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Adem TAŞDEMİR*, Hüseyin ÖZDAĞ*, Güven ÖNAL**

IMAGE ANALYSIS OF NARROW SIZE FRACTIONS OBTAINED BY SIEVE ANALYSIS - AN EVALUATION BY LOG-NORMAL DISTRIBUTION AND SHAPE FACTORS

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In this research, a pure quartz sample was subjected to sieve analysis and nine narrow size fractions were obtained. Polished sections were prepared from the representative samples of each fraction and were examined by image analysis (IA) to determine particle size distributions (PSDs) of fractions. Both size and shape measurements were made on individual quartz particles. Mean Feret diameter (*dF*) and three shape factors measurements, namely chunkiness (*Ch*), roundness (*R*) and form factor (*FF*), were carried out. This study showed that majority of particles in sieved fractions lied outside the nominal openings of the sieves. PSDs in all narrow sieve fractions were found to obey the log-normal distribution function. If number-based distribution of a system is found to be log-normal, the distribution of the derived diameters is also log-normal with the same geometric standard deviation. The number-based means obtained by IA were transformed to the volume (mass)-based means by using this property. The means of number- and volume (mass)-based IA sizes before and after correction by shape factors were compared with their corresponding geometric sieve means. Among the shape factors, *FF* was found as the most relating factor of sieve and IA sizes. The average of mean *FF* values of all size fractions was equal to 0.78. Reciprocal of this value (1.29) was very close to the slope of 1.28 obtained from the volume (mass)-based means of IA versus geometric sieve means relation. This result suggests that the slopes of the lines can provide a measure of differences between sieving and IA and this was related to *FF* values for quartz when d_F was used as IA size.

keywords: particle size distribution, image analysis, sieving, log-normal distribution, shape factor

* Eskişehir Osmangazi University, Dept. of Mining Engineering, Division of Mineral Processing, 26480, Eskişehir-Turkey, atasdem@ogu.edu.tr, Tel: +90 222 239 37 50 ext:3438

** Istanbul Technical University, Faculty of Mines, Dept. of Minerals Engineering, İstanbul-Turkey

1. INTRODUCTION

The quantification of particle size and shape is problematic for researchers because descriptive single parameter measurements of particle morphology do not exist (Meloy, 1984). For a very specific material, a single method of determining size and shape can be sufficient to describe differences between individual particles of that material. However, a combination of methods is often required to provide more precise quantification of the size and shape parameters (Haughton and Amidon, 1992).

There are many different methods available for particle size and shape analysis. Size characterization is simple for spherical particles. For irregular particles it is not, and therefore the assigned size depends on the method of measurement. A comparison of results obtained from different particle size measurement methods is often required. Therefore, it is necessary to determine conversion factors between the methods when it is needed.

Despite a long and successful use in most applications, sieving is still neither a precise nor accurate technique of particle-size measurement. When particle size distribution (PSD) parameters are calculated from the sieve analysis, it is generally assumed that the distributions within the sieve fractions are linear (normal distribution) and produce a mean particle size equal to the average of the sieve intervals or that a more proper number is the geometric mean of these two apertures (Carstensen and Dali, 1999).

Among the methods used for size characterization, microscopy based image analysis (IA) is the only commonly used method in which individual particles are viewed and measured. An advantage of this method is that both various sizes as well as qualitative and quantitative shape information can be obtained simultaneously.

Particle size, like other variables in nature, tends to follow well-defined mathematical laws in its distribution. PSDs have been described by relations between particle mass and size, volume and size, number and size, and number and mass. Different mathematical formulas and approaches have been suggested to express relations against size, and total mass/volume/number is given against increasing size (mass/volume/number) of undersize and to make distribution curves linear, especially the size distribution. PSDs of fragmented geological materials have been described previously by at least seven relations between mass/volume/number and mass/size of fragments (Blenkinsop, 1991).

One major difficulty remaining with the image analysis is that the primary PSDs are number-based. This means that a direct comparison of results from IA with sieving, which yields mass-based size distributions, is generally not possible. In order to compare IA and sieve results, it is necessary to transform the IA results into volume (mass)-based distributions.

The log-normal law is frequently found with particulate systems. Since the particle size is plotted on a logarithmic scale, the presentation of data on a log-probability graph is particularly useful when the range of size is large (Allen, 1997; Svarovsky, 1981). The other usefulness of the log-normal distribution is that if the number distribution of a particulate system is found log-normal, the mass distribution is also log-normal with the same geometric standard deviation (Allen, 1997). Therefore, the other mean sizes and their distributions can be determined by using this property. This method was applied successfully for determination of unbroken grain size distribution of bonded chromites (Taşdemir et al., 1997; Taşdemir and Bozkurt, 2007; Taşdemir, 2008) and the other different types of chromite ores (Taşdemir, 2008).

