Physicochem. Probl. Miner. Process. 49(1), 2013, 121−131

www.minproc.pwr.wroc.pl/journal/

Physicochemical Problems
of Mineral Processing

ISSN 1643-1049 (print)
ISSN 2084-4735 (online)

Received June 5, 2012; reviewed; accepted July 4, 2012

MATHEMATICAL MODELS OF PARTICLE SIZE DISTRIBUTION IN SIMULATION ANALYSIS OF HIGH-PRESSURE GRINDING ROLL OPERATIONS

Daniel SARAMAK

AGH University of Science and Technology, Faculty of Mining and Geoengineering, Department of Environmental Engineering and Mineral Processing, Mickiewicza 30 Av., 30-059 Krakow, Poland, Fax: +48 12 617 21 98, Mobile: +48 692 597 032, Email: dsaramak@agh.edu.pl

Abstract: The high-pressure grinding roll (HPGR) technology is currently one of the most efficient methods of hard ore comminution from the scope of the energy consumption. Throughput and energetic models of performance are quite well developed, but technological models predicting the comminution effects still needs an in-depth research. In the paper author presents the method of modeling of HPGR products' particle size distribution by using suitable mathematical models. A basis of the HPGR performance assessment and HPGR-based crushing circuits design is a product's particle size distribution. In order to precisely determine the particle size distribution of a HPGR product, parameters of product's particle size distributions should be conditional on the main technical parameters of the roller press like operating pressure and the speed of rolls. The most accurate approximation of a distribution of size particle appears to be Weibull's truncated distribution and it is possible to obtain significant relationships between the approximation formula and the value of operating pressure, or feed characteristics. It makes possible a determination of mass recoveries of the product's respective size fractions and the productivity planning. Instead of pressure, the other HPGR operating parameters can be applied, but the pressure value appears to produce the most relevant relationship. A determination of HPGR product's particle size distribution is a basis of the press operation simulation analysis.

Keywords: *HPGR, comminution, particle size distribution, ore processing*

Introduction

Comminuting circuits constitute an integral part of mineral processing technology and a basis of the ore enrichment process. The work efficiency of ore pretreatment circuits for downstream upgrading processes determines in fact the effectiveness of the overall process because it enables proper liberation of useful minerals from the feed to take part. It is especially significant in base metals and non-ferrous ore processing, where liberation of useful components is achieved by obtaining the comminution ratio, suita-

http://dx.doi.org/10.5277/ppmp130112

ble for a given type of ore, together with avoiding feed overgrinding, which increases the paymetal losses (Saramak et al. 2010; Tumidajski 2010).

High-pressure comminution is currently the one of most effective method of the material size reduction. Main advantages of the technology include:

- low unit energy consumption, from 0.8 to 3 kWh (2,880 to 10,800 kJ respectively) per tonne of treated material,
- micro-crack generation, which gives the benefit in downstream grinding processes (usually in ball mill). As a result of the micro-crack formation, the process energy-consumption, measured with using the Bond work index, can be reduced for up to 30%, according to various investigations,
- increasing of useful mineral grade in ore upgrading operations, improvement of the weight recovery index for the entire process,
- dry and wet crushing options, with the feed material moisture up to 10%,
- high availability ($T_e > 95\%$),
- low dust pollution, low noise and vibration emision,
- low footprint comparing to SAG mills or even vertimills

High-pressure comminution operations also affect the grinding kinetics in downstream milling stage, through shortening the grinding time. Another method of enhancing the grinding effectiveness is an increase of grinding media charge (Olejnik 2006), but here the gain is obtained with no extra cost of grinding media.

