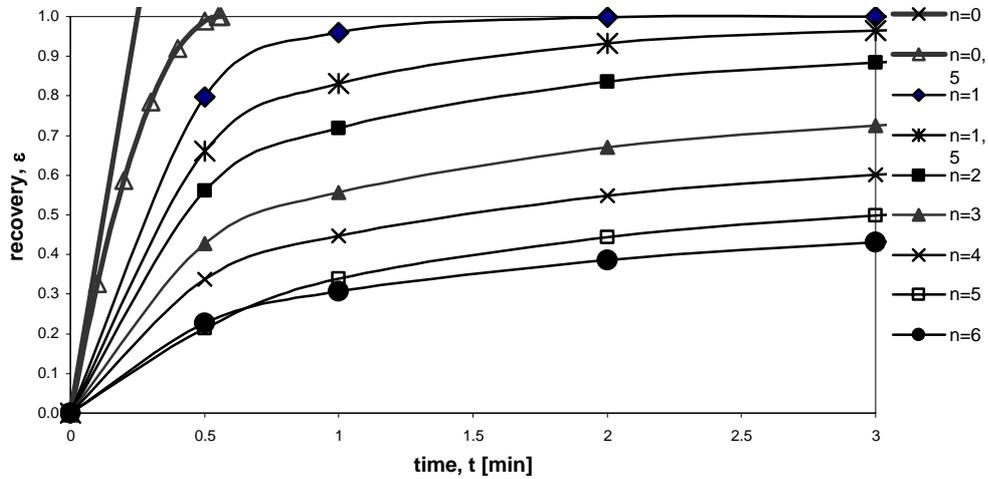


Fig. 1. Pictorial dependence of flotation recovery on time for kinetics orders from $n = 0$ to $n = 6$

ORDER OF FLOTATION KINETICS

In the model of flotation kinetics of the first order based upon the kinetics of chemical reaction the model parameters are the flotation rate constant and equilibrium recovery while in the stochastic model the parameters are the permanent adhesion rate constant and the detachment rate constant of the particle from the bubble (Brozek and Mlynarczykowska, 2006). In the stochastic model both the equilibrium recovery and the flotation rate constant (constant of resultant adhesion) are connected by function with the constants of adhesion and detachment. In both cases these are the two-parameter models. The constants of adhesion and detachment are expressed by probabilities of collision, adhesion and detachment of the particles from the bubble which, respectively, are connected with geometrical and surface properties of particles as well as physicochemical and hydrodynamic conditions in the flotation chamber. In the model of flotation kinetics of the n -th order (Eq. 8), apart from the flotation rate constant and equilibrium recovery, the third parameter occurs, i.e. the order of flotation kinetics. For the flotation carried out under conditions when the surface accessible for adsorption is large (free flotation) Pogorelyj (1962) presented the following expression for the order of flotation kinetics:

$$n = 1 + \frac{\lg \frac{\lambda}{\lambda_0}}{\lg C} \tag{39}$$

where: λ - average value of the adsorption rate constant of particles at the concentration of particles in the flotation chamber C , λ_0 - average value of the adsorption rate constant at the initial moment. Melkich (1964) worked out an

analogical expression for the order of flotation kinetics, starting from the statistical theory of flotation:

$$n = 1 + \frac{\sigma^2}{\lambda^2} \quad (40)$$

where: σ^2 – dispersion (variance) of flotation properties around the average value. It results from Eqs. 39 and 40 that only for the feed strictly homogeneous with respect to flotation properties, the order of flotation kinetics is equal to 1, since $\lambda = \lambda_0$ and $\sigma^2 = 0$. However, such a distribution of flotation properties of the feed is exclusively theoretical in character because even in the narrow size fraction of the pure mineral the distribution of induction time is connected with the statistical distribution of adsorption density of the reagent on the particle surface (Schulze, 1992) and also with the distribution of particle shape and size. Consequently, even for pure minerals, the adhesion rate constant, and also the order of flotation kinetics, will be characterized by fixed distributions. The investigations of flotation kinetics of pure minerals of narrow size fraction proved that under conditions of free flotation, i.e. when the surface of bubbles is not a limitation for their mineralization, the flotation results are consistent with the equations of the first order (Pogorelyj 1961 a,b; Tomlinson and Fleming, 1963).

