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Transformation of equation y=a(100-x)/(a-x) for approximation of separation results plotted as Fuerstenau's upgrading curve for application in other upgrading curves

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Abstract. Equation y= a(100-x)/(a-x), frequently used for approximation of separation results plotted in the Fuerstenau upgrading curve, relating recovery of a selected component of the feed in the concentrate (x = ε) with recovery of another component in the tailing (y = ε_r), can be transformed into one-fitting parameter equation suitable for other upgrading curves. The mathematical formulas for the so-called Luszczkiewicz, Mayer, Henry, Stepinski, Hall, and Halbich separation upgrading curves were derived and presented.

keywords: upgrading curve, upgrading parameter, separation, ore beneficiation

1. Introduction

Material balance, separation graphs and mathematical equations are used for analysis and evaluation of separation results. In case of the Fuerstenau plot (Fig. 1), relating recovery of a selected component of the feed in the concentrate ε with the recovery of another component in the tailing ε_r, the number of available equations is great (Drzymala and Ahmed, 2005). One of them has a form of

$$\varepsilon_r = a\frac{100-\varepsilon}{a-\varepsilon} \tag{1}$$

and is based on a single fitting parameter a. Equation (1) is very useful because it not only approximates numerous literature separation results but also can serve as a single parameter reflecting separation selectivity (efficiency), while in normal situations two upgrading parameters are needed. This is possible due to a special property of the Fuerstenau upgrading curve since its principal lines, that is no upgrading, ideal upgrading, ideal remixing do not change their location with variation of the of feed composition.

Mathematical equations used for other separation upgrading curves are usually more complex and two and more fitting parameters formulas are used. Examples are presented in Table (1).

The goal of this work is to transform Eq. (1) into forms applicable for other than Fuerstenau upgrading separation curves, that is for the Mayer, Halbich, Henry, Luszczkiewicz, Hall, and Stepinski plots (Drzymala, 2006-8).

Table 1. Mathematical equations used for approximation of different upgrading curves

Upgrading curve and parameters	Equation	Source
Halbich ε, β	$\varepsilon = a - b\beta$ *a* and *b* - fitting parameters (for high recoveries) $\frac{\beta_{\max}}{100-\beta} = z(100-\varepsilon)^{z-1}\frac{\alpha}{100-\alpha}$ *z* and β_{max} - fitting parameters	Dell, 1969 Digre, 1960
Hall ε, H	$H = A\frac{100-(100-\varepsilon)}{100+A-\varepsilon}$ (A - fitting parameter)	Hall, 1971
Henry β, γ	$\gamma = c - d\beta$ *c* and *d* - fitting parameters	Foszcz et al., 2010
Halbich (II) $\varepsilon, \beta/\alpha$	$\varepsilon = \varepsilon_{\max} - 2\sinh(k[(\beta/\alpha)-1])$ *k* and $\varepsilon_{\max}$ - fitting parameters	Vera et al., 1999
Mayer III (Dell) $\varepsilon, \gamma/\alpha$	$(a_1x + b_1y + c_1)(a_1x + b_2y + c_2) = K$ a_1, a_2, b_1, b_2, c_1, c_2, and *K* - fitting parameters	Jowett, 1969
Stepinski β, ϑ	$\beta = l + m\vartheta$ *l* and *m* - fitting parameters	Pudlo, 1971; Luszczkiewicz, 1975; Neethling and Cilliers, 2008;
Mayer II ε, γ	$\varepsilon = g + h\gamma$ *g* and *h* - fitting parameters	Nixon and Moir, 1956-7

γ – concentrate yield, , α – content of a considered component in feed, β – content of a considered component in concentrate, ε – recovery of a considered component in concentrate

2. Transformation

Separation results fitted with Eq. (1) in the Fuerstenau separation upgrading plot (Fig.1) form a symmetrical, in respect to the diagonal, curve. A closer literature survey indicates that Eq. 1 has many mathematical forms which depend on the definition of the fitting parameter (Table 2).

