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# Investigation of laboratory conditions effect on prediction accuracy of size distribution of industrial ball mill discharge by using a perfect mixing model. A case study: Ozdogu copper-molybdenum plant

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**Abstract:** In this study, the perfect mixing model was used to investigate its accuracy level, under different laboratory conditions, in predicting the particle size distribution of industrial ball mills discharges. For this purpose, data sets of two laboratory ball mills with eight different compositions of balls and two industrial mills of a copper processing plant for seven different tonnages, which totally included 56 simulation operations, were used. For simulation, the necessary data were obtained through performing the breakage distribution function and kinetic grinding tests using laboratory mills. The results were used to determine the first order grinding kinetics and normalized breakage rate parameters. For the industrial scale, the simulation process was carried out using data, perfect mixing model equations and JKSimMet software. The results showed that the operating conditions of the laboratory mills were quite affected by the predictive power of the desired model. Comparing the measured and simulated values of $P_{80}$, it is clear that 2 minutes of first order grinding using the Bond laboratory ball mill with standard operating conditions and single size ball load of 20 mm provided the best prediction with trivial errors, less than 10%, for all seven tonnages of the industrial mills. The results of this study together with more investigations on different plants can be helpful in optimization, simulation and scale-up procedures of ball mills.

***Keywords****: ball mill scale-up, perfect mixing model, particle size distribution, simulation*

## Introduction

Ball mills have a great role in grinding minerals and other substances in various industries. They are among the most extensive used mills in the mineral processing industry. In the mineral processing industry, more than 50% of costs relate to the

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grinding sector (Fuerstenau and Kenneth, 2003). Then, the correct and systematic design of ball mills is important for efficiency of the processing system.

The main objective of the ball mills scale-up is either to choose appropriate dimensions of a mill or to predict the resulted effects of implementing changes in its design and operational parameters. Scale-up methods are generally divided into two main groups: 1) the Bond model and 2) the mathematical model based methods.

The Bond model (Bond, 1961) is the most widely used and reliable approach in choosing and scaling up the ball and rod mills. This model has gained wide acceptance for many industrial data, which were used in its extraction, simplicity and ease of its lab work. However, the Bond model drawback is its inability to predict the mill performance after making some changes in its design and operating parameters (Napier-Munn et al., 1996; Man, 2000). This method has several deficiencies, which are:

*i)* it cannot predict the particle size distribution of materials and tonnage in the grinding system flows
*ii)* it depends on the material size of $F_{80}$ and $P_{80}$ (80% passing size of a feed and a product, respectively) parameters instead of the particle size distribution
*iii)* its inability to predict the interaction between a mill and a classifier
*iv)* it considers a constant mills circulating load of 250 percent
*v)* it lacks compliance with simulation methods and computer models
*vi)* and it assumes the ideal performance for classifiers.

Gradual development of mathematical models considering the concept of material mass balance in the load (Epstein, 1947) as well as mill energy balance resulted in developing precise grinding models that give the same results after each application (Napier-Munn et al., 1996). On the other hand, the shortcomings of the Bond model can be resolved using mathematical models (Herbst and Fuerstenau, 1980; Kavetsky and Whiten, 1982; Morrell and Man, 1997). The developed models include the matrix (Broadbent and Callcott, 1956; Lynch, 1977), kinetic (Broadbent and Callcott, 1956; Gardner and Austin, 1962; Kelsall and Reid, 1969; Whiten, 1971; Herbst and Fuerstenau, 1980; Austin et al., 1984) and energy models. Considering grinding duration and mill duty, mathematical models can be applied to predict the particle size distribution. Although the model-based methods resolve the major defects of the Bond method, their main drawback is lack of reliability.

In the milling stage of mineral processing plants, one of the greatest problems is its failure in achieving the intended particle size distribution. The aim of this study is to evaluate the effects of controllable operational parameters such as tonnage and ball size on the particle size distribution of industrial mill discharges regardless of the classification system performance.

The paper is structured as follows: (*i*) a full description of the perfect mixing model, (*ii*) laboratory operations and mills modeling, and (*iii*) simulation and scale-up process of laboratory data into industrial ones.