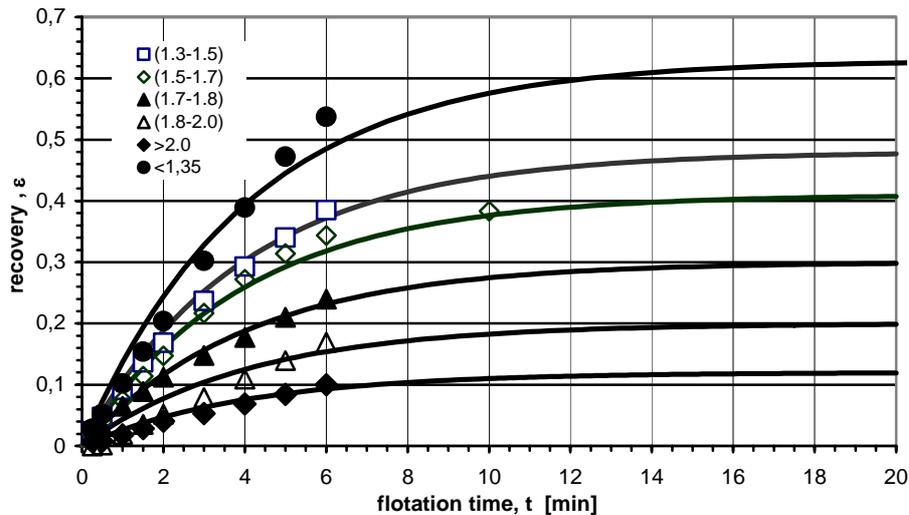


Fig.2. Flotation kinetics (Brozek and Młynarczykowska, 2006):

density fraction <1.35 [Mg/m³], $A=1.06\%$, $\varepsilon=0.63(1-e^{-0.245t})$, $\lambda_0=0.154$ [1/min],
density fraction (1.35-1.5) [Mg/m³], $A=7.91\%$, $\varepsilon=0.48(1-e^{-0.249t})$, $\lambda_0=0.12$ [1/min],
density fraction (1.5-1.7) [Mg/m³], $A=23.18\%$, $\varepsilon=0.41(1-e^{-0.25t})$, $\lambda_0=0.102$ [1/min],
density fraction (1.7-1.8) [Mg/m³], $A=36.32\%$, $\varepsilon=0.30(1-e^{-0.246t})$, $\lambda_0=0.074$ [1/min],
density fraction (1.8-2.0) [Mg/m³], $A=48.71\%$, $\varepsilon=0.20(1-e^{-0.245t})$, $\lambda_0=0.079$ [1/min],
density fraction >2.0 [Mg/m³], $A=77.63\%$, $\varepsilon=0.12(1-e^{-0.249t})$, $\lambda_0=0.03$ [1/min]

Also the investigations of flotation kinetics of narrow size-and-density fractions of intermediately floating coal type 33 proved that the result of flotation fulfill the equation of the first order. Figure 2 shows the curves of flotation kinetics of narrow size-and-density fractions of coal type 33 (Brozek and Mlynarczykowska, 2006). The index of curvilinear correlation, evaluating the rate of agreement of the model with the experiment is larger than 0.96. Therefore it can be said that according to Eq. 40 the rate of non-homogeneity of flotation properties of particle forming narrow size-and-density fractions is within the limit of experiment error and is negligibly little. If the feed is non-homogenous with respect to its flotation properties, as mentioned before, at first the particles of the highest floatability undergo flotation. Respectively, the average value of flotation properties of particle remaining in the flotation chamber decreases with time and also the concentration of floating particles changes.

Therefore, the order of flotation kinetics changes. According to Bogdanov (1959), the order of flotation kinetics changes during the process from 0 to 6 (Bogdanov 1959, according to Pogorelyj 1962). On the other hand, however, there are no theoretical premises stating that in a wide time range the flotation process of the non-homogeneous material will run according to the equation of a fixed order.

ACKNOWLEDGEMENTS

The paper has been written within the European Union grant “Scientific Network-Surfactant and Dispersed System in Theory and Practice”. Contract no INCO-CT-2003-003355, and supported by a grant of the National Research Committee no 4 T12A 035 30

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Flotacja jako zjawisko kinetyczne i termodynamiczne jest procesem losowym. Zmienna losowa oznaczająca liczbę ziaren flotujących, przyczepionych trwale do powierzchni pęcherzyka, a w konsekwencji i uzysk, są zależne od czasu. Dla opisu tej zależności opracowano szereg modeli kinetyki flotacji. Każdy z tych modeli ujmuje wprawdzie inny aspekt zagadnienia, lecz modele te się wzajemnie uzupełniają. W tym artykule podano szczegółową analizę modelu opartego na kinetyce reakcji chemicznej oraz na modelu absorpcji chemicznej. Z analizy równań tych modeli wynika, że w przypadku flotacji nadawy niejednorodnej pod względem własności flotacyjnych, w początkowych chwilach trwania procesu flotują ziarna o najwyższych własnościach flotacyjnych według równania rzędu zerowego a następnie według równania rzędu $\frac{1}{2}$. W miarę upływu czasu flotują ziarna o coraz niższych własnościach flotacyjnych i zwiększa się zarazem rząd kinetyki flotacji. Wąskie klaso-frakcje węgla o średnich własnościach flotacyjnych (typ 33) flotują według równania kinetyki rzędu I. Można więc z teoretycznego punktu widzenia uznać je za materiał jednorodny pod względem własności flotacyjnych.