The application of the high-pressure grinding rolls (HPGR) in comminution circuits is currently a key issue in many mining and mineral processing sectors and HPGR principles are well presented in many works (Schoenert, 1988; Morley, 2003; 2010; Maxton et al., 2003; Daniel and Morell, 2004; Naziemiec and Saramak, 2009; van der Meer and Gruendken, 2010; Saramak, 2011a; 2012). HPGR devices are present in industrial circuits of cement production (Kurdowski, 1998; Gawenda, 2009) and in hard ore processing technology (Schoenert, 1988) since the mid-nineties and the development in HPGR applications in ore pretreatment technology is a current worldwide trend. The investigations of grinding processes in HPGR devices are still in progress and the technical level of current industrial presses was obtained at the beginning of this century (Morley, 2003). The HPGR technology is currently the most effective method, as far as energy consumption and hard minerals comminution are concerned and can be a good alternative for semi-autogenous (SAG) and even tower mills (Kalinowski, 2006).

Taking into consideration the above aspects, the main aim of the paper is a proposal of approximation of HPGR crushing products with using of specific theoretical distribution. All parameters of the selected theoretical particle size distribution will be combined with the main operating parameter – the operating pressure (P). A knowledge of the crushing product's quality described through a particle size distribution curve is significant in terms of the process description and optimization of crushing and grinding circuit's performance.

Crushing product particle size distributions

The feed for HPGR-based crushing circuits can be analyzed with regards to its physico-mechanical properties, mineralogical composition, particle size distribution and other features. These features, in turn, determine the effect of comminution process, assessed on the basis of the quality of grinding products, process energy consumption, process capacity and economic indices. The HPGR feed characteristics usually take into consideration only the d_{max} value (maximum particle size), and that appears to be insufficient for proper modeling of the HPGR comminution processes. Besides d_{max}, knowledge of individual size fraction participations in the feed (product particle size distribution), especially the mass recoveries of fines and coarsest particles, is required (Saramak 2011b).

Particle size distributions of grained material processed in mineral processing plants are important, because they generate distributions of other features of particles, which determine the course of enrichment operations. A modification of material particle size during the comminution process (size reduction), plays a key role in the assessment of process energy-consumption and susceptibility of the ore to downstream upgrading processes. Particle size distributions are also significant in heuristic modeling of various enrichment processes (Tumidajski and Saramak 2010).

A theoretical approach to determination of particle size distribution of crushing products was presented by Kolmogorow (1941) and Epstein (1947), who proved that for a multi-stage comminution process, the particle size distributions of products are log-normal. The product particle size distribution depends on the crushing process intensity (stage). Let’s consider the comminution of crystalline particles. When a full liberation of these grains is achieved (the level of grain boundaries is reached), the size distribution of these particles should tend towards either log-normal or logistic distribution, because the content of very fine particles should be rather low, together with expected lack of coarser (uncrushed) grains. According to the nature of the grained material, the cumulative distribution proposed for description of the empirical data, should reach a value of 1 for the maximum particle size ($d_{\max}$) and therefore there should be applied truncated distributions. A common modification of Rosin–Ramler–Bennets’ distributions, is the truncated distribution, utilizing the expression $w=\left(\frac{d}{d_{\max}-d}\right)$:

$$\Phi(d)=1-\exp\left[-c\left(\frac{d}{d_{\max}-d}\right)^{n}\right],\quad 0\le d\le d_{\max} \tag{1}$$

where: d – particle size;

c, n, $d_{\max}$ – parameters determined on the basis of the empirical data ($d_{\max}$ – maximum particle size in crushing product)

$\Phi(d)$ – cumulative particle size distribution curve.

w –expression converting the regular (untruncated) distribution into truncated one

An advantage of the above formula is using of a third parameter, $d_{\max}$, which denotes the maximum particle size, and the particle size distribution function has its course within a range of <0, $d_{\max}$>, instead of the <0, +∞), which is used for regular Weibull's distribution. The application of $d_{\max}$ parameter also increases the accuracy of approximation. Analogous truncated formulas utilizing the expression w, applied in description of particle size distribution curves, are the log-normal and logistic distributions (Tumidajski, 2012).

The approximation procedure utilizing formula (1) is carried out with using the least sum of squares method. After taking a double logarithm of formula (1) we obtain the following equation:

$$\ln\ln\frac{1}{1-\Phi(d)} = n\ln\left(\frac{d}{d_{\max}-d}\right) + \ln c \tag{2}$$

which has a linear form and where c is a constant.