In this research, PSDs and shape factors of narrowly sieved size fractions of quartz sample were determined for polished sections which were prepared from nine sieved fractions by using IA. The mean Feret diameters were used as the IA size and its PSDs of sieve fractions were evaluated statistically. It was found that all PSDs of size intervals obeyed the log-normal distribution function very well. Using the log-normal distribution properties, number-based means of size fractions were converted into volume (mass)-based means and these IA means were compared with their geometric means of sieve intervals before and after correction by three shape factors, namely chunkiness (*Ch*), roundness (*R*) and form factor (*FF*).

2. MATERIAL AND METHOD

2.1. SAMPLE PREPARATION

This paper presents a comparison of two different methods of measuring particle size distribution of a glass quality quartz sample. Natural quartz is often used as a reference mineral in most mineral processing studies including comminution, flotation, etc. Therefore, a pure quartz sample from Çine/Aydın (Turkey) was chosen and used in the experiments. The quartz sample produced for the glass production was supplied from the Ak Mining Co. The representative sample of quartz was sieved using sieve series including 600, 500, 400, 300, 212, 150, 106, 75, 53 and 38 μm in sizes.

The sieve analysis was initially carried out manually by wet sieving until there was no particle passage under the sieves. At the end of sieving, each sieve was subjected to ultrasonic bath for cleaning. This process also allowed the particles, especially elongated particles trapped between the in the sieve apertures, to pass through the sieve.

The sieve diameter of a particle is usually defined as a geometric mean of openings of the last sieve through which the particle passes and the sieve on which it is retained (Kennedy and Koh, 1961). This is the sieve diameter definition that will be used here. Geometric means of sieve intervals were calculated as:

$$D[S,50]=\sqrt{D^{i}\cdot D^{i-1}}\ , \quad (1)$$

where D^i is the upper sieve size through which a particle can pass and D^{i-1} is the sieve aperture that the particle cannot pass.

Representative samples of dried powders of each sieve fractions were prepared as polished sections by using low viscosity epoxy resin which was mixed with appropriate amount of hardener. The air bubbles which occurred while stirring the epoxy resin/hardener mixture were removed by vacuum. After pouring a small amount of this mixture into a molding assembly, the particles of each narrow-sieved fraction were carefully poured into resin/hardener mixture in the mounting cup where the particles slowly sunk. After ensuring all the surfaces of the particles were wet and particle completely settled, the powders and the resin were stirred, to have them thoroughly mixed. The entrapped air bubbles during this process were also removed by vacuum. Since the epoxy-hardener mixture used cures over eight hours, enough time was given to all grains without preferred orientation on the polished section surface. As a result, the particles on the surface of the polished sections were randomly oriented and laid in their most stable positions. A grinding and polishing flow sheet was developed and applied successfully for the polished sections of coarse and fine sieve fractions.

2.2. SIZE AND SHAPE MEASUREMENTS BY IMAGE ANALYSIS

The size and shape measurements on polished sections of sieve fractions were performed with a Leco 2001 image analyzer. Two-dimensional images of quartz particles were produced by Olympus reflected light microscopy associated with the image analyzer. Each particle was measured individually.

In this research, the measurements such as area (A), perimeter (P) and Feret diameter d_F, which is the distance between two parallel lines tangent to the projected cross-section in eight different directions, i.e. at angular resolution of 22.5^o were performed on the individual quartz particles. In this study. The Mean Feret diameter calculated by averaging these eight measures was used as the IA size. Also, comparisons between D[S,50] and the mean size values, which were obtained from multiplying the mean IA sizes of sieve fractions with their corresponding mean of shape factors, namely chunkiness *(Ch)*, roundness *(R)* and form factor *(FF)*, were carried out. These shape factors were calculated as:

$$Chunkiness(Ch) = [Width(W), min.Feret]/[Feret\ length\ perpendicular\ to\ W]\ , \quad (2)$$

$$Roundness(R) = (4\pi A)/(P^2)\ , \quad (3)$$

$$Form\ Factor\ (FF) = \sqrt{R}\ . \quad (4)$$

2.3. LOG-NORMAL DISTRIBUTION

The probability density function $f(x)$ of the log-normal distribution is given by the following equation (Al-Thyabat and Miles, 2006):

$$f(x) = \frac{1}{\ln x \ln \sigma_g \sqrt{2\pi}} \exp\left[-\frac{(\ln x - \ln x_g)^2}{2 \ln^2 \sigma_g}\right], \tag{5}$$

where x is the particle size.