## Perfect mixing model

The perfect mixing model, is the most widely used formula in simulating the grinding circuit of mineral processing plants and in mills optimization as well as their scale-up process. The model, introduced by Whiten (1974), has been widely used in different types of JKMRC software (Napier-Munn et al., 1996). It is a combination of matrix and kinetic models (Whiten, 1974), and on the other hand, it is a version of the mass balance model (Whiten, 1974; Kavetsky and Whiten, 1982; Whiten and Kavetsky, 1984). This model contains the particle size distributions of a feed and a product, the breakage rate function, the discharge rate and the breakage distribution functions:

$$f_i - p_i + \sum_{j=1}^{i} a_{ij} \left(\frac{r_j}{d_j}\right) p_j - \left(\frac{r_i}{d_i}\right) p_i = 0. \tag{1}$$

To predict the product particle size distribution of an industrial mill using the laboratory mill results, it is necessary to scale-up data using Equations 2 to 7. This can be performed using the breakage rate parameter of the laboratory mill $X_m$ and the operational and designing data of both laboratory and industrial mills. $X_m$ can be obtained from equation:

$$X_m = KD_b^2 \tag{2}$$

where $K$ is $(1 - 0.7) \times 10^{-3}$, $0.44 \times 10^{-3}$ and $1.37 \times 10^{-3}$ in Austin et al. (1984), Morrell (1992) and Man (2000) studies, respectively.

Another relationship proposed by Erdem and Ergun (2009) is:

$$X_m = 0.2971e^{0.0346D_b}. \tag{3}$$

Having the particle size distributions of the feed and the product as well as the breakage distribution function, the breakage rate parameters of laboratory mills can be calculated using the perfect mixing model. During simulating operation through JKSimMet software, the normalized breakage rate parameter, $r/d^*$, is used instead of the breakage rate, $r/d$. By having the $X_m$ values of laboratory and industrial mills and designing and operational parameters of them, $r/d^*$ can be determined for the second mill. Adjustment of $r/d^*$ values from laboratory results to industrial scale, or vice versa, can be done using the following equations:

$$\frac{\left(\frac{r}{d^*}\right)_{ind.}}{\left(\frac{r}{d^*}\right)_{lab.}} = \left(\frac{D_{ind.}}{D_{lab.}}\right)^{0.5} \left(\frac{1-LF_{ind.}}{1-LF_{lab.}}\right) \left(\frac{LF_{ind.}}{LF_{lab.}}\right) \left(\frac{Cs_{ind.}}{Cs_{lab}}\right) \left(\frac{WI_{ind.}}{WI_{lab.}}\right)^{0.8} \tag{4}$$

$$for\ x_i < X_{m\ lab.} : \frac{\left(\frac{r}{d^*}\right)_{ind.}}{\left(\frac{r}{d^*}\right)_{lab.}} = \frac{bd_{lab.}}{bd_{ind.}} \tag{5}$$

$$for\ x_i > X_{m\ ind.}: \frac{\left(\frac{r}{d^*}\right)_{ind.}}{\left(\frac{r}{d^*}\right)_{lab.}} = \left(\frac{bd_{ind.}}{bd_{lab.}}\right)^2. \quad (6)$$

The scale-up procedure of $r/d^*$ for particle sizes ranging from $X_{m\ lab.}$ to $X_{m.ind.}$ is implemented using the Spline function. If necessary, $r/d$ can be calculated from equation:

$$\left(\frac{r}{d}\right)_i = \left(\frac{r}{d^*}\right)_i \left(\frac{D^2 L}{4Q}\right). \quad (7)$$

The particle size distribution of the industrial mill product can be predicted after $r/d^*$ calculation using the simulation software.

## Study objectives and performed operational tasks

The aim of this study is to investigate the effect of laboratory conditions such as the ball size, feed size and mill dimensions on the scale-up process of ball mills and the simulation of the particle size distribution of industrial ball mills.

To assess the anticipated target, two laboratory mills with different compositions of ball loads and an industrial grinding circuit including a primary and a secondary ball mills were used. The laboratory results were used to simulate industrial conditions using JKSimMet V.6 software and were compared to the measured parameters to select the best laboratory conditions.

### Industrial surveying

Industrial data were obtained from a copper - molybdenum mineral processing plant in Turkey. The desired mineral was chalcopyrite with a silicate tailing and a density of 2.85 g/cm$^3$. The plant had a grinding circuit consisting of a primary ball mill (PBM) in an open circuit and a secondary ball mill (SBM) in a closed circuit with hydrocyclones with designing and operational characteristics presented in Table 1. The required industrial data were obtained through sampling at the designated points of the circuit, Fig. 1.