In order to determine n and c parameters, one should accepted such $d_{\max}$ value for which the residual deviation value is minimal:

$$s_r = \sqrt{\frac{\sum_{i=1}^{p_s} \Phi_e(d_i)-\Phi_t(d_i)^2}{p_s-2}} \tag{3}$$

where: p_s – a number of sieves with aperture d_i,

$\Phi_e(d_i)$ empirical cumulative distribution function

$\Phi_t(d_i)$ theoretical cumulative distribution function, determined by using formula (1).

Calculations should be carried out for all $d_{\max}$ values greater from the maximum sieve aperture and the values of n, c and $d_{\max}$, which minimize the s_r value should be accepted in further analyses. When the approximation formula reveal a significant convergence with the empirical data, it is possible to combine its parameters with the feed material properties and the crusher operating parameters. The analysis of relationships between technological and technical operating parameters of the roller press makes it possible to divide them into four groups (Saramak, 2010):

- parameters controlled directly (operating pressure, speed of rolls),
- parameters controlled indirectly (gap, skew, torque),
- parameters combined with the feed material properties,
- indices (i.e. specific energy consumption, specific throughput).

The operating pressure (*P*) has the most significant importance on the roller press operating efficiency. The operating pressure level influences the gap width and, as a result, the $d_{\max}$ value. Relationship between the gap (*s*) and operating pressure is presented in Fig. 1.

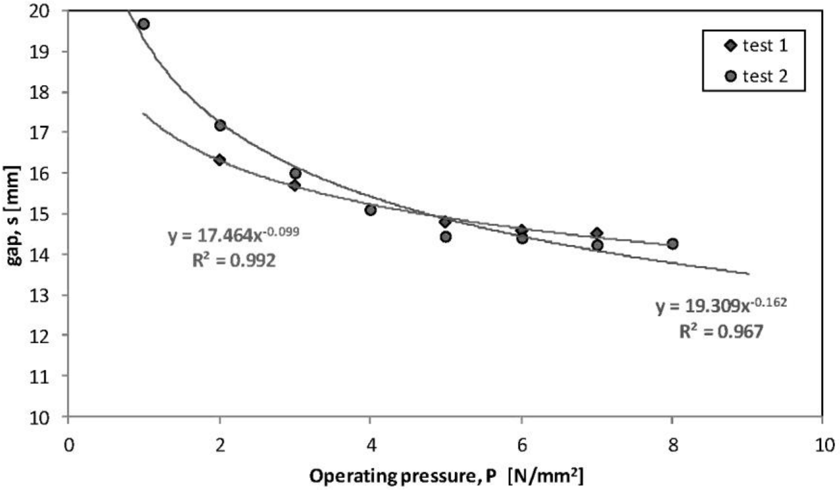

Fig. 1. Relationship between operating pressure and gap in roller press (tests for porphyry) (Saramak, 2011 c) for two series of similar laboratory tests denoted as "test 1" and "test 2".

The shape parameter *n* can be directly combined with the operating pressure value, because the finer product, the lower *n* value in formula (1). The material size reduction, in turn, is directly proportional to pressure level. The scale parameter *c* is combined with the feed material physico-mechanical properties. Changes in *c* value cause a diverse run of the particle size curve approximated with using of formula (1), resulting in existence of the inflection point corresponding to material grindability. The $d_{\max}$ parameter reflects the material size reduction, what can be combined with the gap width and, as a result similarly to *n*, with the operating pressure value. The HPGR crushing process efficiency assessment, considered from the scope of energy, technology or ecology, can be done utilizing the obtained approximation formulas.

Experimental programme

A series of HPGR laboratory crushing tests of copper ore for various operating pressure levels within the range from 1 to 8 N/mm^2 were run. A particle size distribution curve was determined for each crushing product, the results are presented in Fig. 2.

Theoretical cumulative distribution functions were calculated by using truncated Weibull's formula (1). Values of coefficients in the formula are presented in Table 1 and in Figs 3 and 4.