The log-normal distribution is a two-parameter function (geometric mean, $\ln x_g$ and geometric standard deviation $\ln \sigma_g$). The parameters of the log-normal distribution can be calculated by the following equations (Ang and Tang, 1975; Koch and Link, 1970):

$$\ln x_g = \ln \mu - \frac{1}{2} \ln^2 \sigma_g , \tag{6}$$

$$\ln \sigma_g = \sqrt{\ln\left(\frac{\sigma^2}{\mu^2} + 1\right)} , \tag{7}$$

where μ is the normal arithmetic mean of distribution and σ is the standard deviation of the distribution.

If the measured data points show a pattern of linearity in log probability drawing paper, then they can be considered to follow the log-normal distribution.

2.4. RELATIONS BETWEEN MEAN SIZES FOR THE LOG-NORMAL DISTRIBUTION

Since the PSDs measured by an image analysis system are number-based, the geometric mean calculated by Eq. (6) is the geometric mean of number distribution ($\ln x_{gN}$). If a number distribution is found to obey log-normal distribution, length, surface and volume means are calculated by using the same geometric standard deviation found for number distribution. Details of the derivation of these formulas can be found in details in the literature (Allen, 1997 and Svarovsky, 1981). Once a number geometric mean and geometric standard deviation of a log-normal distribution are determined, mean of number-length and volume (mass) means can be calculated by using the following formulas:

$$\ln x_{NL} = \ln x_{gN} + \frac{1}{2} \ln^2 \sigma_g , \tag{8}$$

$$\ln x_{VM} = \ln x_{gN} + 3.5 \ln^2 \sigma_g . \tag{9}$$

Equations (8) and (9) were used as number-based and volume (mass)-based means of log-normal distribution, respectively.

3. RESULTS

3.1. PROPERTIES OF PSDS IN NARROWLY SIEVED FRACTIONS

The PSDs of all sieve fractions of the quartz sample were first evaluated and tested by normal distribution because narrowly sized PSDs are expected to be normally distributed (Carstensen and Dali, 1999). However, it was found that the PSDs of sieved fractions were not neatly normal. The PSDs of sieve fractions are plotted on a log-normal scale plot and resulting graph is presented in Fig. 1. It can be clearly seen, that the data points of size fractions show an appropriate pattern of linearity (Fig. 1), and thus they can be considered to follow the log-normal pattern very well. The log-normal distribution plots of sieve intervals do not deviate from the linearity and provides straight lines for the entire range of measured grain sizes. Coefficients of determination (R^2) were over 0.99 for all sieve fractions. These results suggest that the properties of log-normal distribution can be applied easily to the PSDs of all fractions to obtain various mean diameters and their distributions.

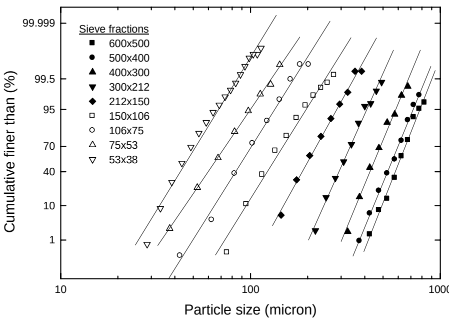

Fig. 1. Size distributions of sieve fractions plotted on log-normal drawing paper