The ordinary operational tonnages of the plant are respectively 240 and 360 Mg/h for PBM and SBM with a circulating load of 1.65. Various tonnage amounts can be achieved by changing the retention time of materials in the primary and secondary mills and by modifying the flow solid percent. In mills, the material retention time is one of the most important parameters, which affects its particle size distribution and mill content. The retention time has a direct relationship with grinding and an inverse relationship with the feed amount, and then, it can be possible to evaluate different tonnages and various product sizes by changing the retention time. The retention times of the PBM and SBM mills can be controlled by changing the feeding amount and its rate into the PBM mill and modifying the hydrocyclon pressure and its operational

parameters. Then, as it was mentioned before, changes in the hydrocyclon operating conditions are not considered. Table 2 shows the considered retention times of the studied mills.

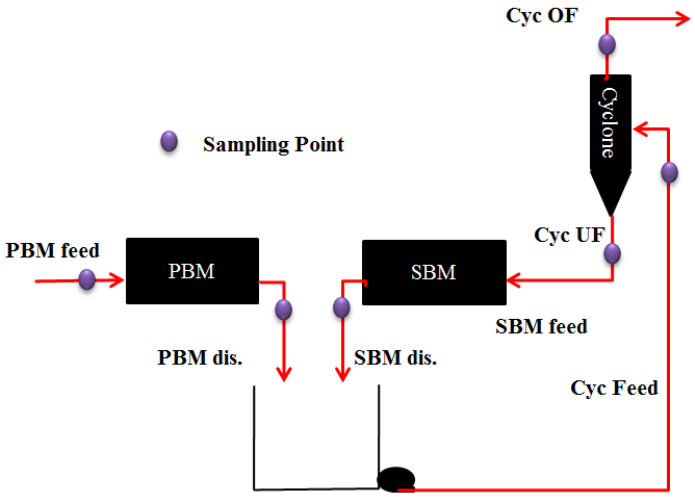


Fig. 1. Flowsheet of copper - molybdenum grinding circuit with sampling points

Table 1. Specifications of industrial ball mills (PBM & SBM)

| Parameter | | Symbol | Value (PBM & SBM) |
|---|---|---|---|
| Mill diameter, m | | $D$ | 4.88 |
| Mill length, m | | $L$ | 7.33 |
| Mill speed, rpm | | $S$ | 15 |
| Mill critical speed, % | | $C_S$ | 78.34 |
| Material discharge system | | | Overflow |
| Ball filling ratio | PBM | $J_B$ | 0.33 |
| | SBM | | 0.30 |
| Maximum ball diameter, mm | PBM | $D_b$ | 100 |
| | SBM | | 40 |

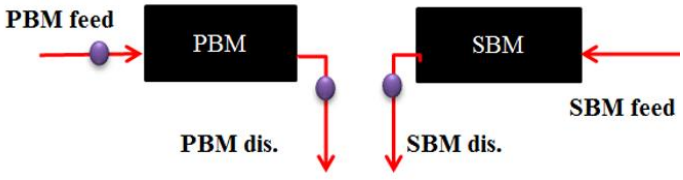


Fig. 2. Diagram of studied mill

Table 2. Material tonnage (Mg/h) and retention time (*RT*, min) in mills

| Mill type | PBM | | | | SBM | | |
|---|---|---|---|---|---|---|---|
| Mg/h | 267.54 | 270.2 | 364.86 | 315 | 667.63 | 458.61 | 395.23 |
| *RT*, min | 6.380 | 6.359 | 5.405 | 5.909 | 2.568 | 3.863 | 4.276 |

For industrial sampling, it was necessary to wait for almost 2 hours to reach the steady state. Then, with respect to the flowsheet of grinding circuit (Fig. 1), for each tonnage the samples were taken at first from the mill products, and then from the feed, underflow and overflow products of the hydrocyclone. Finally, by stopping the input band to PBM and removal of the material existing on the conveyor, 3 meters, the PBM feed sample was collected too. Four surveys containing 20 industrial samples were totally collected from the grinding circuits. Fresh and dry weights of all samples were measured and their solid percentages were calculated. Then, the sieve analysis of all samples was carried out and their corrected values were obtained through the mass balance (Table 3).