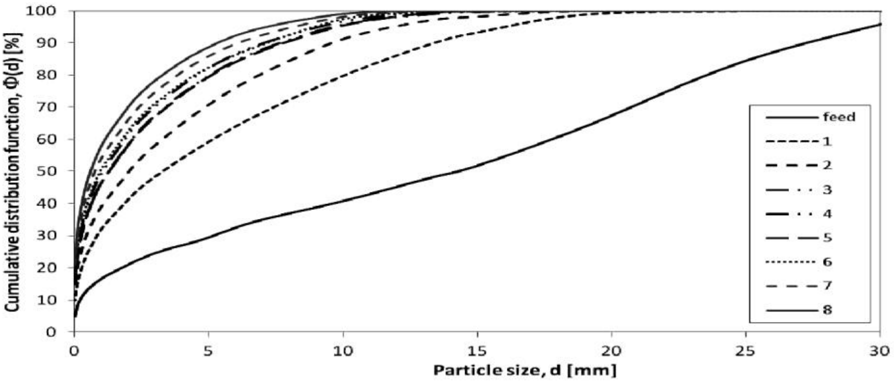

Fig. 2. Particle size distribution curves for all crushing products. Numbers from 1 to 8 denote single batch tests for respective value of operating pressure (from 1 to 8 N/mm^2).

Table1. Values of coefficients in formula (1)

	Operating pressure, P [N/mm^2]							
	1	2	3	4	5	6	7	8
n	0.461	0.443	0.428	0.431	0.432	0.421	0.416	0.413
c	1.868	2.005	2.365	2.359	2.139	2.245	2.467	2.610
d_{max}	25.7	19.6	19.4	16.3	16.0	15.5	15.5	13.9
s_r	2.70	2.37	2.12	1.65	1.38	1.17	1.31	1.21

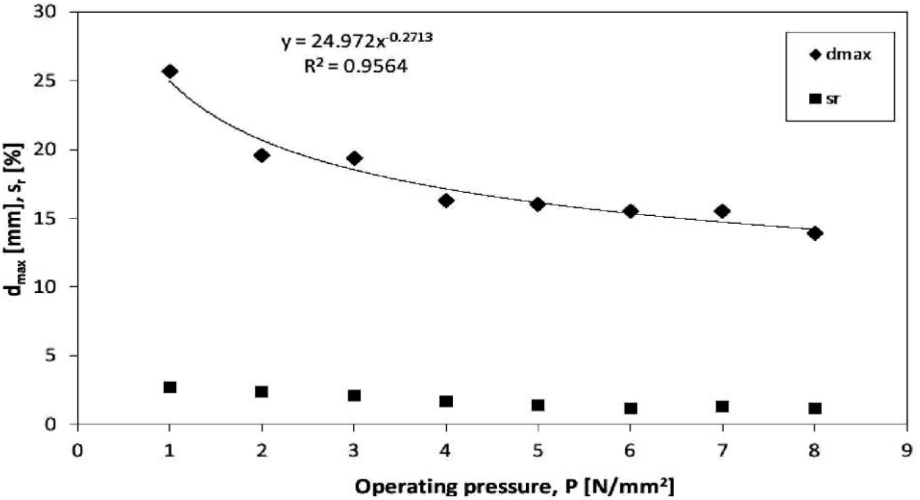

Fig. 3. Course of $d_{\max}$ and s_{r} parameters dependening on the operating pressure P