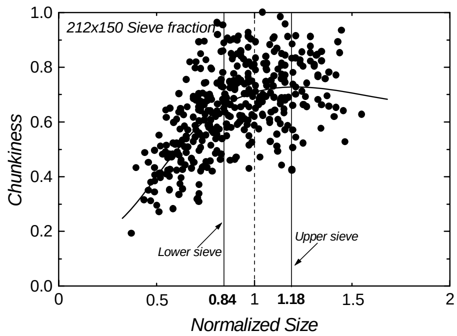

Fig. 2. Chunkiness-normalized size domain for 212x150 μm size fraction

In Fig. 2, chunkiness shape factor against size domain where the information on shape with respect to size for an individual particle can be obtained is presented for 212x150 μm size fraction as an example. Similar results were also obtained for other narrow sieve fractions. In this figure, the width values of individual particles (smallest Feret size) were normalized by the geometric mean of the sieve fraction. It is seen clearly that all of the particles are not within their sieve ranges and actual particle size limits are not confined in this region. This figure also shows different shape properties of particles that are found in the same sieve interval. Particles that were found in the defined sieve range were more regular than other particles. The particles that were smaller than the sieve size limit were more irregular since they had comparatively smaller chunkiness values, i.e. were more elongated. However, the particles which were larger than the sieve ranges were more regular in the shape as the particles lied inside the nominal sieve openings. According to this finding, the particles within a sieve interval might have many different particle shapes, and the *PSD*s of narrowly sieved fractions were wider than the sieve limits and were not narrowly distributed.

3.2 STATISTICAL PARAMETERS OF LOG-NORMAL DISTRIBUTION AND EVALUATION WITH SHAPE FACTORS

In this study geometric means of sieve fractions (*D*[S,50]), arithmetic mean of mean Feret diameters (μ) and standard deviation (σ) obtained by the IA for each sieve fraction were found. For each sieve interval, (μ) and (σ) parameters were converted to number geometric means ($\ln x_{gN}$) and geometric variances ($\ln^2 \sigma_g$) by using Eqs. (6) and (7) to find the log-normal distribution parameters. The x_{NL} and x_{VM} values were calculated by using Eqs. (8) and (9). Also means of *Ch*, *R* and *FF* shape factors of each sieve fraction were determined by Eqs. (2-4). The statistical data of the nine sieve intervals for size and shape are shown in Table 1. According to these results, the mean particle shapes varied in sieve fractions and were not constant, indicating that they were generally size dependent. Both number- and volume (mass)-based IA means of all sieve intervals are larger than the calculated *D*[S,50].

Table 1. Statistical parameters of mean Feret diameters and mean shape factors measured in narrowly sieved intervals by IA

Sieve fractions, μm	Geometric means D[S,50]	μ	σ	$\ln x_{gN}$	$\ln \sigma_g$	x_{NL}	x_{VM}	*Ch*	*R*	*FF*
600x500	547.72	610.94	94.69	6.40	0.16	611.64	658.24	0.74	0.56	0.75
500x400	447.21	557.58	91.85	6.31	0.17	557.78	606.69	0.71	0.61	0.78
400x300	346.41	443.67	74.59	6.08	0.16	443.65	481.05	0.73	0.61	0.78
300x212	252.19	309.07	55.68	5.72	0.18	309.08	339.58	0.72	0.62	0.79
212x150	178.33	201.46	44.22	5.28	0.21	201.38	230.07	0.65	0.59	0.76
150x106	126.10	129.59	32.69	4.84	0.24	129.48	153.25	0.65	0.61	0.78
106x75	89.16	91.98	21.93	4.49	0.23	91.97	108.09	0.63	0.62	0.78
75x53	63.05	66.34	18.06	4.16	0.27	66.34	81.53	0.64	0.58	0.75
53x38	44.88	45.29	10.65	3.79	0.22	45.24	52.26	0.65	0.69	0.83
							Mean	0.68	0.61	0.78
							1/Mean	1.47	1.64	1.29

The plots of x_{NL} and x_{VM} versus D[S,50] are shown in Figs (3-6). It should be noticed that the slope values on all plots in the figures were obtained from simple linear equations but the results were presented as log-log graphs for all plots given here to see the differences more clearly for the smaller sieve fractions. There was a simple linear correlation between D[S,50] and x_{NL} as well as x_{VM} sizes in all cases. These basic relations gave straight lines. The slopes of the lines may provide a measure of the discrepancy between the IA sizes and sieve sizes.

For investigation of possible relations between calculated shape factors and x_{NL} as well as x_{VM} slopes value, the number- and volume (mass)-based IA sizes were multiplied by their corresponding shape factor values and their means were compared with the corresponding D[S,50]. The slopes obtained before and after correction were evaluated as the deviations from the ideal 1:1 line.

Figs (3) and (4) give the x_{NL} and x_{VM} IA means against their corresponding D[S,50], respectively. The values obtained from x_{NL} and x_{VM} calculations deviated from the 1:1 line with slopes of 1.21 (Fig. 3) and 1.28 (Fig. 4), respectively.