Table 3. Corrected values of industrial surveying

| Survey No. | 1 | | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|---|---|
| | Mg/h | Solid, % | Mg/h | Solid, % | Mg/h | Solid, % | Mg/h | Solid, % |
| PBM Feed | 267.54 | 96.90 | 270.20 | 98.00 | 364.86 | 96.00 | 315 | 98 |
| PBM Dis. | 267.54 | 62.86 | 270.20 | 65.52 | 364.86 | 68.35 | 315 | 66.5 |
| SBM Dis. | 667.63 | 63.04 | 458.61 | 66.58 | 395.23 | 62.50 | | |
| CYC. Feed | 935.17 | 53.35 | 728.81 | 54.60 | 760.09 | 52.23 | | |
| CYC. Under | 667.63 | 63.04 | 458.61 | 70.87 | 395.23 | 76.98 | | |
| CYC. Over | 267.54 | 38.57 | 270.20 | 39.29 | 364.86 | 38.73 | | |

## Laboratory tests

The laboratory operations, which were implemented using the industrial grinding circuit samples, consist of two different phases, that is calculation of breakage distribution function, and kinetic grinding tests.

The breakage distribution function was calculated considering the Austin and Luckie BII method (1972) and the primary ball mill feed given by equation:

$$B_{i1} = \frac{\log\left[\frac{1-P_i(0)}{1-P_i(t)}\right]}{\log\left[\frac{1-P_2(0)}{1-P_2(t)}\right]}. \tag{8}$$

The kinetic grinding operations were performed using two laboratory Bond ball mills (BD) with *ΦD/L*= 0.3048/0.3048 m in dimensions and the laboratory ball mill (LD) designed for this study with *ΦD/L*= 0.47/0.80 m dimension (Fig. 3), as well as considering the operational and designing conditions which are listed in Table 4.


Fig. 3. BD and LD laboratory ball mills

Table 4. Specifications of laboratory ball mills and their operational conditions

| Parameter | | | Symbol | LD | BD |
|---|---|---|---|---|---|
| Mill diameter (m) | | | $D$ | 0.47 | 0.3055 |
| Mill length, m | | | $L$ | 0.80 | 0.3055 |
| Mill critical speed, % | | | $C_S$ | 0.80 | 0.80 |
| Lifter | | | | No | No |
| Ball diameter, mm | 1. Condition | 100% | *bd* | 50 | 38 |
| | 2. Condition | 100% | | 40 | 30 |
| | 3. Condition | 100% | | 25 | 20 |
| | | 40% | | 50 | Bond standard ball size distribution |
| | 4. Condition | 45% | | 40 | |
| | | 15% | | 25 | |
| Ball filling ratio | | | $J_B$ | 0.2 | 0.2 |
| Grinding time, min | | | $t$ | 1, 2, 4 | 1, 2, 4, 8 |

Laboratory mills with different sizes and operating conditions were chosen to investigate the effects of various parameters on simulations. In both kinetic experiments, except for the balls composition, all operating conditions such as the mill load, ball fraction and mill speed were consistent with the standard Bond conditions.

The Bond laboratory mill was chosen because of being reliable and gaining wide acceptance among all researchers. For the Bond standard method, Man (2000) discussed the validity reason of the mentioned values.

The ball composition in the BD mill included the Bond standard composition and single size balls of 20, 30 and 38 mm. The ball composition in the LD mill included single size balls of 25, 40 and 50 mm and their combination. The required samples for kinetic grinding were obtained from the material resulted from industrial operations. For the BD mill 15 kg of material, less than 3.35 mm in size, and for the LD mill almost 40 kg of material, less than 5 mm in size, were prepared.

In the BD mill, the kinetic grinding operation included 16 combinations regarding 4 different ball loads and 4 grinding periods. For each ball load, 700 $cm^3$ of the prepared material with a weight of 1.144 kg was ground considering 1, 2, 4 and 8 minute periods. For the LD mill, the kinetic grinding operation included 12 combinations considering 4 different ball loads and 3 grinding periods. In each ball load, 4300 $cm^3$ of material with a weight of 8.75 kg was ground choosing 1, 2 and 4 minute periods. Then, for each combination, the particle size distributions of the feed and the product were measured.