In this work we will use only Eq. (1) with *a* as the fitting parameter. Transformation of Eq.(1) to forms useful for other separation upgrading curves was performed manually by replacing ε and ε_r with appropriate formulas based on the original upgrading parameters, that is contents of a considered component in the feed (α), concentrate (β) and in the tailing (ϑ) (Table 3), and next rearranging the obtained equation until a relation between the two characteristic, for a given separation curve, parameters were obtained.

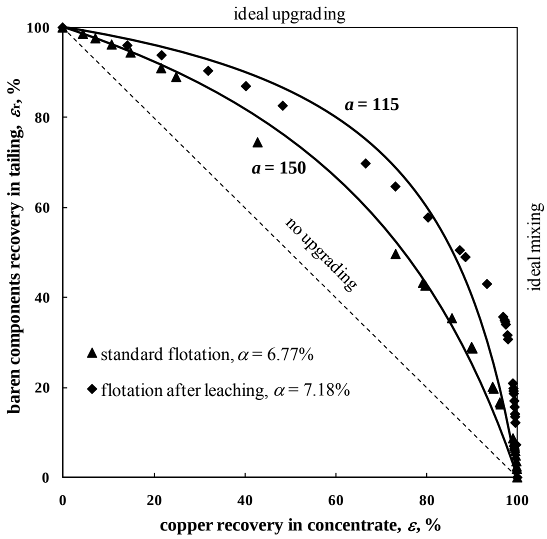

Fig. 1. The Fuerstenau separation upgrading curve showing fixed background lines and real separation data adopted from Luszczkiewicz and Chmielewski (2008)

Table 2. Mathematical forms of Eq. (1) depending on definitions of fitting parameter

Formula	Fitting parameter	Limits	Relation to *a*	Source
$\varepsilon = a\dfrac{100-\varepsilon_r}{a-\varepsilon_r}$	*a*	$a = \infty$ (no) $a = 100$ (ideal)	$a = a$	Drzymala and Ahmed, 2005
$\varepsilon_r = A\dfrac{100-\varepsilon}{A+\varepsilon}$	*A*	$A = \infty$ (no) $A = -100$ (ideal)	$a = -A$	Hall, 1971
$Z = \dfrac{(100-\varepsilon_r)(100-\varepsilon)}{\varepsilon\varepsilon_r}$	*Z*	$Z = 1$ (no) $Z = 0$ (ideal)	$a = \dfrac{1}{-0.01Z+0.01}$	Laplante, 1989
$\varepsilon = F^2\dfrac{100-\varepsilon_r}{(2F-100)(\frac{F^2}{2F-100}-\varepsilon_r)}$	*F*	$F = 50$ (no) $F = 100$ (ideal)	$a = \dfrac{F^2}{2F-100}$	this work
$y = 100\Phi(a + b\Phi^{-1}(x))$ Φ – distribution function	*a, b*	$a = 0$ (no) $a = \infty$ (ideal) $b = 1$ (for symmetrical curve)		Krzanowski and Hand, 2009; Wlodarski, 2009