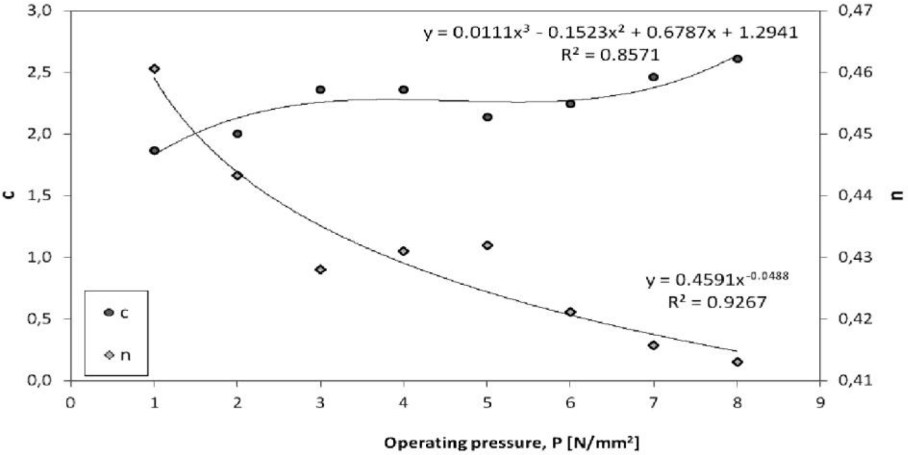

Fig. 4. Course of *n* and *c*parameters dependening on the operating pressure *P*

The next stage included the approximation of *n*, *c* and d_{max} parameters course. It was possible to determine an approximation function for each parameter on satisfactory accuracy level. The course of *c* parameter is well described by polynomial of third degree (Fig. 4) what has its practical justification. Following ranges can be distinguished in the course of *c* parameter approximation function:

- range from 1 to 4 N/mm^2, where *c* values increase – it illustrates the entire comminution process at relatively lower pressure values. Size reduction is directly proportional to the pressure, but comminution efficiency is less intense together with the further increase of *P* value,
- range from 4 to 6 N/mm^2, where *c* values are generally stable. In comminution process this stage corresponds to the situation where almost full liberation of particles, (up to the grain boundary) was achieved. The size of most particles was reduced along the grain boundaries, micro-cracks formation is also observable, but the pressure is not high enough to disintegrate a single grain structure,
- range above 6 N/mm^2. The highest pressure values cause crushing of single structures of grains, size reduction index increases again, until the maximum operating pressure for the device is achieved.

The d_{max} parameter course is very well approximated by the hyperbolic function (Fig. 3). Significant decrease of d_{max} for lower pressure values is observed, while for the higher *P* values, d_{max} course stabilizes itself, reflecting relatively lower size reduction index for excessive pressure (maximum compression of the material bed between the rolls).

Values of the *n* parameter decrease together with an increase the operating pressure *P* values (Fig. 4). At the entire stage of the process (lower *P* values) the *n* values drop significantly (a relatively easy increase in finer size fractions of crushing product), next a kind of stabilization is observed and finally, for highest P values, a further decrease on n can be noticed (renewed increase of finer particles in the product).

Practical implementations

It appears that the course of n, c and $d_{\max}$ parameters can be described as functional relationships of pressure (Table 2). A selection of respective pressure level in press results in particular values of n, c and $d_{\max}$ which, in turn, give us the information about the particle size distribution of crushing products.

Table 2. Coefficients of function (1) as functional relationships of pressure P

Parameter	Function	R^2
n	$n = 0.459 \cdot \frac{0.46}{P^{0.05}}$	0.956
c	$c = 0.01 \cdot P^3 - 0.15 \cdot P^2 + 0.68 \cdot P + 1.29$	0.857
$d_{\max}$	$d_{\max} = \frac{24.97}{P^{0.27}}$	0.927

As a result of the above it is possible to control the work of entire crushing and grinding circuit in terms of technological, economic or ecological effects. A general three-stage crushing and grinding circuit, under examination, is presented in Fig. 5.

Following variables can be regarded as controllable:

- crusher's outlet gap (s)
- press operating pressure (P)
- screening cut point (d_T)
- grinding time (t).

A generalized model (target function) from a technological (product size reduction) or economic (process energy consumption) scope will have a following form:

$$F = f(P, s, d_T, t) \tag{4}$$

where F (target function) can denote either S_{50} or S_{80} (average or eighty percent product's size reduction index), or E_{sp} (unit energy-consumption of the process) depends of the accepted point of view.