## Modeling of laboratory mills

The laboratory mills were modeled using the kinetic grinding results. To model the laboratory mills, the cumulative based kinetic model was used for the ease of laboratory and calculation operations in comparison to the other complex models:

$$W_i(t) = W_i(0)e^{-k_i t}. \quad (8)$$

Using Equation 9 and the results of kinetic tests, the required time for linear breakage or first order grinding of materials can be calculated (Austin et al., 1981; Austin et al., 1984). This time, which results from the linear part of the log [$W_{i(t)}/W_{i(0)}$] – $t$ graph, is obtained to be equal to 2 minutes for all tests. Now, the breakage rate values of either $k$, $r$, or $r/d$* parameter can be calculated through the cumulative based kinetic model or JKSimMet V.6 software using the first order grinding kinetics. Due to the dominance of the plug flow current in the batch grinding and the lack of material transportation to out of the mill, the values of $r/d$, $r/d$* and $k$ would be equal.

### Scale-up operations and simulation process

JKSimMet V.6 was used to scale-up the laboratory data into the industrial ones. In this software, the input data are the laboratory mills characteristics and the simulations are the predictions for particle size distributions of industrial mills.

The simulation results, which consisted of 56 possible combinations, are shown in Table 5 for comparing $P_{80}$ values. In Table 5, for each tonnage value, the measured $P_{80}$ and predicted $P_{80}$' were used in evaluation of simulation error ($Error(\%) = \frac{(P'_{80}-P_{80})}{P_{80}} \times 100$). For example, for BD 20 mm and a tonnage value of 267 Mg/h, the simulated value and error percentage were 0.648 μm and 7.714%, respectively. Moreover, Fig. 4 shows the measured and the predicted particle size distribution of mill discharge of BD 20 mm balls.

As it was mentioned before, the main objective of this study was to scale-up the laboratory data into the industrial ones such that it would be possible to determine controllable operating parameters to obtain the aimed size distribution in the mill discharge. The most important controllable parameters were the ball size, feed size and feed tonnage. Moreover, the relationship between the ball size and feed size

should be found in a way that balls were able to grind the largest particle of the feed (Bond, 1961). Then, according to Eqs. 2 to 6, which were consistent with the simulation results, the discharge size was a function of ball and feed sizes.

Figure 5-top shows the estimation errors of particle size distribution of industrial mills with different capacities and mentioned ball sizes. Estimation errors of 20 mm and 30 mm balls, which were quite small in comparison to the other ball sizes, are clearly demonstrated in Fig. 5-bottom using a different vertical scale.

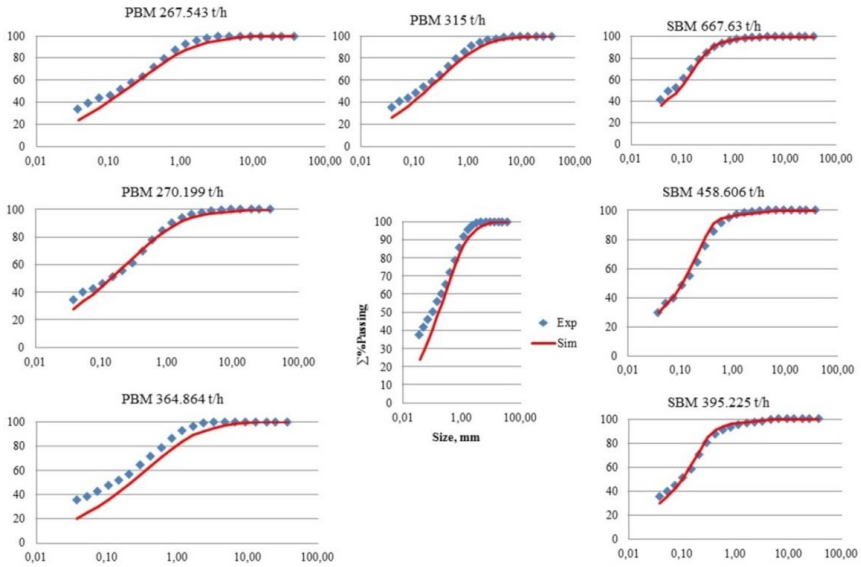


Fig. 4. Comparison of measured and predicted PSD of mill discharge. Estimated values were obtained through a scale-up process of BD 20 mm balls to industrial mills