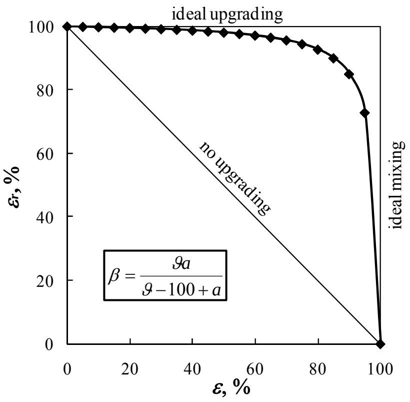

Fig. 2. Fuerstenau's curve

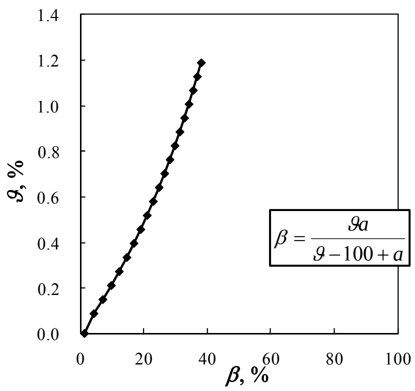

Fig. 3. Stepinski's (I) curve

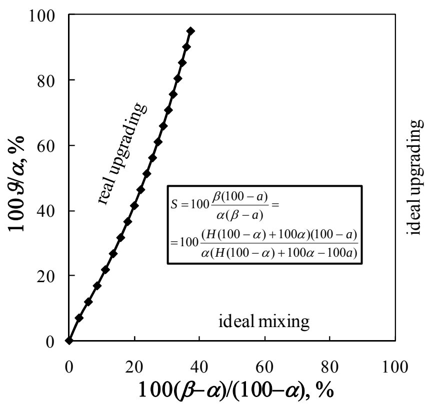

Fig. 4. Stepinski's (V) curve

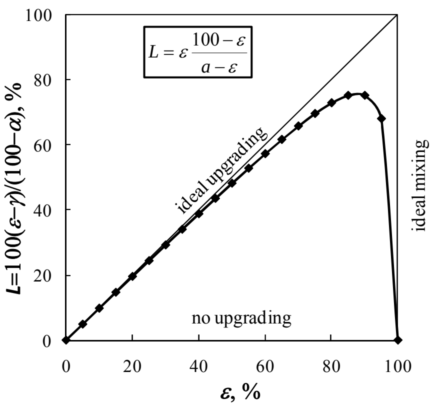

Fig. 5. Luszczkiewicz's curve

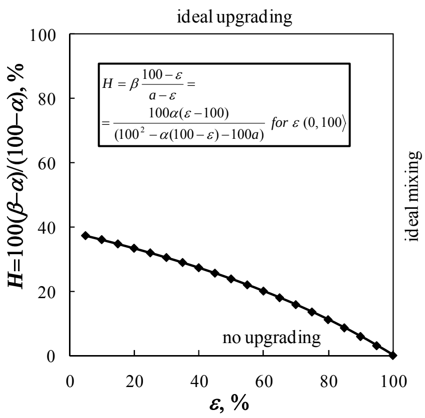

Fig. 6. Hall's curve

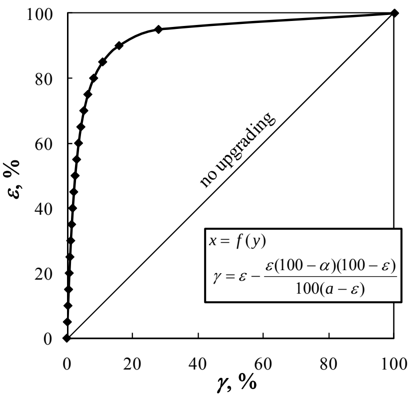

Fig. 7. Mayer's curve

Table 3. Upgrading curves, their characteristic parameters based on α, β, ϑ, and equations for approximating separation results based on fitting parameter *a*

Upgrading plot	Upgrading parameters	Formulas based on α, β, ϑ	Form of Eq. 1 suitable for considered upgrading plot
Fuerstenau (1988/1992)	ε, ε_r	$\varepsilon = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}100$ $\varepsilon_r = (100 - \frac{\alpha-\vartheta}{\beta-\vartheta}100)\frac{100-\vartheta}{100-\alpha}$	$\varepsilon_r = a\frac{100-\varepsilon}{a-\varepsilon}$
Luszczkiewicz (2002)	ε, L	$\varepsilon = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}100$ $L = \frac{\alpha-\vartheta}{\beta-\vartheta}100(\frac{\beta}{\alpha} - \frac{(100-\beta)}{(100-\alpha)})$	$L = \varepsilon\frac{100-\varepsilon}{a-\varepsilon}$
Mayer (1950)	ε, γ	$\varepsilon = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}100$ $\gamma = \frac{\alpha-\vartheta}{\beta-\vartheta}100$	$\gamma = \varepsilon - \frac{\varepsilon(100-\alpha)(100-\varepsilon)}{100(a-\varepsilon)}$
Henry (1905)	β, γ	β $\gamma = \frac{\alpha-\vartheta}{\beta-\vartheta}100$	$\gamma = \frac{100(\beta(100-\alpha) - a(\beta-\alpha))}{\beta(100-\beta)}$
Stepinski V (1964, 1965); Drzymala, (2005, 2006)	ϑ/α, S	ϑ/α, $S = 100\frac{\beta(100-a)}{\alpha(\beta-a)}$	$S = 100\frac{\beta(100-a)}{\alpha(\beta-a)} =$ $= 100\frac{(H(100-\alpha)+100\alpha)(100-a)}{\alpha(H(100-\alpha)+100\alpha-100a)}$
Hall (1971)	H, β	β $H = \frac{100-\beta}{100-\alpha}100$	$H = \beta\frac{100-\varepsilon}{a-\varepsilon} =$ $\frac{100\alpha(\varepsilon-100)}{(100^2 - \alpha(100-\varepsilon) - 100a)}$
Halbich (1934)	β, ε	β $\varepsilon = \frac{\alpha-\vartheta}{\beta-\vartheta}\frac{\beta}{\alpha}100$	$\beta = \frac{100\alpha(\varepsilon-a)}{100^2 - \alpha(100-\varepsilon) - 100a}$
Stepinski I	β, ϑ	β $\vartheta = \frac{100\alpha - \gamma\beta}{100-\gamma}$	$\beta = \frac{\vartheta a}{\vartheta - 100 + a}$