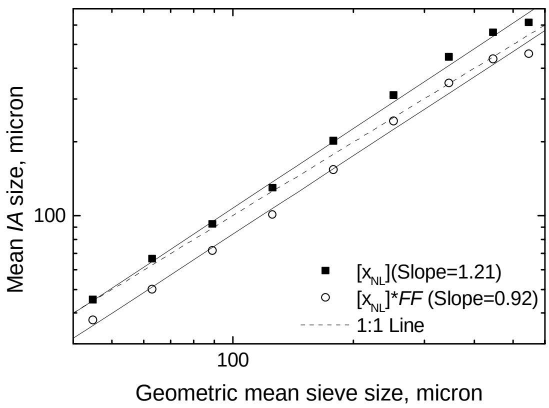

Fig. 3. Relationship between number-based IA means versus mean sieve sizes (■) and corrected number-based IA means by *FF* shape factor against geometric mean of sieve fractions (○)

After correction with the *FF* values, the deviations are smaller and have the values of 0.92 (Fig. 3) and 0.97 (Fig. 4) for x_{NL} and x_{VM} respectively. The close proximity of the deviation for x_{VM} to the ideal slope (1) shows that the volume (mass)-based values transformed from the log-normal distribution and corrected by *FF* agrees closely with that of sieving. The reciprocal of the average of the mean *FF* values of all sieve fractions was found as 1.29 (Table 1), which was very close to the slope value calculated before the correction by the shape factors for the D[S,50] versus x_{VM} plot (1.28) (Fig. 2). In a previous work, geometric mean of the minimum Feret (width) and maximum Feret (length) were used as the IA size and compared with the D[S,50] (Taşdemir et al., 2009). The slope was 1.23 for the sieve size means against the volume (mass)-based means which were calculated in a different manner from this study. It was found that the mean Feret diameter (d_F) gave better results than the previous work and improved the results.

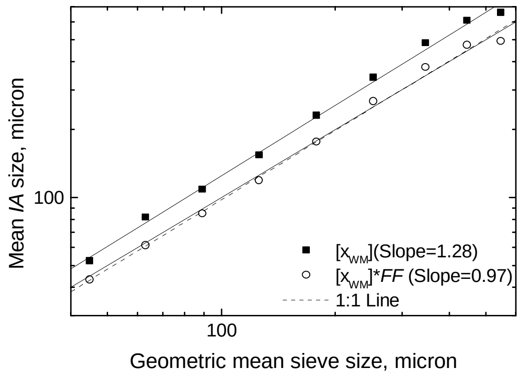

Fig. 4. Relationship between volume (mass)-based IA means versus mean sieve sizes (■) and corrected volume (mass)-based IA means by *FF* shape factor against geometric mean of sieve fractions (○)

The IA values corrected by *Ch* and *R* versus *D*[S,50] are given in Fig. 5 and Fig. 6, respectively. The result was also good when the x_{NL} and x_{VM} values were multiplied with their corresponding *Ch* values since the slopes were 0.90 and 0.96 for these relations, respectively, and the deviations from the 1:1 line were quite small (Fig. 5). But these results were not as good as the one which was obtained by using the *FF* values since fewer points were on the 1:1 line. In addition, neither the mean *Ch* (0.69) nor reciprocal of the *Ch* value (1.47) of sieve fractions give an approximate value of the slope as in the *FF*.

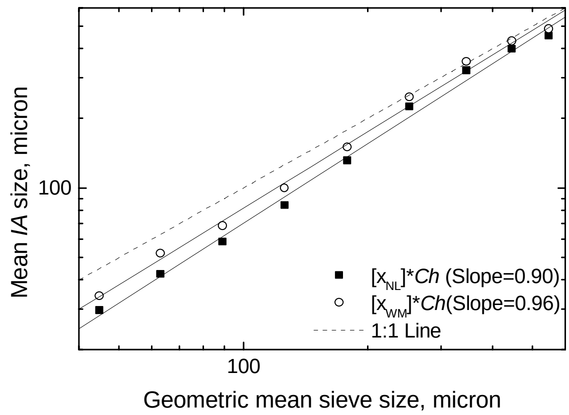

Fig. 5. IA means corrected by *Ch* against geometric mean of sieve fractions for number-based means (■) and volume(mass)-based means (○)

After correction of the IA results by R values of particles, the slopes were 0.70 and 0.75 for the x_{NL} and x_{VM} means respectively and the deviations were larger than the ones which were obtained by the *FF* and *Ch* factors. Although the relations were also linear, when R was used as a correction factor, there were no points on the 1:1 line and the points were very far from the 1:1 line in both cases (Fig. 6). From these results, the differences between IA of polished section and sieving for quartz sample can be attributed to the effects of particle shape and this relation is related mostly to the *FF* of particles.