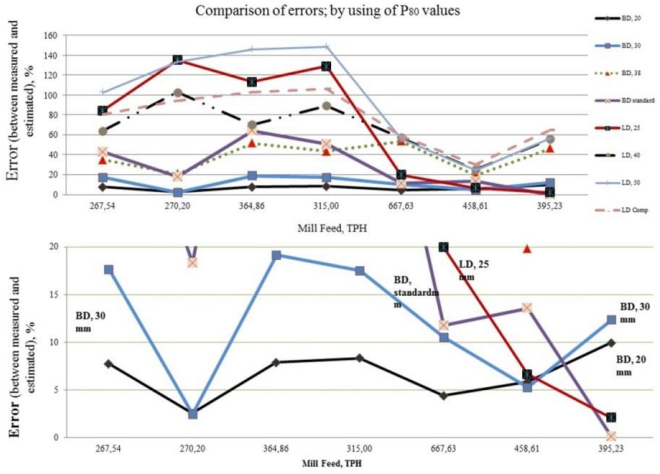


Fig. 5. Comparison of errors using $P_{80}$ values

Table 5. Comparison of the simulated and the meausred results for $P_{80}$ values

| Tonnage, tph | Mill tyape | Measured | $P'_{80}$, mm Simulated (from Lab. Conditions) | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | BD, 20 mm | | BD, 30 mm | | BD, 38 mm | | BD, Standard comp. mm | | LD, 25 mm | | LD, 40 mm | | LD, 50 mm | | LD, comp. mm | |
| | | | Value | Error,% | Value | Error,% | Value | Error,% | Value | Error,% | Value | Error,% | Value | Error,% | Value | Error,% | Value | Error,% |
| 267,54 | PBM | 0,602 | 0,648 | 7,714 | 0,708 | 17,648 | 0,815 | 35,377 | 0,863 | 43,264 | 1,111 | 84,604 | 0,988 | 64,084 | 1,225 | 103,413 | 1,089 | 80,942 |
| 270,20 | PBM | 0,686 | 0,704 | 2,526 | 0,703 | 2,481 | 0,832 | 21,224 | 0,812 | 18,307 | 1,615 | 135,424 | 1,392 | 102,889 | 1,603 | 133,644 | 1,335 | 94,597 |
| 364,86 | PBM | 0,660 | 0,712 | 7,861 | 0,786 | 19,125 | 1,004 | 52,231 | 1,085 | 64,520 | 1,408 | 113,465 | 1,124 | 70,420 | 1,627 | 146,650 | 1,340 | 103,092 |
| 315,00 | PBM | 0,603 | 0,653 | 8,303 | 0,708 | 17,514 | 0,867 | 43,766 | 0,910 | 50,963 | 1,382 | 129,211 | 1,142 | 89,458 | 1,503 | 149,308 | 1,244 | 106,385 |
| 667,63 | SBM | 0,231 | 0,241 | 4,380 | 0,206 | 10,522 | 0,355 | 53,953 | 0,258 | 11,776 | 0,277 | 19,952 | 0,363 | 57,377 | 0,360 | 56,234 | 0,366 | 58,899 |
| 458,61 | SBM | 0,360 | 0,381 | 5,808 | 0,379 | 5,251 | 0,431 | 19,812 | 0,409 | 13,555 | 0,336 | 6,631 | 0,451 | 25,199 | 0,451 | 25,230 | 0,468 | 30,106 |
| 395,23 | SBM | 0,296 | 0,267 | 9,904 | 0,332 | 12,341 | 0,434 | 46,849 | 0,295 | 0,147 | 0,302 | 2,104 | 0,462 | 56,082 | 0,464 | 56,971 | 0,488 | 65,020 |

The results can be evaluated by considering equivalent of combined balls which were consequently equal to 24.41 and 37.95mm for BD and LD mills, respectively. Then, for both laboratory mills, data can be put in order according to ball sizes. For the PBM and SBM mills, a comparison of the simulated results and the measured values through this approach are presented in Figs. 6 and 7, respectively.

For all PBM, the Bond mill had smaller estimation errors than the LD mill. In the SBM mills, all the ball sizes except for LD 25 mm resulted in better predictions of the Bond mill with smaller errors. In both mills, the largest balls resulted in the largest error values. The results showed that the most important factors that led to the estimation error through scale-up process were the ball size, feed size and laboratory mills dimensions. In the other words, the results showed the importance of laboratory conditions in the scale-up process.