α – content of a considered component in feed, β – content of a considered component in concentrate, ϑ – content of considered component in tailing, γ – yield, ε – recovery of the considered component in concentrate, ε_r – recovery of remaining (100% - considered component) in tailing, H – Hall parameter, L – Hancock (ε - ε_2) parameter where ε_2 denotes recovery of a second (here other than first component) in concentrate

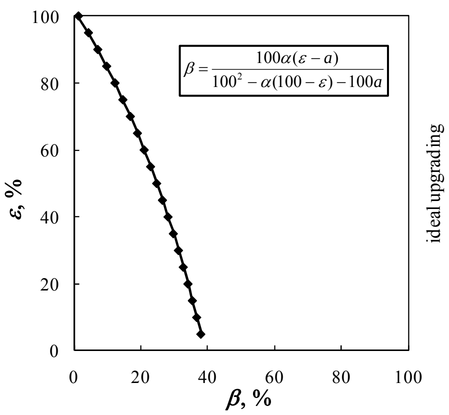

Fig. 8. Halbich's curve

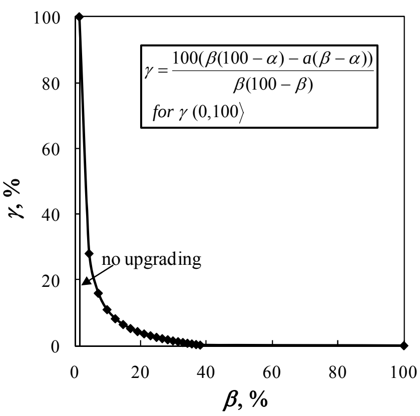

Fig. 9. Henry's curve

Arbitrary separation data with a = 102 and α =1.25% were fitted with Eq. 1 and plotted in Fig. 2 as the Fuerstenau upgrading curve. Presented in Fig. 2 data can be now re-plotted in other separation upgrading plots (Figs 3-9) and approximated with the newly derived (Table 3) equations.

3. Conclusions

The symmetrical in relation to diagonal, one-fitting parameter y = a(100 - x)/(a-x) equation used for approximation of separation results in the Fuerstenau upgrading plot can be transformed into one-fitting parameter equations consisting of a for any other separation upgrading plot. Some of them have been presented in this paper.

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References

Dell, C.C., 1969. An expression for the degree of liberation of an ore, Trans. Inst. Min. Metal., Sec. C, Mineral Process Extr. Metal., 78, C152

Digre M., 1960. Separation factor analysis for mineral dressing processes, IMPC, Group IX, Paper 49, p.999-1113, IMM publisher, London

Drzymala J, 2005. Evaluation and comparison of separation performance for varying feed composition and scattered separation results, Int. J. Miner. Process., 75, s. 189-196.

Drzymala J. 2006-8. Atlas of upgrading curves used in separation and mineral science and technology. Physicochem. Probl. Miner. Process., Part I, 40, 19-29, 2006; Part II, 41, 27-35, 2007; Part III, 42, 75-84, 2008.