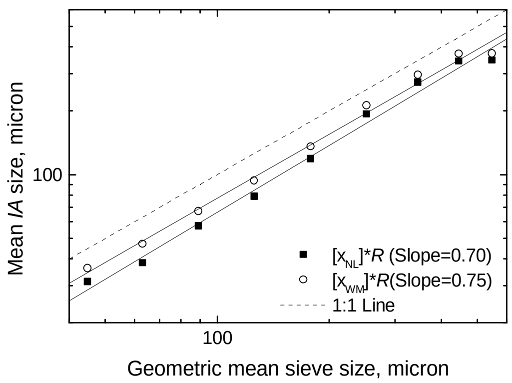

Fig. 6. IA means corrected by R against geometric mean of sieve fractions for number-based means (■) and volume (mass)-based means (○)

4. CONCLUSION

In this work, particle size and shape measurements of narrowly sieved fractions were measured on polished sections for pure quartz by using image analysis. Mean Feret diameter and three shape factors namely chunkiness, roundness and form factor, were obtained for individual particles in the sections. The PSDs of all sieve fractions were found to obey the log-normal distribution, not normal distribution as expected. The number-based distributions were converted into volume (mass)-based distributions by log-normal distribution properties and their means were compared with the geometric means of sieve intervals before and after correction by the shape factors.

This study was also focused on the possible conversion of factor between the IA size measured on polished sections of particulate materials and sieve size by using measured shape factors. It was concluded that the mass-based means multiplied by the *FF* values of sieve fractions gave the best results with the geometric sieve means. The slope of the mass-based means against sieve means was 1.28 and this value was very near to reciprocal of the mean value of *FF* shape factors of all sieve fractions (1.29).

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Taşdemir, A., Özdağ, H., Önal, G., *Analiza obrazu wąskich klas ziarnowych otrzymanych za pomocą analizy sitowej – ocena z użyciem log-normalnej dystrybucji oraz współczynnika kształtu*, Physicochem. Probl. Miner. Process., 46 (2011) 95-106, (w jęz. ang), http://www.minproc.pwr.wroc.pl/journal

Próbki czystego kwarcu poddano analizie sitowej w celu wydzielenia dziewięciu wąskich klas ziarnowych. Klasy te użyto do przygotowania zgładów, które analizowano za pomocą komputerowej analizy obrazu (IA) w celu wyznaczenia składu ziarnowego (PSDs) każdej klasy. Określano zarówno rozmiar jak i współczynniki kształtu poszczególnych ziarn. Dokonano pomiarów średniej średnicy Fereta (d_F) oraz trzech współczynników kształtu (krępość *Ch*, zaokrąglenie *R*, wskaźnik kształtu *FF*). Wykazano, że wymiary większości ziarn frakcji z analizy sitowej znajdowały się poza nominalnymi rozmiarami sit. PSDs wszystkich wąskich klas ziarnowych można było opisać log-normalną funkcją rozkładu. Jeżeli oparta na liczbie ziarn dystrybucja spełnia rozkład log-normalny, wynikająca średnica jest także opisywana funkcją log-normalną z tym samym geometrycznych odchyleniem standardowym. Średnie średnice oparta o liczbę ziarn, otrzymana za pomocą IA, zostały przeliczone na średnią objętościową (masową). Oznacza to, że wymiary oparte o liczbę i objętość (masę) IA przed i po korekcie za pomocą współczynnika kształtu były porównywalne z ich odpowiednimi geometrycznymi średnimi średnicami sitowymi. Stwierdzono, że wśród współczynników kształtu, wskaźnik *FF* okazał się najlepszy dla powiązania rozmiaru sitowego z rozmiarem z IA. Średnia wartość *FF* dla wszystkich frakcji wyniosła 0.78. Odwrotność tej liczby (1.29) jest bardzo bliska nachyleniu 1.28 otrzymanemu z średniej średnicy objętościowej IA wykreślonej jako funkcja geometrycznej średniej średnicy sitowej. Sugeruje to, że nachylenie tych zależności może dostarczyć miary różnicy wyników otrzymanych z przesiewania a z IA. Wyjaśniono to wartościami *FF* kiedy d_F jest użyte jako rozmiar oparty o IA.

słowa kluczowe skład ziarnowy, komputerowa analiza obrazu, przesiewanie, dystrybucja log-normalna, współczynnik kształtu