Figures 6 and 7 show the effects of laboratory ball sizes on the prediction accuracy of particle size distribution of the industrial mill discharges at different tonnage amounts. In Figures 6-7, for each tonnage value, the measured $P_{80}$ and predicted $P'_{80}$ were used in the evaluation of simulation error ($Error(\%) = \frac{(P'_{80}-P_{80})}{P_{80}} \times 100$). The red and blue lines consequently demonstrate the estimation errors for the BD and the LD laboratory mills. For both of the laboratory mills, the larger balls showed higher estimation errors. This can be resulted from the higher ratios of ball size to mill diameter for the laboratory mills rather than the industrial ones, the disproportion of ball size to the mill diameter. The larger balls reduced the number of impacts and increased their intensity that eventually reduced the grinding rate. In both mills, the smallest balls demonstrated the best estimations with the lowest errors for the secondary mills due to compatibility between the laboratory and the industrial balls sizes (Table 5, Figs. 6 and Fig. 7).

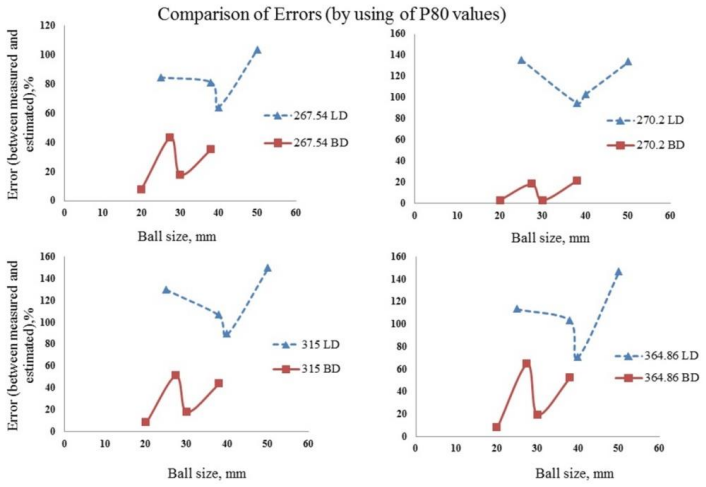


Fig. 6. Comparison of errors using $P_{80}$ values for PBM

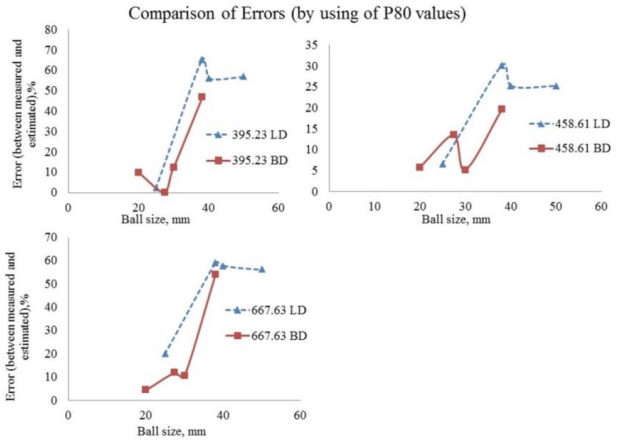


Fig. 7. Comparison of errors using $P_{80}$ values for SBM

Comparing $P_{80}$ values, it is clear that the estimated particle size distribution for the industrial mills, based on the first order grinding simulation using the Bond laboratory mill and 20 mm balls, gives the best results with acceptable errors that are less than 10% (Table 5, Fig. 6 and 7).

## Conclusions

The aim of this study was to investigate effects of various parameters of laboratory mills, such as the ball size, feed size and mill dimensions, on the prediction accuracy of the perfect mixing model. An industrial mill was designed and built beforehand so that the changeable parameters were the feed and ball sizes and the purpose was to optimize them in order to achieve the intended particle size distribution in the industrial mill discharge. The optimization process was performed using BD and LD laboratory mills considering different combinations of balls and grinding times. The remained operating parameters were chosen regarding the standard Bond method.

Finally, using the obtained parameters for BD and LD mills, the scale-up relationships of the perfect mixing model and JKSimMet V.6 software for industrial mills were implemented under various conditions and the results were compared with the measured values. Estimation of the particle size distribution of industrial mills using first order grinding kinetics in the Bond laboratory mill and 20 mm balls gave the best results with the smallest errors that were less than 10% for $P_{80}$ values.

The results showed that the laboratory conditions had great importance in the scale-up procedure. For example, using large balls which were unproportional to laboratory mills diameter and, also, using laboratory mills with unconfirmed operating and design parameters led to large errors in estimating the industrial mill particle size distribution.

By using the results of this paper and also running more tests on the different plants, a new fast method, based on the perfect mixing model and simple laboratory operations, can be developed to simulate the particle size distribution of industrial ball mills discharges. The prediction algorithm and its cross-validation results will be presented in the next paper.

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