Drzymala J., Ahmed H. A. M., 2005. Mathematical equations for approximation of separation results using the Fuerstenau upgrading curves, Int. J. Miner. Process., vol. 76 (1–2), s. 55–65.

Foszcz D., Niedoba T., Tumidajski T., 2010. Analiza mozliwosci prognozowania wynikow wzbogacania polskich rud miedzi uwzgledniajacego stosowana technologie, Gornictwo i Geoinzynieria z. 4/1, s. 25–36

Fuerstenau D.W., et al., 1988-1992, Coal surface control for advanced fine coal flotation, Final Report, University of California, Berkeley, Final Report DOE/PC/88878-T13, DE92 015625 for U.S. Dept. of Energy. Prepared by Univ. California, Columbia Univ., Univ. of Utah, and Praxis Engineers Inc.

Halbich W., 1934. Über die Anwendungsmöglichkeiten einiger Netzmittel in der Flotation, Kondrad Triltsch, Würzburg

Hall W.B., 1971. The mathematical form of separation curves based on two known ore parameters and a single liberation coefficient, Trans. IMM., Sec.C, 80, C213–C222.

Henry, 1905. Le lavage des charbons, Revue Universelle, ser. 4, vol. V, p.274 (Information after H. Czeczott, Przerobka mechaniczna uzytecznych ciał kopalnych, Krakow, 1937).

Jowett A., 1969. A mathematical form of minerals separation curves, Trans. IMM, Sec. C., 78, C185.

Krzanowski W. J., Hand D. J., 2009. ROC curves for continuous data, CRC Press/Taylor & Francis Group, Boca Raton, London, New York.

Laplante A., Kaya M., Smith H. W., 1989. The Effect of Forth on Flotation Kinetics-A Mass Transfer Approach, Mineral Processing and Extractive Metallurgy Review, vol. 5, s. 147-168.

Luszczkiewicz A., 1975. Korelacje pomiedzy podstawowymi wskaznikami przemyslowego procesu flotacji miedzi w swietle laboratoryjnych charakterystyk wzbogacalnosci, Physicochem. Prob. Miner. Process., 9, 121-131

Luszczkiewicz A., 2002. Evaluation of efficiency of separation of multi-component concentrates of disseminated elements, Prace Naukowe Instytutu Politechniki Wroclawskiej, nr 101, Konference 35, s. 87-103 (in Polish).

Luszczkiewicz A., Chmielewski T., 2008. Acid treatment of copper sulfide middlings and rougher concentrates in the flotation circuit of carbonate ores, Int. J. Miner. Process., vol.88, nr 1-2, s. 45-52.

Mayer F.W., 1950. Die Mittelwertkurve, eine neue Verwacksungskurve, Glückauf, 26, 498–509.

Neethling S.J., Cilliers J.J., 2008. Predicting and correcting grade recovery curves. Theoretical aspects, Int. J. Miner. Process., 89, 17-22

Nixon J.C., and Moir D.N., 1956-7. The assessment of flotation results, Trans. IMM (Bull. IMM), 66, 453-469.

Pudło W., 1971. O pewnej metodzie aproksymacji krzywych wzbogacania, Zeszyty Problemowe Gornictwa PAN 2, 83–103.

Stepiński W., 1964, Krzywe srednich wartosci jako miernik oceny dwustadialnej flotacji, Rudy i Metale Nieżelazne R9, nr 10, 532-535

Stepiński W., 1965, Krzywe srednich wartosci dwustadialnej flotacji rudy ubogiej, Rudy i Metale Nieżelazne, R10, nr 3, 117-120

Vera M.A., Franzidis J_P., Manlapig E. V. An empirical equation for recovery - enrichment ratio curve (AREV model), Coppper 99-Cobre 99 International Environment Conference, Volume II - Mineral Processing/Environment, Health and Safety, B.A. Hancock and M.R.L. Pon Eds, The Minerals, Metals & Materials Society, 1999, 69-82

Włodarski M., 2009. Porownanie parametrycznej i nieparametrycznej metody obliczania krzywej ROC na przykladzie zbioru sygnalow elektroretinograficznych, Metody Informatyki Stosowanej, nr 2 (19), s. 177-192.