Determining the value of the target function (4) is possible after calculating the conditional extremum of multi-variable function within the constrained area. This area respects a changeability range of key-operational parameters of devices incorporated into the circuit and below relationship constitute the limitations supplementing the target function formula:

- $P < P_{\max}$
- $Q_{\min} < Q < Q_{\max}$
- $S_x > S_{x\_\min}$

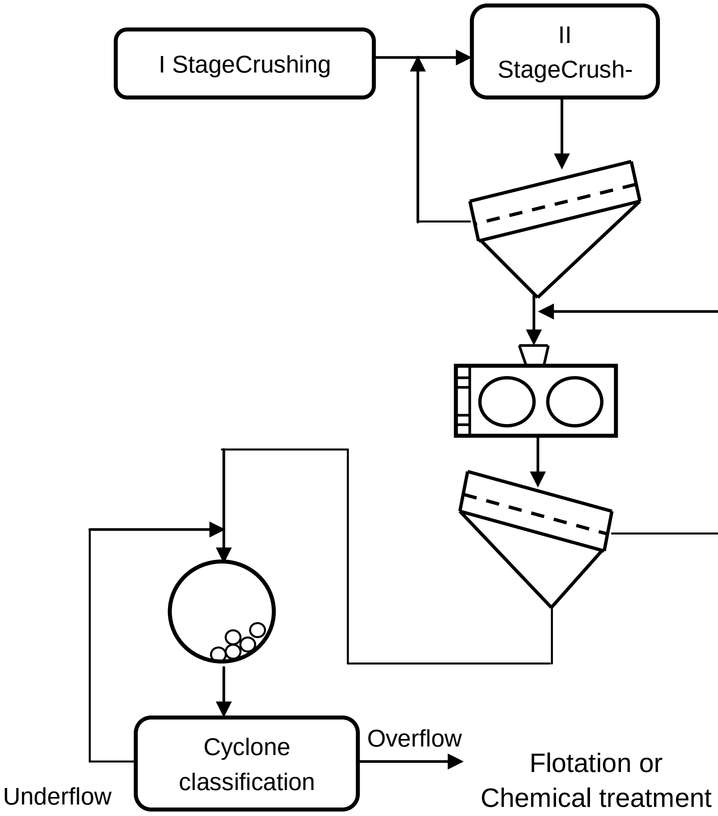

Fig. 5. Exemplary three-stage crushing and grinding HPGR based circuit

- $\gamma_x > \gamma_{x\_\min}$
- $E_{sp} < E_{sp\_\max}$

where S_x – given size reduction ratio (i.e. average or 80%), γ_x – the yield of given size fraction in crushing products, Q [Mg/h] – press throughput, E_{sp} [kWh/Mg] – unit energy-consumption.

The model built in the above manner is a classical issue of mathematical programming, i.e. finding the minimum (or maximum) of target function at presence of given limitations. The conditional extremum of the target function, in turn, gives us information of the controllable variables' values, which optimize the device's performance.

Summary and final conclusions

The issue of modeling the particle size distribution curves of HPGR crushing products, presented in the article, is significant from the point of view of both HPGR and the modern enrichment circuits work optimization. The methodology presented appears to be accurate, because the truncated distribution applied in the work, gives a very precise approximation, accuracy of which increases together with increasing the operating pressure value. The control of comminution results is then possible through a selection of particular value of the pressure which, in turn, is combined with parameters of the approximation formula (1) through equations presented in Table 2. It also appears that the major influence on the crushing process course has the operating pressure P. The process efficiency, measured with chosen indices, like the specific energy consumption, throughput or the content of desirable size fractions in product, can be then dependent on the operating pressure value. Conditioning the parameters of formula (1), approximating the particle size distribution function of crushing products, on the device work characteristics, permits the simulation to be carried out, which leads to determination the optimal operating conditions. It also enables the HPGR crushing process course to be better understood, as well as emphasizing the role of individual operating parameter of the device and the feed properties.

The investigations were carried out for copper ore and no additional analyses concerning the particle structure and the size of grain boundaries were made. Relationships obtained in the above investigations should have different values of coefficients depending the type of material, but it is accepted that the forms of functional relationships (presented in Table 2) are the same. It is obvious that the relationship $d_{\max} = f(P)$ is a hyperbole, $c = f(P)$ – polynomial of third degree, while $n = f(P)$ can be either the hyperbole or the straight line. Parameters n and c are responsible for the shape of distribution function (convexity) and it is claimed that the c parameter is connected with the material characteristics (structure). It might be accepted a hypothesis that the type of material determines a distribution type, i.e. either Weibull's distribution or the one of two others, namely log-normal or exponential, not considered in this work.

Acknowlegdements

The article was written within the frames of Ministry of Science Research Grant N N524 466139.

References

DANIEL M.J., MORRELL S, 2004, *HPGR model verification and scale-up*, Minerals Engineering, vol. 17.

EPSTEIN B., 1948, *Logaritmico-normal distributions in breakage of solids*. Ind. Eng. Chem., 40.

GAWENDA T., 2009, *Główne aspekty rozdrabniania twardych surowców mineralnych w wysokociśnieniowych prasach walcowych*. ZN AGH Górnictwo i Geoinżynieria, rok 33 (4).

KALINOWSKI W., 2006, *Modernizacja procesów przemiałowych w przemyśle cementowym w świetle wymagań najlepszych dostępnych technik*. Surowce i Maszyny Budowlane, nr 1/2006.

KOŁMOGOROW A.N., 1941, *O łogarifmiczieski normalnom zakonie razpriedielienija razmierow czastic pri droblieniji*, dokł. AN SSSR, 31.

KURDOWSKI W., 1998, *Stan techniki w zakresie przemiału cementu*. Cement Wapno Beton nr 3/1998.

MAXTON D., MORLEY C., BEARMAN R., 2003, *A quantification of the benefits of high pressure rolls crushing in an operating environment*. Minerals Engineering, vol. 16.

MORLEY C., 2003, *HPGR in hard rock applications*, Mining Magazine, IX/2003.

MORLEY C., 2010, *HPGR-FAQ*, The Journal of Southern African Institute of Mining and Metallurgy, vol. 110.

NAZIEMIEC Z., SARAMAK D., 2009, *Analiza zmian obciążenia materiału w strefie zgniotu pras walcowych*. ZN AGH Górnictwo i Geoinżynieria, rok 33 (4).

OLEJNIK T.P., 2006, *Grinding kinetics of selected minerals with reference to the number of contact points*, Physicochemical Problems of Mineral Processing, vol. 40.

SARAMAK D., TUMIDAJSKI T., BROŻEK M., GAWENDA T., NAZIEMIEC Z., 2010, *Aspects of comminution flowsheets design in processing of mineral raw materials*. Gospodarka Surowcami Mineralnymi, Mineral Resources Management, vol. 26 (4).

SARAMAK D., 2012, *De-agglomeration in high pressure grinding roll based crushing circuits*, Physicochemical Problems of Mineral Processing vol. 48 (1).

SARAMAK D., 2011c, *High-pressure comminution technology* (in Polish), Surowce i Maszyny Budowlane, 2011/3.

SARAMAK D., 2011a, *Technological issues of High-Pressure Grinding Rolls operation in ore comminution processes*, Archives of Mining Sciences, vol. 56 (3).

SARAMAK D., 2011b, *The influence of chosen ore properties on efficiency of HPGR-based grinding circuits*, GospodarkaSurowcamiMineralnymi Mineral Resources Management, vol. 27 (4).

SCHOENERT K., 1988, *A first survey of grinding with high-compression roller mills.* International Journal of Mineral Processing, vol. 22.

TUMIDAJSKI T., SARAMAK D., 2009, *Metody i modele statystyki matematycznej w przeróbce surowców mineralnych*, AGH University of Science and Technology Press, Krakow.

TUMIDAJSKI T., 2010, *Actual tendencies in description and mathematical modeling of mineral processing* (in Polish), Gospodarka Surowcami Mineralnymi - Mineral Resources Management, vol. 26 (3).

TUMIDAJSKI T., 2012, *Heuristic models of comminution processes as a basis for simulation and optimization of their course* (in Polish), Gospodarka Surowcami Mineralnymi - Mineral Resources Management, vol. 28.

Van der MEER F.P., GRUENDKEN A., 2010, *Flowsheet considerations for optimal use of high pressure grinding rolls*, Minerals Engineering, Volume 23 (9).